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An empirical comparison of the performance of different stock market indices

ABSTRACT

The performance on The Johannesburg Stock Exchange of five different types of stock market indices (those based on the Dow Jones, Standard and Poors, United Press International Market Indicator, Value Line, and ESE philosophies) are compared for four different types of markets. It is shown that, after the actual type of market, the method of computing the index is a more important source of variation in the index than is the actual sample of securities chosen as constituents. Pairwise comparisons are made between the different types of indices, and the performance of the indices based on price is contrasted with that of the indices based on return.

1 INTRODUCTION

In this paper, the performance on The Johannesburg Stock Exchange of five different types of stock market indices is empirically examined. An attempt is made to compare the relative performance of the different types of indices in both bull and bear markets, and some remarks are made about their relative volatility. In addition, some other interesting results are highlighted, and some of the subjective remarks made in the literature relating to the performance of particular types of stock market indices are shown to hold in practice.

2 TYPES OF STOCK MARKET INDICES

The five methods of constructing stock market indices which will be examined in this paper, may be defined as follows:

(i) Arithmetic average of price (DJ index)

This type of index is constructed as follows:

$I_t = 1/n \sum_{j=1}^{n} P_{j;t}$

where

I_t is the level of the index at period t;

n is the number of securities included in the index;

$P_{j;t}$ is the price of the j^{th} security in period t.

This is perhaps the most intuitive type of stock market index and is the methodology upon which the Dow Jones Averages are based. As far as investment is concerned, this type of index corresponds to an investor who buys one share in each of the n constituent companies.

(ii) Market capitalization index (SP index)

This type of index is computed as follows:

$$I_t = \frac{\sum_{j=1}^{n} N_{j;t} * P_{j;t}}{\sum_{j=1}^{n} N_{j;o} * P_{j;o}} * L.F.$$

where

$N_{j;t}$ is the number of shares issued in the j^{th} security at time t;

$P_{j;o}$ is the price of the j^{th} security at some base period, t = o;

L.F. is a linking factor to preset the index at some desired level; and the remaining symbols are as previously defined.

This method of construction, which gives greater weight to the larger companies (those with bigger market capitalizations), has become extremely popular, especially with large investors such as mutual funds, since it takes the "availability" of the security into account. As far as investment is concerned, it is equivalent to an investor who spreads his money among the n constituents in proportion to the market capitalization of each security relative to the total market capitalization of all n companies. Some of the better known indices which use this approach are the Standard and Poor's Indices, the New York Stock Exchange Indices, the Financial Times Actuaries Index (London Stock Exchange) and the Rand Daily Mail Indices (The Johannesburg Stock Exchange).

(iii) Arithmetic average of return (UP index)

It has been argued by Cohen and Fitch[2] that since investors are generally interested in return and not usually in price *per se,* stock market indices should be based on return and not price. Most of the empirical work pertaining to stock market indices based on return has not used return in the traditional sense of the word (difference in price over some period divided by price at the beginning of the period) but have used a related measure, the price relative ($P_{j;t}/P_{j;t-1}$). Thus, this type of index is usually contructed as follows:

$I_t = 1/n \sum_{j=1}^{n} (P_{j;t}/P_{j;t-1}) * I_{t-1}$

where the symbols are as previously defined. This index is equivalent to the performance of an investor who invests equal monetary amounts in each security and reallocates back to equal amounts at the start of each new period (whether a day, a week, a month or a year). The United Press International Market Indicator is constructed in this manner (New York Stock Exchange).

(iv) Geometric average of return (VL index)

This method has received a fair amount of attention in the more recent literature and is also based on price relatives (as for (iii) above). A Geometric Average index is constructed as follows:

$I_t = (\pi_{j=1}^{n} P_{j;t}/P_{j;t-1})^{1/n} x I_{t-1}.$

This type of index has become known as the continuous reallocation type index since it reflects the behaviour (theoretical) of an investor who continuously reallocates his resources so as to maintain a portfolio with equal monetary amounts in each security. The most famous index using this type of methodology is the Value Line index (NYSE).

(v) The ESE indices (ESE index)

The fifth type of index which will be considered in

this paper, is an index which is peculiar to The Johannesburg Stock Exchange. It is the ESE type index, which is constructed in the following manner:

$$I_t = I/n \Sigma_{j=1}^{n} \left(\frac{1\,000}{(P_{j;o}} * P_{j;t} \right) * L.F.$$

where

$P_{j;o}$ is the price of the j^{th} security at the beginning of the current year, and the remaining symbols are as previously defined.

This type of index reflects the performance of an investor who allocates his funds equally (i.e. equal rand amounts) in each of the n constituent securities at the beginning of the year, and maintains that portfolio until the end of the year when he sells all his securities and reallocates equal rand amounts to each security. Thus, in effect, he sells part of his holdings in those securities which have performed best in the year, and purchases more of those securities which have performed worse. Clearly, this index is very similar to the UP type index with the difference being that the UP index reallocates each period that the index is computed, while the ESE index reallocates annually. It can be argued that this makes the ESE a more realistic index for the average investor, since it is unlikely that an investor would generally reallocate his funds every period (especially if the index is constructed daily, or weekly) whereas he might reallocate annually. In any case, except for a very active trader, the yearly reallocation is probably closer to reality than daily or weekly reallocation. While on the subject of reallocation and reality, it is worth noting that the market capitalization type indices, which appear to be the most popular at the present time, are not very realistic in terms of mirroring the performance of the average investor. As Levy[3] points out, the average investor is much more likely to buy either an equal number of the securities he selects (i.e. the DJ type index) or an equal number of dollars (or rands) worth (as assumed by the UP, VL and ESE indices).

3 THE DATA

In order to examine the performance of the different methods of constructing stock market indices, it was decided to draw eight random samples of 50 securities each from the 203 securities quoted on The Johannesburg Stock Exchange for which data were available (over the period 22 March 1968 to 20 June 1975). Then, for each sample, five indices were constructed – one for each method of construction considered. Thus, 40 indices in all were constructed.

In order to analyze the performance of these indices, it was decided to break the data into four distinct periods:

(i) **General bull market:** 22 March 1968 to 30 April 1969
(ii) **General bear market:** 30 April 1969 to 5 November 1971
(iii) **Gold bull market:** 5 November 1971 to 5 April 1974
(iv) **Gold bear market:** 5 April 1974 to 24 January 1975

For each of the 40 indices, the percentage return on the index was computed for each of the four periods above, using the formula

$$\frac{I_t - I_o}{I_o} \times 100$$

where

I_t was the level of the index at the end of the period; and

I_o was the level at the beginning of the period.

These 160 values (returns on each of the 40 indices for each of the 4 types of markets) were used as the data for the analysis in the next section[4].

4 TESTS AND RESULTS

Initially, the data were analyzed by performing a three-way analysis of variance where the factors were:

(i) The method of construction (DJ, SP, UP, VL and ESE) which will be called the **index** effect.
(ii) The sample drawn (sample number 1 to sample number 8) – the **sample** effect.
(iii) The type of market (bull, bear, gold bull, gold bear) – the **market** effect.

It must be mentioned that in order to perform this analysis of variance, the returns on the various indices must be assumed to be normally distributed. However, while this assumption is questionable since it has been shown (Fama[5] and Affleck-Graves[6]) that the distributions appear to be Stable Paretian, it can nevertheless be argued that the data are not "too far removed" from normality (except for the tails of the distribution) and hence the analysis of variance should yield acceptable results.

The analysis of variance table is given in Table 1 below:

Table 1: Anova table for three factor model

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-ratio
Index	5 051,2	4	1 262,8	10,29**
Sample	2 537,5	7	362,5	3,53**
Market	1 046 708,4	3	348 902,8	1 486,59**
Index * sample	3 435,4	28	122,7	1,19
Index * market	14 285,2	12	1 190,4	11,58**
Sample * market	4 928,3	21	234,7	2,28**
Error[7]	8 636,7	84	102,8	
Total	1 085 582,7	159		

*Indicates significance at the 5% level
**Indicates significance at the 1% level

It should be noted that the three-factor model used above is a "mixed effects" model with the **"index"** and **"market"** effects being fixed and the **"sample"** effect being random. This accounts for the entries in the "F-ratio" column in Table 1.

On examining Table 1 it can be seen that all three main effects are significant at the 1% level. However, as the **index*market** and **sample*market** interactions are also significant, interpretation is rather difficult. Moreover, as the **index*sample** interaction is not significant (while both interactions with the **market** are significant) it was decided to analyze the data for each of the four types of markets separately. Thus, for each of the types of market, a two-way analysis of variance was performed with the two factors being **index** and **sample.** These results are presented in Tables 2 to 5 below.

Table 2: Anova table for general bull market

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-ratio
Index	162,3	4	40,6	2,72*
Sample	157,4	7	22,5	1,51
Error[8]	417,2	28	14,9	
Total	736,9	39		

Table 3: Anova table for general bear market

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-ratio
Index	1 563,1	4	390,8	45,44**
Sample	732,7	7	104,7	12,17**
Error	241,3	28	8,6	
Total	2 537,1	39		

Table 4: Anova table for gold bull market

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-ratio
Index	17 521,0	4	4 380,3	10,80**
Sample	6 295,8	7	899,4	2,22
Error	11 356,6	28	405,6	
Total	35 173,4	39		

Table 5: Anova table for gold bear market

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-ratio
Index	89,8	4	22,5	10,71**
Sample	279,9	7	40,0	19,05**
Error	57,3	28	2,1	
Total	427,0	39		

($F_{0,95;4;28}$ =2,71; $F_{0,95;7;28}$ =2,36; $F_{0,99;4;28}$ =4,07; $F_{0,99;7;28}$ =3,36)

Examination of Tables 2 to 5 reveals that the **index** effect is significant at the 5% level in all four types of markets and is significant at the 1% level in three of the four markets. The **sample** effect, however, is significant (at both the 1% and 5% level) in only two of the four markets examined. Also, with the exception of the "Gold Bear Market" (Table 5), the mean square for the **index** effect is considerably larger than the mean square for the **sample** effect. This is also true for these respective mean squares in Table 1 and therefore it is argued that the **index** effect, that is the method used in constructing the index, is a more important source of variation than the **sample** effect, that is the actual sample of securities chosen.

5 COMPARISONS OF THE DIFFERENT INDICES

In order to distinguish between the five different methods of constructing indices, it is necessary to perform pairwise multiple comparisons on the **index** effect. This was done using Tukey's method (c.f., for example, Miller[9]) and the results are summarized in Table 6 below.

Table 6: Pairwise comparison of the index effect

Type of market	Significant difference in means	Mean return on index DJ	SP	UP	VL	ESE	Result of comparison
General bull	5,62	21,25	22,62	24,25	21,37	26,62	No significant difference
General bear	4,28	−59,62	−61,12	−45,37	−54,37	−47,75	SP =DJ< VL< ESE =UP
Gold bull	29,34	175,63	168,13	154,38	115,88	162,88	VL< UP =ESE =SP =DJ
Gold bear	2,08	−29,75	−30,25	−27,12	−31,75	−29,37	VL< ESE< UP; SP =DJ< UP VL =SP =DJ and ESE =SP =DJ

The analysis of the results presented in Table 6 above, is fairly difficult and subjective since the results are not identical for each type of market. In addition, in the fourth type of market (the "Gold Bear Market") the results are somewhat contradictory – a problem which frequently occurs when doing pairwise comparisons. However, those results which it is felt are most important, are summarized below:

(i) Indices based on price (the DJ and SP type indices)

These indices are of interest because at present they are by far the most popular methods of constructing stock market indices. In all four types of market, no statistically significant difference in the performance of the indices constructed using those two methods, was detected.

(ii) Indices based on return (the UP, VL and ESE type indices)

On examining the results presented in Table 6 it can be seen that in the first three markets, no significant difference between the UP and ESE type indices was observed, while in the fourth type of market, the average return on the UP type indices was found to be greater than that on the ESE type indices. However, on studying the actual data and the means presented in Table 6, it was found that for all eight samples, in both **bull** markets (that is, markets 1 and 3) the ESE indices had higher return than the corresponding UP indices. Also, in the two **bear** markets, the ESE indices had lower return (that is, fell more) than the corresponding UP indices. Thus, since the ESE type indices appear to rise more in a bull market and fall more in a bear market, it can be argued that they are more volatile than the UP indices. Hence, if one equates volatility in an index with information (the more volatile an index, the more informative it is – Feeney and Hester[10]), then it would appear that the ESE type indices are more informative than the UP type indices.

Since the geometric mean will always be less than the arithmetic mean it follows that the VL type indices will always have lower return than the UP or ESE type indices (cf. for example, Marks and Stuart[11], Latané, Tuttle and Young[12]). Thus in a rising market the VL index will rise less than the other two indices while in a falling market the VL will fall more.

(iii) Return vs price indices

In recent years interest in indices based on return rather than price has grown in the United States with the United Press International Market Indicator and the Value Line index being fairly widely quoted. It is therefore of interest to investigate whether there is any difference in the performance of an index based on price as opposed to an index based on return. As has been argued in (ii) above, the Value Line forms a floor for all other return indices and therefore in the analysis below it is not considered. Thus, the performance of the DJ and SP type indices is contrasted with that of the UP and ESE type indices. This can be done by testing the null hypothesis[13] that

$$\mu_{DJ} + \mu_{SP} = \mu_{UP} + \mu_{ESE}$$

against the alternative of inequality, for each of the four markets examined, using an ordinary t-test (since only one comparison is being made for each type of market). The results are presented in Table 7 below.

Table 7: Comparison of DJ and SP with UP and ESE type indices[14]

Type of market	T	D
General bull	5,59	– 7,00*
General bear	4,25	– 27,62*
Gold bull	29,17	26,50
Gold bear	2,10	– 3,51*

From Table 7 it can be seen that the two bull markets provide conflicting results. In the general bull market a significant difference between the return and price indices was found. As the sign of D is negative this implies that the return indices rose more on average than the price indices. In the gold bull market, however, no significant difference was detected, but it should be noted that the sign of D changed to positive and was close to the critical value.

The results for the two bear markets are similar with the price indices falling significantly more on average than the return indices in both instances.

Since the results above appear to indicate that there is no consistent difference between return and price indices in bull markets and that the price indices fall more on average in bear markets, these results can be said to support the oft quoted remark that indices based on return "outperform" indices based on price because of an upward bias in the return indices (cf. for example, Latané, Tuttle and Young[15]). On the other hand, if one equates volatility with information then it would appear that the price indices are superior – they rise as much as return indices in bull markets and fall by more in bear markets and are hence more volatile. It should be noted that the above is a rather subjective remark and is not based on statistical significance.

6 CONCLUSIONS

In this paper an empirical study of some of the more popular methods of constructing stock market indices has been presented for various market conditions. A brief summary of the main conclusions which can be drawn from the above is presented below.

Firstly, the "type of market" has the most influence on the performance of an index, as is to be expected. The study revealed that for a particular type of market all methods of construction provided indices which moved in the same general direction.

Secondly, for a particular type of market, the method used in constructing the index was a more important factor than the actual sample of securities chosen. In view of this it is surprising to find on studying the historical development of the major indices that often the method of construction was fairly arbitrarily

chosen while a great deal of research was devoted to the selection of the constituent securities. This result should be carefully studied by anyone contemplating construction of a new stock market index.

Thirdly, no significant difference in the performance of the two indices based on price was detected. In fact, in the 32 cases examined (eight samples in each of four markets), the DJ (arithmetic average) type indices performed better (in terms of volatility) than the SP (market capitalization) type indices in 15 cases and vice versa in 16 cases, with one case producing identical performance. Thus there does not appear to be any empirical evidence to support preference for either of these two methods over the other.

Fourthly, study of the indices based on return reveals that the VL (geometric average) type indices are always at a lower level than either the UP (arithmetic average) or ESE (arithmetic average with yearly reallocation) type indices thus forming a "floor" for the other return indices. While this can be a useful index in certain circumstances this property is unlikely to make it popular with the average investor. It appears as if the ESE type indices are superior to the UP type indices in performance and, as the ESE indices are probably closer to the behaviour of the average investor than the UP type indices (as discussed in Section 2), it is argued that the ESE type indices are the most suitable of the three return indices examined.

Finally, comparison of the price and return indices reveals that the former are possibly slightly more volatile and hence could be considered preferable. However, this difference is not very marked (especially as far as the ESE type indices are concerned) and the individual investor should choose the index which most clearly matches his investment strategy – DJ type if he purchases equal numbers of each security; SP if a large investor and concerned about "availability"; and ESE (or UP) if equal monetary amounts are invested in each security.

In conclusion, it can be said that South African investors are fairly well catered for with the RDM indices (market capitalization (SP) indices) catering for those investors with a preference for a price index while the ESE indices are probably the most suitable for those investors requiring an index based on return.

Footnotes

1 The authors wish to thank the Council for Scientific and Industrial Research for their financial support.

2 Cohen, K. J. and Fitch, B. P. (1966): The Average Investment Performance Index. Management Science, Vol. 12, pp 195-215.

3 Levy, R. A. (1968): The Relative Strength Concept of Common Stock Price Forecasting. Investors Intelligence, N.Y.

4 Listings of this set of data are available from the authors as are lists of the actual 50 securities chosen for each of the eight samples and the population of 203 securities.

5 Fama, E. F. (1965): The Behaviour of Stock Market Prices. Journal of Business, Vol. 38, pp 34-105.

6 Affleck-Graves, J. F. (1974): Portfolio Selection on The Johannesburg Stock Exchange. Unpublished M.Sc. thesis, University of Cape Town.

7 Since there is only one observation per cell it is not possible to check for a three factor interaction.

8 Since there is only one observation per cell it is not possible to check whether the **index*sample** interaction is still zero.

9 Miller, R. G. (1966): Simultaneous Statistical Inference. McGraw-Hill, N.Y.

10 Feeney, G. J. and Hester, D. D. (1967): Stock Market Indices: A Principal Components' Analysis. In Hester, D. D. and Tobin, J.: Risk Aversion and Portfolio Choice. John Wiley and Sons, N.Y.

11 Marks, P. and Stuart, A. (1971): An Arithmetic Version of the Financial Times Industrial Ordinary Share Index. Journal of the Institute of Actuaries, Vol. 97, pp 297-324.

12 Latané, H. A., Tuttle, D. L. and Young, W. E. (1971): Market Indexes and their Implications for Portfolio Management. Financial Analysts Journal, Vol. 27, pp 75-85.

13 μ_{DJ} denotes the true mean of the returns on the DJ type indices, with μ_{SP}, μ_{UP} and μ_{ESE} being similarly defined.

14 $T = t_{28;0,975}(4*MS_E/8)^{\frac{1}{2}}$

$D = (\overline{DJ} + \overline{SP}) - (\overline{UP} + \overline{ESE})$ where $\overline{DJ}$ is the average of the returns on the DJ type index over all samples and $\overline{SP}$, $\overline{UP}$ and $\overline{ESE}$ are similarly defined.

*Indicates a significant difference between the price and return indices – if $|D| > T$.

15 Latané, Tuttle and Young, op. cit.