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Expanded ratio analysis

How can the tactical causes of inadequate return on investment (ROI) be systematically diagnosed? What kind of tactics and control are likely to improve low ROI? It is somewhat surprising, considering the common use of ratio analysis, to realize that its usefulness can be extended to provide more complete answers to the above questions; it is even more surprising to realize that this increased usefulness depends on no more than the application of an old technique.

SIMPLE TECHNIQUE

The technique in question is implicit in the widely used relationship[1] between ROI, profit margin, and turnover of total capital employed (TCE) in sales:

$$ROI \equiv \frac{\text{Profit}^2}{\text{TCE}^3} \equiv \frac{\text{Profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{TCE}}$$

$$\equiv \text{Profit margin} \times \text{TCE turnover} \quad \ldots\ldots\ldots (1)$$

Both outside and inside the firm, equation (1) is used to evaluate financial performance. Components of expense and working capital are often expressed as additive percentages of sales and TCE, respectively. These secondary ratios are used, in turn, to pinpoint weaknesses in the profit margin and TCE turnover ratios.[4] For the outside analyst, however, additive ratio expansion is but one approach; he generally complements it with another equally simple technique, which consists in generating an alternative definition of a ratio by dividing both the numerator and denominator by the same variable.

Exhibit I contains a common set of ratio relationships[5] used by outside analysts. The earnings per share ratio is expanded to yield its relationship with return on equity and equity per share, by dividing both numerator and denominator by equity. The return on equity, in turn, is expanded into the product of after-tax profit margin and equity turnover, by using sales as the common divisor. Similarly, using TCE as the expansion variable, equity turnover is expanded into the product of TCE turnover and leverage. Combining these relationships, earnings per share can be defined as the product of after-tax profit margin, TCE turnover, leverage, and equity per share.[6]

By selecting appropriate variables as common divisors of both numerator and denominator, any ratio can be expanded into a network of interrelated ratios. In theory, such networks can be expanded indefinitely;[7] in practice, the technique, referred to below as multiplicative ratio expansion, is limited only by the usefulness of the resulting relationships.

Neglected application

An important, neglected application of the multiplicative technique is its use, in conjunction with additive ratio expansion, as an *internal* tool for systematic diagnosis of the profit margin and TCE turnover ratios. Using both the additive and multiplicative approaches, the profit margin ratio can be expanded as shown in Exhibit II. This particular expansion is based on the classification of expenses as either variable or fixed, the former depending largely on the volume of goods sold and the latter on plant capacity.

The resulting equation (2) reveals the overall structure of the underlying relationships which define profit margin. By localizing the impact of volume[8] in the capacity utilization ratio, equation (2) highlights the more constant components of profit margin. It demonstrates that, at constant volume, improvements in profit margin require decreases in the ratios of (variable costs per unit)/price and (fixed costs per unit capacity)/ price.

This is not the whole story, however, because the structures of both the variable cost per unit and fixed cost per unit capacity ratios can be further elaborated by breaking them down into their component cost ratios and, multiplicatively, expanding these latter with related operational variables. It turns out that both are sums of input/output efficiency ratios, such as (raw material price)/(raw material output) and (average fixed salary)/ (capacity per fixed personnel). Substituting the ratio sums into equation (2) one obtains a more complete expansion, equation (3) which shows how profit margin depends on the efficiency with which different inputs into the operational mix are employed. The number of dependent input/output ratios depends, of course, on the manner in which variable and fixed costs are broken down. To avoid overcomplicating the example, expenses such as those for R&D and marketing, which strictly speaking depend on neither

1 For a history of ratio analysis and the origins of Equation (1), see James C. Horrigan, *Accounting Review,* XLIII (April, 1968) p. 284.

2 Unless otherwise stated, profit refers to earnings before interest and taxes.

3 TCE is defined as equity plus long-term debt, or alternatively, working capital plus fixed assets.

4 E. I. Dupont Nemours and Company, *Executive Committee Control Charts,* Wilmington Delaware, 1959
Business Week, Nov. 24, 1962, p. 128

5 F. B. Cohen and E. D. Finberg, *Investment Analysis and Portfolio Management,* Richard D. Irwin Inc., 1967, p. 313

6 It is assumed here, for the sake of simplicity, that after tax profit and equity earnings are one and the same. It should be noted, however, that minority interests and preference dividends have often to be deducted from after tax profit to arrive at equity earnings necessitating further analytical refinement.

7 In methodological terms, these networks are no more than alternative expanded definitions of the original ratio.

8 Volume refers to the volume of goods sold; it is assumed that this approximates the volume of goods produced.

volume nor plant capacity, have been regarded as fixed and included in service charges. There is, however, no reason why appropriate variables cannot be used to expand the corresponding ratios separately.

In Exhibit III, the TCE turnover ratio has been expanded to illustrate its relationship to price, degree of utilization of productive capacity, productivity of fixed assets, and the intensity (fraction) of total investment funds allocated to fixed assets, or alternatively, the major components of working capital. Equation (4) demonstrates that the TCE turnover ratio can be regarded as the product of these management control ratios.[9]

Substitution of the intermediate profit margin and TCE turnover expansions, equations (2) and (4), into equation (1) provides an expanded definition of the ROI ratio:

$$\text{ROI} \equiv k \left[\text{Capacity utilization} \left(\text{Price} - \text{Variable costs per unit} \right) - \text{Fixed costs per unit capacity} \right]$$

$$\text{where } k \equiv \frac{\text{Capacity}}{\text{TCE}} \qquad (6)$$

Except for working capital control, the factor, k, is determined largely by the technological requirements of the industry, so that once a strategic choice of industry has been made, k remains relatively constant and beyond management control. ROI can be regarded, therefore, as proportional to (capacity utilization) x (contribution margin per unit) — (fixed costs per unit capacity).

The relationship between ROI and the input/output ratios is obtained by combining the more complete profit margin and TCE turnover expansions, equations (3) and (6):

$$\text{ROI} \equiv k \Bigg[\left(\text{Capacity utilization} \times \text{Price} \right) - \text{Capacity utilization} \times \left(\frac{\text{Raw mat. price}}{\text{Raw mat. output}} + \frac{\text{Average direct wage}}{\text{Direct labour productivity}} + \frac{\text{Average sales salary}}{\text{Sales productivity}} \right) - \left(\frac{\text{Service charges}}{\text{Capacity}} + \frac{\text{Average fixed salary}}{\text{Capacity per fixed personnel}} \right) \Bigg] \qquad (7)$$

Equation (7) suggests that ROI can be considered proportional to two multipliers, capacity utilization and price, and two series of input/output efficiency ratios, one related to variable costs per unit and the other to fixed costs per unit capacity. To the extent that the variables are independent of one another,[10] equations (6) and (7) can be used to operationally compute the ROI impact of variations in any of the management variables shown: for example, a unit increase in price will increase ROI by k x (capacity utilization); a unit increase in raw material costs will decrease ROI by k x (capacity utilization)/(raw material output).

DIAGNOSTIC TOOL

Like most diagnostic techniques, expanded ratio analysis of the profit margin and turnover ratios depends for its application on a comparative standard of reference. As in the case of ratio analysis in general, the expanded ratio profile of a single operation can be compared with itself over time, or alternatively, a cross-sectional comparison can be made of different operations in the same industry.

Cross-sectional diagnosis

If comparable data are available,[11] analysis of different operations in the same industry permits the diagnosis of two general kinds of management problem. The first kind are tactical (or control) problems which reveal themselves in relatively inefficient input/output ratios. Since the ROI impact of tactical problems can be directly traced by means of expanded ratio analysis, the technique is especially suited to their detection. Examples of such problems are: low productivity reflected by a high wage/productivity ratio, or poor working capital control reflected by a high working capital intensity ratio.

The second kind of problem, which can be detected by way of its indirect impact on ROI within the expanded ratio framework, is more strategic in nature. It involves comparative differences in mix with respect to variables like direct manufacturing costs, personnel salaries, or fixed asset productivity. Such strategic mix differences, while not necessarily showing up directly in the corresponding ratio, tend to be reflected indirectly in other ratios: for example, a low wage, low productivity labour

9 Bela Gold, *Foundations of Productivity Analysis*, Univ. of Pittsburgh Press, 1955, p. 274

10 Ratio expansion takes no account of any underlying interdependence between variables. It ignores feedback relationships, the determination of which requires empirical research. For an empirical study of the factors affecting ROI see *Harvard Business Review*, April/May 1974, p. 137

11 In the case of cross-sectional analysis, the expanded ratio approach is subject to the usual ratio distortions associated with differences in size of operation and accounting procedure, such as the treatment of inflation.

mix, while not affecting labour costs per unit, might adversely affect product quality and hence indirectly price, volume, and ROI.

As an illustration of how expanded ratio analysis can be used to identify these two problem types, consider three hypothetical operations in the same industry with the returns on investment shown in Table 1.

Table 1
Return on investment anajysis

Operation	Profit margin	TCE turnover	ROI
A	16,7%	3,0	50,1%
B	1,1%	3,0	3,3%
C	5,0%	1,5	7,5%

Relative to A, which may be regarded as the desirable norm, the poor ROI performance of B is apparently due to a low profit margin, whereas C is weak in terms of both profit margin and TCE turnover. To probe the origins of these relative weaknesses, the profit margin first can be expanded additively to obtain the commonly used percentage of sales analysis (Table 2).

Table 2
Percentage of sales analysis

	R'000			%		
	A	B	C	A	B	C
Sales	R360	R360	R240	100,0%	100,0%	100,0%
Raw materials	60	86	30	16,7	23,9	12,5
Direct labour	60	90	30	16,7	25,0	12,5
Selling personnel	60	60	60	16,7	16,7	25,0
Total variable costs	R180	R236	R120	50,0%	65,6%	50,0%
Service charges	40	40	36	11,1	11,1	15,0
Fixed personnel	80	80	72	22,2	22,2	30,0
Total fixed costs	R120	R120	R108	33,3%	33,3%	45,0%
Profit before interest and taxes	R 60	R 4	R 12	16,7%	1,1%	5,0%

B's problem almost certainly is high percentage raw material and direct labour costs relative to sales, although whether this is due to higher costs *per se*, or inefficient productivity and usage, cannot be deduced from the percentage of sales analysis. C's problem, however, is not as transparent. Its total variable costs comprise the same percentage of sales as A's, but whereas raw materials and direct labour are a smaller percentage, selling personnel costs are relatively greater. C's fixed costs also are high relative to its sales. The possible causes for C's position are many — inefficiency and low capacity utilization immediately spring to mind, but to isolate the problem with some certainty, further analysis is necessary. In Table 3, profit margin is expanded further multiplicatively (see Exhibit II) to obtain a unit cost analysis.

Table 3
Unit cost analysis

	A		B		C	
Sales volume (000 units)	30		30		30	
Price		R12		R12		R8
Variable costs						
Raw material volume (000 units)	100		143		100	
Raw material price	R0,6		R0,6		R0,3	
÷ Raw material output	0,3		0,21		0,3	
= Raw material cost		R2		R2,9		R1
Number of direct labour units	10		15		5	
Average direct wage (R000)	R6		R6		R6	
÷ Direct labour productivity	3		2		6	
= Direct labour cost		R2		R3		R1
Number of sales personnel	5		5		6	
Average sales salary (R000)	R12		R12		R10	
÷ Sales personnel productivity	6		6		5	
= Sales personnel cost		R2		R2		R2
Variable costs per unit		R6		R7,9		R4
Fixed costs						
Plant capacity (000 units)	40		40		36	
Fixed costs per unit capacity	R3		R3		R3	
÷ Capacity utilization	0,75		0,75		0,83	
= Fixed costs per unit		R4		R4		R3,6

The variable cost analysis in Table 3 pinpoints B's problem; the high variable costs are not due to higher input costs *per se*, but to inefficient raw material usage (raw material output, that is, the volume of finished goods produced per unit of raw material, is only two-thirds of what it is in A and C) and low labour productivity (two-thirds of A's). By concentrating on increased efficiency in raw material usage and improved labour productivity, B should be able to improve its ROI.

By contrast, C's problem is more fundamental, as is evident from some of the key differences between C and A, highlighted by Table 3: the average price of C's product line is only two-thirds that of A's; C's raw materials are only half the price of A's; and its labour productivity is double that of A's. Thus, although C's total variable costs are the same percentage of sales as A's, its product mix is obviously different. This conclusion is corroborated by the difference in fixed asset productivity, or plant capacity per unit of fixed asset investment (see Table 4). Assuming similar dates of installation as well as depreciation methods, C's plant apparently cost more than A's; moreover, fixed assets comprise a higher proportion of C's TCE (compare fixed asset intensities).

Table 4
TCE turnover analysis

	A	B	C
Price	R12	R12	R8
x Capacity utilization	0,75	0,75	0,83
= Capacity turnover $\frac{(\text{Sales})}{(\text{Capacity})}$	9	9	6,6
x Fixed asset productivity $\frac{(\text{Capacity})}{(\text{Fixed assets})}$	0,5	0,5	0,3
= Fixed asset turnover $\frac{(\text{Sales})}{(\text{Fixed assets})}$	4,5	4,5	2,0
x Fixed asset intensity $\frac{(\text{Fixed assets})}{(\text{TCE})}$	0,67	0,67	0,75
= TCE turnover	3,0	3,0	1,5

C is probably making cheaper products with more machine and less labour content than A or B. But C's fixed costs per unit are only slightly less than A's, so that when C's lower price is taken into account, its fixed costs represent a considerably higher percentage of sales than those of A. In order to increase its ROI, supposing little improvement in efficiency is possible, C must raise prices while maintaining sales volume (capacity utilization).

General diagnosis

Although the diagnosis and tactics for a particular operation are best derived from comparative analysis of other operations in the same industry, a number of general tactical principles are implicit in the expanded ratio equations. In cases where a cross-sectional comparison is not possible, these principles provide a guide to the evaluation of expanded ratio trends over time.

The well-known equation (1) indicates that ROI is the product of profit margin and TCE turnover. These two ratios are interdependent, however, to the extent that both are functions of capacity utilization and price. Profit margin and TCE turnover, therefore, cannot be regarded as basic components of ROI.

Equation (6), on the other hand, comprises more fundamental components of ROI:- k (that is plant capacity per unit of TCE), capacity utilization, contribution margin per unit, and fixed costs per unit capacity. Insofar as k is determined by technological factors, management control is much more likely to improve ROI by focussing on the other terms in equation (6). These terms are, of course, all the more important, the lower k; if their ROI is to be comparable, firms in low k industries must perform better than average with respect to the other ratios. Some general comments on the evaluation of these ratios are provided below:

(i) Capacity utilization, the most volatile of the variables because it embodies the volume of goods, is directly related to market share:

$$\text{Capacity utilization} \equiv \frac{\text{Volume}}{\text{Capacity}} \equiv \frac{\text{Volume}}{\text{Market}} \times \frac{\text{Market}}{\text{Capacity}} \equiv \frac{\text{Market share}}{\text{Capacity market share}} \quad \ldots\ldots (8)$$

The other ratios being equal, a healthy ROI requires high average capacity utilization, or alternatively, a market share close to that corresponding to full capacity utilization; in brief, little excess capacity. However, if both ROI and growth are company objectives, the negative impact of excess capacity on ROI must be carefully balanced against its frequent necessity for growth.

(ii) The price structure affects not only the contribution margin per unit but also capacity utilization. Market forces permitting, prices obviously should be set with a view to maximizing ROI by maximizing (capacity utilization) x (contribution margin per unit).

(iii) Variable costs per unit, as shown in Exhibit II, can be expressed as the sum of input/output efficiency ratios. Of the corresponding input variables, raw material costs, wages and salaries are subject to relatively little management control, except for periodic contract negotiations. If the variable cost structure is to remain the same, changes in these input variables must be offset by corresponding changes in the output variables, raw material output, direct labour productivity, and sales productivity, respectively. Therefore, if ROI is to be maintained, variable expenses inputs should be monitored in terms of both cost and efficiency.

(iv) Fixed costs per unit capacity, which include overhead, administration, marketing, and R&D expenses, are subject to more management control than are variable costs. Overhead and administration include discretionary expenses such as those associated with automation and data processing. Clearly, these expenses are less easily supported the lower k, capacity utilization, and contribution margin per unit. Marketing and R&D expeness

are generally used by management to improve one of the other ratios in equation (6). Their optimal level depends on the extent to which they have the desired impact. Marketing and R&D expenses should be monitored, therefore, both with respect to their negative impact on fixed costs per unit capacity as well as their potential positive impact on k x (capacity utilization) x (contribution margin per unit).

CONCLUSION

Internal management's use of equation (1) (usually as a link between ROI, the income statement, and the balance sheet) can be greatly enhanced by applying the multiplicative ratio expansion technique which is implicit in equation (1) itself. With this technique, expanded ratio analysis can be used to connect ROI, an input/output ratio, to other input/output ratios made up of management variables. Thus, ROI can be defined directly in terms of decision rather than accounting variables.

The ease with which the corresponding profit margin and TCE turnover expansions can be applied to information generally available within any company make them appropriate as tools for internal control and diagnosis. As such they are particularly suited to the detection of tactical problems, involving variations in the efficiency with which available resources are being used. To the extent that comparable information is available on other operations in the same industry, a cross-sectional analysis of expanded ratio profiles can be used to diagnose strategic problems of resource mix.

More generally, the application stressed here provides an example of the results which can be obtained from multiplicative ratio expansion, provided it is applied within a logical framework appropriate to the problem under study. (In this case, the problem of ROI analysis for purposes of internal diagnosis and control suggested the use of a break-even type framework, or the classification of expenses as either variable or fixed.) Haphazard ratio expansion without a guiding framework is likely to lead to a redefinition of the original ratio in terms of a meaningless set of secondary ratios, of much less use than the arithmetically unrelated ratios used in conventional analysis.

Within an appropriate framework, expanded ratio analysis can be used to probe the normative relationship between any ratio and corresponding secondary ratios. Given its inherent simplicity, the technique is limited only by the ingenuity of the user in creating useful expansions.

Exhibit I Common multiplicative ratio expansions

$$\text{Earnings per share} \equiv \frac{\text{After-tax profit}}{\text{Number of shares}} \equiv \frac{\text{After-tax profit}}{\text{Equity}} \times \frac{\text{Equity}}{\text{Number of shares}} \equiv \text{Return on equity} \times \text{equity per share}$$

$$\text{Return on equity} \equiv \frac{\text{After-tax profit}}{\text{Equity}} \equiv \frac{\text{After-tax profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Equity}} \equiv \text{After-tax profit margin} \times \text{equity turnover}$$

$$\text{Equity turnover} \equiv \frac{\text{Sales}}{\text{Equity}} \equiv \frac{\text{Sales}}{\text{TCE}} \times \frac{\text{TCE}}{\text{Equity}} \equiv \text{TCE turnover} \times \text{leverage}$$

$$\text{Earnings per share} \equiv \frac{\text{After-tax profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{TCE}} \times \frac{\text{TCE}}{\text{Equity}} \times \frac{\text{Equity}}{\text{Number of shares}}$$

$$\equiv \text{After-tax profit margin} \times \text{TCE turnover} \times \text{Leverage} \times \text{Equity per share}$$

Exhibit II Profit margin expansion

$$\frac{\text{Profit}}{\text{Sales}} \equiv \frac{\text{Sales} - \text{Variable costs} - \text{Fixed costs}}{\text{Sales}}$$

$$\equiv \frac{\text{Sales}}{\text{Sales}} - \left(\frac{\text{Variable costs}}{\text{Volume}} \times \frac{\text{Volume}}{\text{Sales}}\right) - \left(\frac{\text{Fixed costs}}{\text{Capacity}} \times \frac{\text{Capacity}}{\text{Volume}} \times \frac{\text{Volume}}{\text{Sales}}\right)$$

$$\frac{\text{Profit}}{\text{Sales}} \equiv 1 - \frac{\text{Variable costs per unit}}{\text{Price}} - \frac{\text{Fixed costs per unit capacity}}{\text{Capacity utilization} \times \text{Price}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Variable costs per unit and fixed costs per unit capacity can be further expanded as follows:

$$\text{Variable costs per unit} \equiv \left(\frac{\text{Raw mat. costs}}{\text{Raw mat. volume}} \times \frac{\text{Raw mat. volume}}{\text{Volume}}\right) + \left(\frac{\text{Direct labour costs}}{\text{Direct labour}} \times \frac{\text{Direct labour}}{\text{Volume}}\right) + \left(\frac{\text{Sales personnel costs}}{\text{Sales personnel}} \times \frac{\text{Sales personnel}}{\text{Volume}}\right)$$

$$\text{Fixed costs per unit capacity} \equiv \frac{\text{Service charges}}{\text{Capacity}} + \left(\frac{\text{Fixed personnel costs}}{\text{Fixed personnel}} \times \frac{\text{Fixed personnel}}{\text{Capacity}}\right)$$

Substitution into equation (2) yields:

$$\frac{\text{Profit}}{\text{Sales}} \equiv 1 - \frac{1}{\text{Price}} \times \left(\frac{\text{Raw mat. price}}{\text{Raw mat. output}} + \frac{\text{Average direct wage}}{\text{Direct labour productivity}} + \frac{\text{Average sales salary}}{\text{Sales personnel productivity}}\right) - \frac{1}{\text{Price} \times \text{Capacity utilization}} \times \left(\frac{\text{Service charges}}{\text{Capacity}} + \frac{\text{Average fixed salary}}{\text{Capacity per fixed personnel}}\right) \quad \ldots\ldots\ldots (3)$$

Exhibit III TCE turnover expansion

$$\frac{\text{Sales}}{\text{TCE}} \equiv \frac{\text{Sales}}{\text{Volume}} \times \frac{\text{Volume}}{\text{Capacity}} \times \frac{\text{Capacity}}{\text{Fixed assets}} \times \frac{\text{Fixed assets}}{\text{TCE}}$$

$$\frac{\text{Sales}}{\text{TCE}} \equiv \text{Price} \times \text{Capacity utilization} \times \text{Fixed asset productivity} \times \text{Fixed asset intensity} \quad \ldots\ldots\ldots\ldots (4)$$

Fixed asset intensity can be further expanded as follows:

$$\text{Fixed asset intensity} \equiv 1 - \frac{\text{Working capital}}{\text{TCE}}$$

$$\equiv 1 - \frac{\text{Accounts receivable}}{\text{TCE}} - \frac{\text{Inventories}}{\text{TCE}} - \frac{\text{Other current assets}}{\text{TCE}} + \frac{\text{Current liabilities}}{\text{TCE}}$$

Substitution in equation (4) yields:

$$\frac{\text{Sales}}{\text{TCE}} \equiv \text{Price} \times \text{Capacity utilization} \times \text{Fixed asset productivity} \times \left(1 - \text{Accounts receivable intensity} - \text{Inventory intensity} - \text{Other current asset intensity} + \text{Current liability intensity}\right) \quad \ldots (5)$$