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A note on the rule of 72 or how long it takes to double your money

Many investors are no doubt familiar with this rule which can be used for quick mental calculations. The rule states:

For any amount which is compounding periodically at a fixed growth rate, the number of periods required for that amount to double can be estimated by dividing the percentage growth rate into 72.

For example, if R100 is invested at an interest rate of 8% per annum compounding annually, how many years will it take for the investment to double in value?

$$\text{Number of years} = \frac{72}{\text{interest rate}} = \frac{72}{8} = 9 \text{ years}$$

The aim of this note is to comment on the accuracy of this calculation.

Mathematical derivation

For a principal investment P, invested at a compound interest rate of r% per period, the amount A after n periods is given by:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

For the principal investment to double, i.e. A = 2P,

$$2 = \left(1 + \frac{r}{100}\right)^n$$

Taking the natural logarithm of both sides of the equation

$$\log_e 2 = n \log_e\left(1 + \frac{r}{100}\right)$$

$$\text{i.e. } n = \frac{\log_e 2}{\log_e\left(1 + \frac{r}{100}\right)}$$

From natural logarithm tables $\log_e 2 = 0{,}693$.

Applying a power series expansion to $\log_e\left(1 + \frac{r}{100}\right)$ this is:

$$\log_e\left(1 + \frac{r}{100}\right) = \left(\frac{r}{100}\right) - \frac{1}{2}\left(\frac{r}{100}\right)^2 + \frac{1}{3}\left(\frac{r}{100}\right)^3$$

$$\text{for } -100 < r \leqslant 100$$

$$\text{Thus } n = \frac{0{,}693}{\frac{r}{100} - \frac{1}{2}\left(\frac{r}{100}\right)^2 + \frac{1}{3}\left(\frac{r}{100}\right)^3 \text{----}}$$

$$= \frac{69{,}3}{r\left(1 - \frac{r}{200} - \frac{r^2}{30\,000} \text{----}\right)} \qquad 1$$

As shown below, for low values of r this may be approximated by

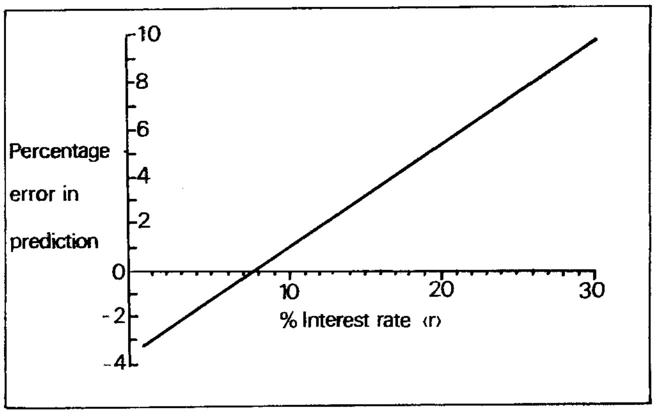
$$n = \frac{72}{r} \qquad 2$$

72 is chosen as it is easily divisible by most numbers for mental calculations. It will also, as shown below, give the correct relationship at an 8% interest rate.

Comparing some values predicted by the rule — equation 2 — with the true values as obtained from equation 1, we get the following:

Rate % per period	Predicted number of periods (eq. 2)	Actual number of periods (eq. 1)
1	72,0	69,7
2	36,0	35,0
4	18,0	17,7
8	9,0	9,0
16	4,5	4,7

Or, for the graphically minded:



In the case of continuous compounding, the amount is given by:

$$A = Pe^{\frac{r\,n}{100}}$$

For the principal to double, i.e. A/P = 2, the equation becomes:

$$2 = e^{\frac{r\,n}{100}}$$

$$\text{or} \quad n = \frac{\log_e 2}{r/100} = \frac{69{,}3}{r}$$

Thus if the rule of 72 is used for continuous compounding, there is a constant error for all values of r, viz:

$$\% \text{ error using rule} = \frac{72 - 69{,}3}{69{,}3} \times 100 = 3{,}9\%$$

Reference

Abromowitz, M. and Segun, I. A.
'Handbook of Mathematical Functions', Dover Publications, Inc., New York, 1965