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The re-investment assumption in capital budgeting examined

Capital budgeting is gaining an increasingly prominent place in investment analysis. The many assumptions that it introduces are often overlooked or not fully appreciated. This article sets out to examine a few of them. It begins by demonstrating that both the net present value method and the internal rate of return method (sometimes referred to as the discounted cash flow method) give identical answers for acceptance-rejection decisions on single projects. The apparent conflict between these two methods in comparing mutually exclusive projects is then examined. It is shown that this conflict arises from assumptions regarding the re-investment of funds generated by the project. Analysis by calculation of terminal values is suggested to enable the restrictions imposed by these assumptions to be removed. The article closes with a fairly detailed example of a terminal value calculation, which is free of implicit assumption.

Van Horne (1971) suggests that 'in general, the present value and internal rate of return of methods lead to the same acceptance or rejection decision' in capital budgeting proposals. However, 'when two investment proposals are mutually exclusive, so that we can select only one, the two methods may give contradicting results.' The reason for this he ascribes 'to different assumptions with respect to the re-investment rate on funds released from the proposal.' (pp. 61-63.) As these assumptions are important to the decision as to which proposal to choose and seem not to be well understood, this note seeks to analyse them in some detail.

We start with a simple acceptance-rejection decision involving a project whose benefits are constant for simplicity's sake. This last does not affect our analysis in any way. The internal rate of return which we wish to calculate is that rate which will discount the benefits to the initial cost of the project. If we take as our example a project costing R23 616 and yielding R10 000 a year for four years this rate is 25% as can easily be verified, where 0,8 is the annual discounting factor for 25%.

TABLE I

Year	Balance carried down	Receipt on last day of year	Year end balance	Discounted balance
4	0	10 000	10 000	8 000
3	8 000	10 000	18 000	14 400
2	14 400	10 000	24 400	19 520
1	19 520	10 000	29 520	23 616

In other words the present value of R10 000 received at the end of each year for four years is equal to R23 616 if the discount rate is 25%. If we now assume that our required rate of return is 10% we should accept this proposal.

It should be obvious that if the required rate of return is less than the internal rate and if this required rate is used to discount the benefits, a positive present value **must** result. Consider our second table where 0,9091 is used as the appropriate discounting figure for a required rate of 10%.

TABLE 2

Year	Balance carried down	Receipt on last day of year	Year end balance	Discounted balance
4	0	10 000	10 000	9 091
3	9 091	10 000	19 091	17 356
2	17 356	10 000	27 356	24 869
1	24 869	10 000	34 869	31 699
				Less 23 616
				R8 083

This last is the normal net present value calculation where the so called cost of capital is assumed to be 10%. The internal rate of return is merely that rate of interest that makes the net present value equal to zero. In a sense it is a special case of the general net present value calculation. Provided the cost of capital used to calculate the net present value is less than this internal rate, the discounted benefits must exceed the initial cost and the net present value must be positive. On both methods the decision is identical — accept. If the cost of capital were greater than the internal rate we would reject the proposal. If we calculated the net present value we would find it negative and again reject the proposal. Thus in straight acceptance-rejection decisions the methods give identical results.

We now turn to the question of re-investment of the benefits from a project which as Van Horne states create conflicts when two projects are compared one with the other. When we speak of re-investment we are viewing the project from the front, so to speak, as opposed to the discounting calculations where we can be said to be working backwards. In effect we are saying that if our internal rate of return is 25%, then the initial capital outlay invested at 25% must result in the same terminal value as the accumulated receipts from the project. This is seen most clearly in the two tables that follow. Table 3 shows the effect of investing R23 616 at 25% over four years while Table 4 accumulates the benefits at the same rate of interest.

TABLE 3

Year	Balance at start of year	Interest at 25%	Balance at end of year
1	23 616	5 904	29 520
2	29 520	7 380	36 900
3	36 900	9 225	46 125
4	46 125	11 531	57 656

TABLE 4

Year	Balance at start of year	Interest at 25%	Receipt at end of year	Balance at end of year
1	0	0	10 000	10 000
2	10 000	2 500	10 000	22 500
3	22 500	5 625	10 000	38 125
4	38 125	9 531	10 000	57 656
		17 656	40 000	

To re-state our results — if we invest our outlay on the project at 25% our accumulated sum at the end of four years will be the same as the accumulated benefits received from the project when re-invested at 25%.

To achieve this result we re-invested the entire benefits from the project at 25%. But, and this is the point often overlooked, re-investment is not an inherent assumption of the discounting model. It is done only to compare it with the usual compound interest calculation. We can quite easily drop this assumption and allow full or part consumption of the benefits and arrive at the same terminal value as the compound interest model, provided we apply the same consumption policy there. For instance let us assume that R5 000 is consumed annually. Our two tables will now appear as follows.

TABLE 5

Year	Balance at start of year	Interest at 25%	Consumption at end of year	Balance at end of year
1	23 616	5 904	5 000	24 520
2	24 520	6 130	5 000	25 650
3	25 650	6 413	5 000	27 063
4	27 063	6 766	5 000	28 829

TABLE 6

Year	Balance at start of year	Interest at 25%	Receipt at end of year	Consumption at end of year	Balance at end of year
1	0	0	10 000	5 000	5 000
2	5 000	1 520	10 000	5 000	11 250
3	11 250	2 813	10 000	5 000	19 063
4	19 063	4 766	10 000	5 000	28 829

Any other amounts may be re-invested or equally consumed in each case yielding the same terminal values. We can allow for full consumption of the benefits too, in which case the terminal values would be zero. Thus re-investment *per se* is not inherent in the discounting model. The only assumption made is that whatever part of the benefits are re-invested, they must be re-invested at the rate of interest being used for the project. This assumption is made purely to enable a direct comparison with the compound interest model which itself contains this assumption.

A clear appreciation of this point enables one to understand far more clearly the implications of the various techniques for evaluating capital projects. Let us consider by way of contrast a second project which costs the same i.e. R23 616 but yields one benefit only — an amount of R48 970 in the fourth year. If we calculate its net present value with a cost of capital of 10% we will find it is R9 831. This is higher than the first project which was R8 083 (see Table 2), and suggests that the second is to be preferred. However, if we calculate the internal rate of return of this new project we find that it is 20% only, as opposed to our earlier 25% which suggests that the first is to be preferred.

If the previous analysis was clear it should be apparent how the conflict has arisen. Let us calculate terminal values for the second project using the internal rate of return and compare this with Table 4.

TABLE 7

Year	Balance at start of year	Interest at 20%	Receipt at end of year	Balance at end of year
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	48 970	48 970
		0	48 970	48 970

The difference between the two is startlingly obvious. Receipts from operations in the first (see Table 4) were only R40 000 compared to R48 970 for the second (see Table 7), but in the first these were re-invested to produce R17 656 in interest or further benefits (see Table 4 again), while in the second no such extra benefit accrued. The question of re-investment and the benefits that result is therefore crucial to the choice of an investment. It is this that leads to the conflict between the two methods since in the net present value method we are effectively using a rate of interest of 10%. If we re-invest our benefits at this rate we find that the interest earned is not sufficient to outweigh the added receipts from operations. This is clearly seen in the next table when interest on the first project now totals R6 410.

TABLE 8

Year	Balance at start of year	Interest at 10%	Receipt at end of year	Balance at end of year
1	0	0	10 000	10 000
2	10 000	1 000	10 000	21 000
3	21 000	2 100	10 000	33 100
4	33 100	3 310	10 000	46 410
		6 410	40 000	

We are now in a position to draw our initial conclusions together. These may best be set out in point form where NPV and IRR are used as abbreviations for the net present value method and the internal rate of return method respectively.

1 Neither method makes any assumption regarding the actual quantity of funds re-invested.
2 Both methods assume that whatever funds are re-invested are invested at the rate of return used in the calculation, and that this rate is constant over the life of the project.
3 NPV and IRR give identical results for a simple accept-reject decision.
4 The IRR will tend to favour the project which generates funds early as these may be re-invested at the relatively high IRR rate. By contrast the NPV will tend, with its generally lower rate, to favour the project which produces the greatest overall inflow of receipts from operations.
5 NPV is the more conservative as it assumes re-investment of benefits at a generally lower rate than the IRR.

We are also in a position to state quite strongly that the timing of the cash flows from a project must be carefully analysed. This has been said many times in connection with ensuring a steady flow of cash to meet dividend and/or interest payments, but is not realised sufficiently in regard to the actual evaluation of a project. It is quite possible for example to analyse the purpose to which the benefit will be put and to calculate the amount that will be consumed and the rate of interest that will be earned on the balance. A terminal value may then be calculated and compared with another project. For instance let us assume that given our first project we could re-invest its entire proceeds at 20%. Our new terminal value would be as follows.

TABLE 9

Year	Balance at start of year	Interest at 20%	Receipt at end of year	Balance at end of year
1	0	0	10 000	10 000
2	10 000	2 000	10 000	22 000
3	22 000	4 400	10 000	36 400
4	36 400	7 280	10 000	53 680

We have used in effect the assumptions of Table 7 here i.e. a re-investment rate of 20%. The result is that the first proposal is preferable.

By calculating the terminal value of a project we are seeking to establish the wealth that it will generate at a point in time. In the words of Ezra Solomon, 'The valid comparison is not simply between two alternative courses of action. The ultimate criterion is the total wealth that the investor can expect from each alternative by the terminal date of the longer lived project . . .' (quoted in Cairns and Taylor, 1965). To stress the obvious, if two projects have different lives, it is necessary to extend the analysis of the shorter project to the end of the life of the longer project to make the wealth comparisons meaningful.

The concept of calculating future wealth is one that is easily understood and interpreted. There is no difficulty in altering the re-investment ratio from year to year — one does not therefore assume a fixed re-investment policy. Equally, there is no difficulty in altering the interest rates earned by those portions of the funds re-invested. It would thus appear that wherever mutually exclusive projects are compared, the calculation of terminal values is the most preferable of the three methods considered in this paper.

Let us, to close, use our first example again. We assume that 100% of the first year's funds are re-invested at 25%. 75% of the second year's funds are re-invested at 20% and 50% of the third year's funds are re-invested at 10%. Our calculation would then appear as follows:

TABLE 10

Year	Balance at start of year	Interest rate	Interest	Receipt at end of year	Consumption at end of year	Balance at end of year
1	0	—	—	10 000	0	10 000
2	10 000	25%	2 500	10 000	2 500	20 000
3	20 000	20%	4 000	10 000	5 000	31 500
4	31 500	10%	3 150	10 000	7 500	37 150

Our investment of R23 616 has led to wealth of R37 150 at the end of four years. This implies an effective average rate of return of some 12% which is above our cost of capital.

The above examples indicate that where the investment problem involves the acceptance or rejection of a single project, either an NPV or IRR evaluation would be acceptable. With mutually exclusive projects, however, evaluation on the basis of terminal values has much in its favour. The ease with which terminal values can be calculated and the minimum of rigid assumptions they involve is apparent from the cases given.

References

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