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Portfolio selection: a non-technical overview

The modern theory of portfolio selection is now a quarter of a century old and a great deal of scientific work has arisen from it. However, it remains a somewhat obscure area of knowledge for most practitioners in the field. This is probably because of its technical and mathematical character which makes it difficult to absorb for those who are not at home with mathematical approaches. All the same the subject can give many insights, even to those who will not readily comprehend its technical niceties. This article is an attempt to spell out the principles that are involved and to summarise some of the important results while eschewing mathematical exposition.

Affluence is preferable to poverty but the rich are not without their problems. Amongst other things they need to decide in what form to keep their wealth. That requires selection, in the modern world, from a bewilderingly large array of possibilities, resulting in a choice problem that is far from trivial. Neither is this a problem of the wealthy alone. Those entrusted with the stewardship of the assets of others face the same bewildering problem of choice. Readers of the 'Investment Analysts Journal' scarcely need to be reminded of this.

CHOOSING A PORTFOLIO

A range of choices immediately raises the question of what principles should be applied in selecting a portfolio of assets. Part of the problem is solved by the requirements of use. A house or motor car is very largely purchased to meet the life style needs of the purchasers without much reference to formal rules of choice. For most assets, however, the benefits of ownership are less direct and it is necessary to analyse and assess these in order to make sensible choices. Such assessments require the specification of a basis and standard of choice.

A basis which is most likely to spring to mind is that of holding the assets which will give the highest rate of return, i.e. which will in some sense be most profitable to hold. Such an approach is cognate with the general economic principle of maximisation, and offers a single and apparently unambiguous objective to be pursued by asset holders.

However, this principle cannot be applied in the most literal sense. The rate of return on different assets is frequently not known in advance. Consequently there is no way of actually seeking to attain it even though it may be reached by chance.

This can be illustrated by a simplified example. Consider an investor who faces two alternatives:

(a) He can acquire a bank deposit which will yield precisely 10% over the coming year;

(b) He can acquire ordinary shares which offer an equal chance of a return of 60% or of minus 20%.

If the ordinary shares do yield 60% the investor will have maximised his return by holding them. But if they yield minus 20% the alternative policy — holding the bank deposit — will maximise return. Since the investor does not know in advance what will actually be the yield on the shares, he has no way of choosing the alternative which will give the highest return.

The prescription that return should be maximised is useful only to those who can foretell the future. It is of no use to ordinary mortals.

EXPECTED RETURN

The reader may well protest, at this point, that this is an over-literal interpretation. What is actually meant is that probable or expected return should be maximised.

We can rapidly ascertain that the holding of shares gives an expected return of 20% $\{(\frac{1}{2} \times 60) + (\frac{1}{2} \times -20)\}$. This is higher than the 10% promised on the bank deposit and a clear choice can be made. The principle has been rendered operational.

However, it is still not clear that the principle will always give 'sensible' prescriptions. Many asset holders faced with a choice of this type would still prefer the certainty of 10% to an equal chance of a 20% loss and a 60% gain. One can certainly think of cases where it would be most unacceptable advice to tell an investor to hold all his or her wealth in a form which might easily lead to a 20% (or some higher) loss. Indeed, it would usually be unacceptable to advise any investor to hold one asset only. A 'reasonable' asset holding rule must provide for this, or else provide a convincing reason why it does not.

Does this mean that expected return in its various manifestations, and the body of financial technology which depends upon the concept, must be abandoned? Fortunately it does not. Return (in a broad sense) is a matter of great importance to asset holders. Other things being equal its maximisation will be sought. The issues to be clarified are those of the nature of those 'other things' and what is to be done when they are not equal. This is the substance of portfolio theory.

INTRODUCING RISK

In portfolio theory those 'other things' are usually described by the summary term 'risk'. Asset holders prefer higher expected returns to lower ones (rationality) and generally they also prefer less risk to more (risk aversion). In choosing assets both risk and return must be considered, and the terms of a tradeoff must be developed.

Before doing this it is necessary to define 'risk' and 'return'. In the case of return the definition is intuitively straightforward. Return is the percentage accretion with respect to an asset over a year (or other period). It is income received plus capital profits (or less capital losses) adjusted for taxes and measured as a percentage of assets at the start of the year. There may be practical difficulties about measuring return. This is particularly so for assets where the capital value is not established in a continuous and active market. Nonetheless, the principle is clear. (It becomes more complex in a multi-period context.)

When it comes to risk such conceptual simplicity is absent. There is no commonly accepted and measurable risk concept which may be drawn from ordinary 'commonsense', although the idea of 'risk' as an undesirable aspect of an investment is certainly one that is understandable in general terms.

Some obvious definitions of risk are:

(a) The probability that an asset will yield less than its expected return;

(b) The probability that an asset will yield a negative return (show losses);

(c) The extent to which the worst outcome contemplated falls below the expected return.

All of these are probability concepts. The definition that portfolio theory uses is that of the 'standard deviation' of return. This is a somewhat technical statistical concept. It may be thought of as a measure of the difference between the expected return and the average shortfall that is contemplated if circumstances turn out less favourably than is expected (the average 'bad' outcome expected). The definition has the advantage that it is a basic concept of statistical theory and is thus highly usable. Other measures have been tried, but by and large they do not compare with 'standard deviation' in fruitfulness, at least in the present state of knowledge.

One way of interpreting the concept is to describe it as a measure of 'unreliability' or 'variability' of return, rather than of risk. A certain return would have a zero standard deviation. Certainty is, fairly obviously, synonymous with the absence of risk.

ADJUSTING RETURN FOR RISK

The general rule of asset choice that is applied by portfolio theory is that a rational risk averse asset holder should seek to maximise expected return after adjusting it for risk. With the definitions used it turns out that this rule usually leads to the choice of a diversified portfolio of assets, which is an intuitively satisfying result.

The precise nature of the adjustments for risk that must be applied to expected return are complex. Fortunately portfolio theory generates certain results which make these more tractable than might otherwise have been the case.

A possible formalisation of the risk adjusted expected return objective might be to maximise expected return reduced by a proportion of its standard deviation. This could be spelt out as an equation.

For example:

$$\text{Objective (1)} = \bar{r} - 0{,}2\sigma \qquad (1)$$

or

$$\text{Objective (2)} = \bar{r} - 0{,}5\sigma \qquad (2)$$

where $\bar{r}$ is the expected rate of return, σ is its standard deviation, and the overall expression is that which is to be maximised.

The objective function set out in the first example may be described as moderately risk averse. The asset holder reduces his expected return by one fifth (0,2) of the risk factor (standard deviation) to determine the objective or adjusted return expected. The objective function set out in the second example may be described as highly risk averse. A much larger weight is placed on the risk factor to determine the objective.

Table 1
Risk adjusted returns (utility indices) which arise from applying objective (1) and objective (2) to the two portfolios

Asset	Bank deposit	Shares
Objective (1) (Moderate risk aversion)	10	12
Objective (2) (High risk aversion)	10	0

The asset holder who is moderately averse to risk prefers the share portfolio. Its risk adjusted expected return is higher. The second asset holder, who is highly averse to risk, prefers the bank deposit. His aversion to risk is so weighty that it wipes out the value of the high expected return.

PORTFOLIO DIFFERENCES

It is most important to note that, by introducing the concept of risk adjustment, it has emerged that different asset holders may seek to hold different portfolios, even though they are all rational and risk averse. However, it has not yet been seen how individual sset holders may decide to hold mixed portfolios (i.e. consisting of more than one type of asset) and how they would carry out such mixing.

In order to demonstrate how this might happen it will be convenient to expand the example to the case where the asset holder faces three alternatives. The first two will be the same as those set out earlier. The third will be a holding of shares which, like the others, offer an equal chance of a 60% gain and a 20% loss. However, these chances are independent of those affecting the other shares. The fact that one share yields 60% or minus 20% will be unrelated to whether the other yields 60% or minus 20%. It will also be assumed that the asset holder can hold mixed portfolios, consisting of different proportions of the three assets. Table 2 then sets out a few of the possible alternatives and their implications in terms of various outcomes, expected returns, standard deviations of expected return, and the utility indices that emerge in terms of objective (1) and objective (2).

The portfolios are illustrated in Figure 1, where their risk/return characteristics are shown graphically. The figure also indicates, by way of arrows, the directions of preference of a rational risk averse asset holder in these respects.

The first three portfolios listed (I, II and III) are single asset holdings. The others are made up of combinations of those three basic components. Portfolio V, for example, is made up of equal proportions of Portfolios II and III. It has the same expected return as its components, but its risk is lower. Thus the combined portfolio will be preferred to its component portfolios by all rational risk averse asset holders.

Figure 1

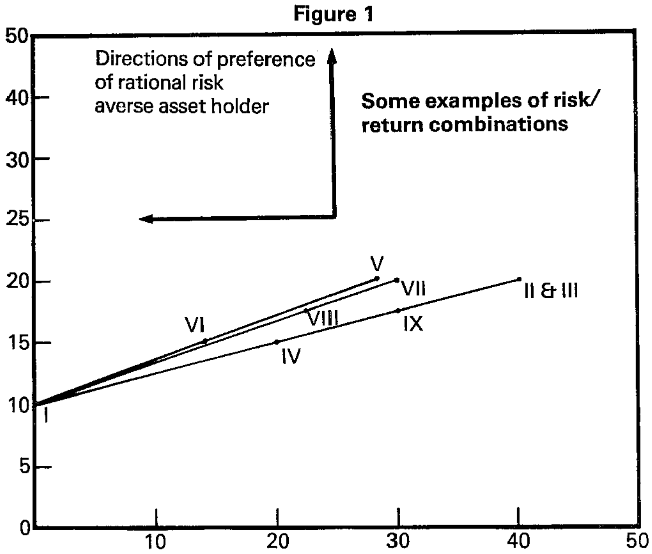
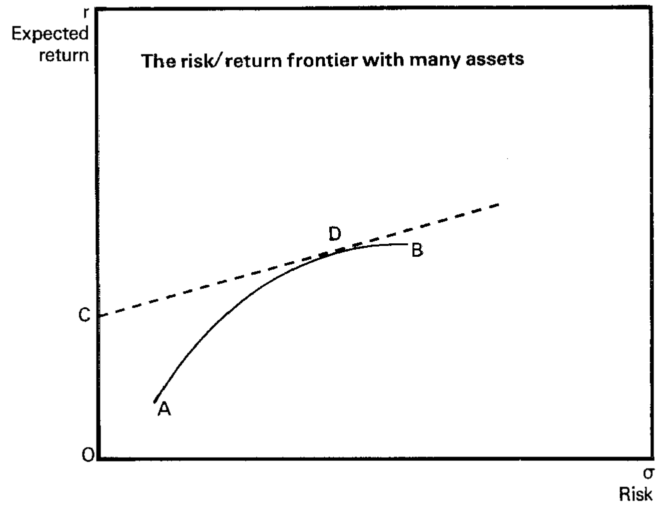


Figure 2



Portfolio IV is made up of equal proportions of Portfolios I and II. In this case both expected return and standard deviation are the arithmetic means (simple averages) of those applying to the component parts.

Portfolios V, VII, VIII and IX are made up of different combinations of the three basic assets. Portfolio VI is in effect a 50/50 combination of Portfolios I and V. Portfolio VII is a non-equal combination of the same two components as make up Portfolio V (which is preferable). Portfolio VIII is a combination of Portfolios I and VII.

Of interest is the fact, illustrated in Figure 1, that several groups of the portfolios listed lie on a single ray from the return axis, at the point where Portfolio I is represented. This makes elucidation a little easier, and facilitates the choice process.

Consider Portfolios I, V, and VIII. The ordering of their expected return/risk characteristic is such that any one of them might be most preferred by some rational risk averse asset holder. Between any two of them higher return is purchased only at the cost of higher risk to be borne. Thus, at first sight, all are acceptable portfolios on some assumption about the tastes of the asset holder. As Table 2 shows, Portfolio V is preferred to all others by the moderately risk averse asset holder with objective (1), and he would prefer Portfolio VIII to Portfolio I. Yet the highly risk averse asset holder with objective (2) gives his first preference to Portfolio I, and his last preference to Portfolio V. It certainly seems plausible to postulate that some other asset holder, perhaps one who had an intermediate amount of risk aversion, might prefer Portfolio VIII to either of the other two.

However, it can be shown that such an asset holder would still not choose Portfolio VIII. Instead he would choose a combination of Portfolios I and V.

HOME-MADE PORTFOLIOS

Consider an example of such a 'home-made' portfolio. Let this Portfolio X (not shown) consist of 76% Portfolio V and 24% Portfolio I. It would then have an expected return of 17,6% and a standard deviation of about 21,5% which makes it clearly preferable to Portfolio VIII. Its return is higher and its risk is lower. Thus such a 'home-made' portfolio would be preferred to Portfolio VIII by any rational risk averse asset holder.

It is of some importance to note that the return and risk applicable to Portfolio X is given by the weighted arithmetic means of those applicable to its component parts. In the case of return this relationship always holds. In the case of standard deviation of return, however, it only holds in the case where one of the two components has a zero standard deviation. In general the standard deviation of a composite portfolio will be less than the weighted arithmetic mean of those of its component parts (as in the case of Portfolio VI). It is this fact which justifies diversification and it is a most important principle of portfolio theory.

In the examples given the line joining Portfolios I and V represents the locus of optimal combinations. Portfolios made up of those two components dominate (are preferred to) all others that can be made up from the basic set of assets available. In effect all rational risk averse asset holders will choose portfolios that consist of a combination of the bank deposit (Portfolio I) and equal holdings of the two shares (Portfolio V).

This principle, in an expanded form, is an important result of portfolio theory. It has the effect of reducing substantially the range of portfolios which asset holders need to consider in making their choices.

THE SEPARATION THEOREM

To derive the expanded form of this principle, which is known as the 'separation theorem' it is necessary to treat portfolio theory in somewhat more depth than has been done thus far. The approach will still be fairly general, however.

Two points should be noted. The first is that the number of assets to be considered by asset holders in the real world is actually very large indeed. The second is that returns from risky assets are not, generally speaking, strictly independent. Consequently the variance (and, thus, the standard deviation) of expected return on a portfolio of assets depends not only on the variances of the individual components and their proportions but also on the covariances (association) of return between the various component assets. This has to be calculated by a formula which is somewhat complex in its structure though not inherently difficult. That formula requires, however, a rather large amount of information as input.

If a portfolio is to be selected from among 50 assets, for example, there are 2 550 variances and covariances to be considered (as well as 50 rates of return).

Then, when the assets being considered are the shares of large quoted companies this is a great deal of information and much of it is of a kind which is not readily available, and is not readily interpretable into everyday concepts. In the practical application of portfolio theory such information is frequently generated in a mechanical way from historic data. In addition, certain 'short cuts' have been developed to facilitate the analysis of such data.

Figure 2 shows the general shape of the risk/return frontier that will rule. No combinations will lie above and to the left of the line A B, which slopes upwards at a diminishing rate (or at least at a non-increasing rate). Consequently it is only the characteristics of frontier portfolios that need be calculated. And it can be shown that the frontier portfolios can be devised from the characteristics of a limited number of points along it. The computational problem is by no means as great as first appeared. Indeed, with a modern electronic computer, the computation is a matter of seconds even for fairly large populations of assets.

It can be simplified even more, in most cases. It will be recalled that in the example used earlier it was pointed out that a portfolio can be treated as a combination of other portfolios. It was also stated that where portfolios consist of one security which offers a zero standard deviation and another which has some risk, the risk/return characteristics of such portfolios can be given by a straightforward line on a graph such as in Figure 1 or Figure 2. That continues to be the case in the more general situation which is now being considered.

Consequently, as asset holder who faces the opportunity set given by the frontier A B in Figure 2, and also has the opportunity to hold a risk-free asset illustrated by point C, actually has an opportunity set of the share C D B. The existence of asset C has expanded his opportunity set, possibly to a substantial extent. For a wide range of asset holders, the most preferred portfolios will consist of the same two components. One of these will be the risk-free asset C. The other will be a holding of risk portfolio D. Some (the very risk averse) will hold a very high proportion in form C. Others will follow different policies. But the same components will make up all their portfolios. The speculator will not hold different shares from the cautious widow. He will merely hold a larger proportion of his portfolio in those shares.

SOME QUALIFICATIONS

There are, however, two qualifications to this result. The first is that it depends on the assumption that the opportunity set (in terms of risk/return possibilities) is the same for all asset holders. That may not be the case.

Asset holders in different countries for example, face genuinely different opportunity sets. Even within the same economy differences in effective factors such as: the scale of assets, knowledge, and above all, tax liabilities may result in differences in the set of opportunities available to different asset holders. In addition, different asset holders facing the same objective opportunity set, with similar personal situations, may nonetheless have different assessments of those opportunities. If one individual believes firmly that a doubling of the copper price is imminent while another believes with equal firmness that a halving is imminent, their portfolios are likely to reflect these differing assessments in the form of differing proportions of copper shares held in their portfolios.

The other qualification is somewhat different in character. In fact it arises from a body of thought which largely discounts the significance of the first qualification. The bottom line of that qualification is that the curve A B is likely, over most of its length, to be virtually flat; for all practical purposes it will coincide with the tangent C D. Consequently, a wide range of risk portfolios will be virtually equivalent from the point of view of asset holders. Thus, different holders may quite reasonably hold materially different portfolios although their risk return characteristics will be such that the differences are more apparent than substantive.

The reason for this situation, it is argued, is that the interaction of asset holders in the capital market will bring into being a price structure for different assets which will ensure that a wide range of portfolios are substitutes for what may be called 'the market portfolio' — that portfolio which is made up of all available assets.

A BIT ABOUT BETA

A by-product of this theoretical approach is the enigmatic beta coefficient which is the best known manifestation of portfolio theory among financial practitioners.

The beta coefficient is a measure of the responsiveness of the probable return on a given asset to variation in the return on the market portfolio. Any reasonably well spread portfolio with an average beta of one will be a good surrogate for the market portfolio, or so the theory indicates. Thus, the portfolio manager is able to work with yet less information; he needs only the expected returns and betas of his potential assets. And he can process it in a fairly simple manner in order to generate an efficient portfolio.

The conclusion that every risk portfolio will have the same composition has the somewhat disconcerting implication that all asset holders will hold a portfolio containing all risk assets. This is because the result does not allow for any asset to be held only by a few people.

However, while the calculation of beta is easier than that of covariances it still may be a major task. And the procedures that have been used to estimate beta values have not shown these to have as much stability as theory would require if they were to be very useful. In addition the implication that you cannot really beat the market, or at any rate that you cannot do it consistently, is unattractive to those who trade on the stock exchange and other markets where the prices are patently volatile. Such individuals and institutions often depend to a substantial extent on a clientele which seeks rapid and spectacular profits. The fact that some people do indeed make spectacular profits reinforces this approach although it is probable that such spectacular profits represent luck or special situations which are not susceptible to analysis and which are balanced by many failures, some of them equally spectacular, and many which are disastrous to the individual concerned.

Portfolio theory and its developments remain the best worked out logical approach to asset selection that is known. This is illustrated by the very fact that the approach invented by Markowitz has sole claim to this appellation. Nonetheless, in the quarter century since Markowitz's first article appeared, the approach has not gained acceptance in the financial community and it does not show many signs of doing so. The reasons are no doubt complex and have partly to do with the preference of clients for the speculative and spectacular, and partly with the fact that the most widely used approaches demand less intellectual and physical effort.

Table 2
The alternatives facing asset holders with three basic investment opportunities open to them

Portfolio		I	II	III	IV	V	VI	VII	VIII	IX
	A	1	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{4}$
Composition in terms of assets	B	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{3}{4}$
	C	0	0	1	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	0
There is a 25%	25%	10	−20	−20	−5	−20	−5	−20	$-12\frac{1}{2}$	$-12\frac{1}{2}$
probability that each	25%	10	−20	−20	−5	20	15	$6\frac{2}{3}$	$7\frac{1}{2}$	$-12\frac{1}{2}$
of the listed rates	25%	10	60	60	35	20	15	$33\frac{1}{3}$	$27\frac{1}{2}$	$47\frac{1}{2}$
of return C% will materialise	25%	10	60	60	35	60	35	60	$47\frac{1}{2}$	$47\frac{1}{2}$
Expected return (%)		10	20	20	15	20	15	20	$17\frac{1}{2}$	$17\frac{1}{2}$
Standard deviation of expected return (%)		0	40	40	20	28,3	14,1	29,8	22,4	30
Utility index in terms of Objective (1) (Moderate risk aversion)		10	12	12	11	*14,3	12,2	14,0	13,0	11,5
Utility index in terms of Objective (2) (High risk aversion)		*10	0	0	5	5,8	8,0	5,1	6,3	2,5

*Indicates portfolio that maximises objective function.