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The market model and The Johannesburg Stock Exchange

ABSTRACT

The mining sector and the industrial sector each comprise a significant proportion of the total market capitalisation of The Johannesburg Stock Exchange. The factors affecting the fortunes of these two sectors can be quite different over extended periods of time and this should be taken into account in "market model" analyses of share price behaviour. This is quite easily done via multiple regression analysis. Although portfolio analysis based on the resulting share risk parameters is slightly more complicated than that conventionally performed, its results are intuitively acceptable and perhaps of greater value in practical applications.

1. CONCEPTS

Modern portfolio theory ("MPT") has provided important concepts to facilitate the analysis of investment portfolios. One of its more important constructs – the so-called "market model" – seeks to explain realised security returns by postulating a linear relationship with realised returns on "the market", viz:

$$R_j = \alpha_j + \beta_j.R_m + e_j \quad (1)$$

where

- R_j, R_m are the realised returns on share j and "the market" respectively
- α_j, β_j are constants
- e_j is a random variable uncorrelated with R_m and having a (Gaussian) distribution with zero expected value

The assumption underlying equation (1) is that all shares are affected to a greater or lesser extent by a common "underlying factor", typically developments in the "overall market". In "boom-times", most companies tend to do well and their share prices will tend to rise; the opposite will tend to occur in "depressed times". Of course, developments specific to a particular company may cause its shares either to move against such broad economic trends or to emphasise them. In equation (1), such specific factors can be accounted for by the random disturbance term, e_j.

A further notion from MPT is that the "risk" associated with any investment might be conceptualised as the likelihood that the return ultimately realised will be less than expected. It is typically assumed that the distribution of possible future returns is Gaussian and hence a convenient measure of risk is the variance (σ^2) or the standard deviation (σ) of the distribution. The overall prospects for the investment are then defined completely by two distribution parameters, namely the expected return, $E(R_j)$, and the standard deviation, σ.

The assumed independence of R_m and e_j in equation (1) permits the variance of the investment returns to be partitioned into a "market-related" component and a "non-market-related" component, viz:

$$\sigma^2(R_j) = \beta^2_j.\sigma^2(R_m) + \sigma^2(e_j) \quad (2)$$

The second quantity on the right-hand side, $\sigma^2(e_j)$, quantifies that part of the total share risk, $\sigma^2(R_j)$, that arises from factors unrelated to changes in the overall market, and hence specific to the share concerned. The quantity $\sigma(e_j)$ is accordingly termed "specific risk". The beta-coefficient, β_j, on the other hand, provides a measure of that part of the share return that is perfectly correlated with the market return. The risk arising from this relationship, $\beta_j.\sigma(R_m)$, is termed the "systematic risk".

The concepts set out above have important practical applications in portfolio analysis. Given the availability of three share risk parameters – α_j, β_j and $\sigma(e_j)$ – for each share comprising the universe of feasible investments, it is possible to estimate the risk properties of any portfolio constructed from that universe. The risk parameters for the individual shares are usually generated by regressing a large number of historical share returns on the corresponding market returns and assuming that the resulting statistics provide a good estimate of the risk parameters for the following (future) period. Typical examples of such parameters are given in Table 1A. (These estimates were generated by regressing the realised weekly share returns on the realised JSE Actuaries All Market Index returns.)

2. A PROBLEM PECULIAR TO THE SOUTH AFRICAN SITUATION

It is questionable whether the conventional market model formulation of equation (1) provides a satisfactory explanation of share returns in the South African context. Unlike most overseas stock markets, The Johannesburg Stock Exchange ("the JSE") is distinguished by the fact that mining companies – mainly gold but also diamonds – comprise a major proportion of the total market capitalisation. The fortunes of the gold mining companies in particular depend critically on a gold price that is established by international political and economic events that are frequently quite divorced from developments in the local South African economy. Consequently, it is not unreasonable to assume that the returns of mining shares on the one hand and industrial shares on the other, will, at times, be influenced by different "underlying factors". Consequently, there is a prima facie case for reformulating the market model so as to incorporate at least a "mining factor" and an "industrial factor".

The existence of "two factors" would account for some of the observed properties of conventionally estimated share risk parameters (i.e. as estimated against a single "market factor"). Table 2 records the findings of Campbell[1] that the beta-coefficient of the JSE Industrial Sector (as estimated against the JSE Actuaries All Shares Index) declined dramatically from the mid-1960s to the mid-1970s; the reverse was true of the JSE Gold Sector. This is exactly the kind of behaviour that would have been expected in a situation where the returns over the period from one sector (Gold) had been greatly in excess of the returns from the other sector (Industrial) due to a fundamental structural change in the underlying economic factors (the establishment of a two-tier pricing system in 1968 and the end of dollar convertibility into gold in 1971). It is clearly desirable that the effects of such different factors be separated if at all possible.

TABLE 1
Share risk statistics

Share code	Market model	Specific risk % pw	Correlation coefficient
Table 1A Conventional one-factor risk measurements			
EDR	$R_{EDR} = -0{,}006 + 1{,}773\ R_{AS}$ (9,29)	4,09	0,731
AAC	$R_{AAC} = -0{,}001 + 1{,}297\ R_{AS}$ (11,50)	2,42	0,799
BAR	$R_{BAR} = +0{,}003 + 0{,}784\ R_{AS}$ (5,33)	3,15	0,524
Table 1B Two-factor risk measurements			
EDR	$R_{EDR} = -0{,}002 + 1{,}655\ R_{MI} - 0{,}302\ R_{IN}$ (11,15) (–1,39)	3,6	0,806
AAC	$R_{AAC} = +0{,}000 + 0{,}763\ R_{MI} + 0{,}436\ R_{IN}$ (6,47) (2,53)	2,9	0,713
BAR	$R_{BAR} = -0{,}001 + 0{,}060\ R_{MI} + 1{,}197\ R_{IN}$ (0,55) (7,42)	2,7	0,701

EDR = East Driefontein Gold Mining Company Limited
AAC = Anglo American Corporation of South Africa Limited
BAR = Barlow Rand Limited
σ_{AS} = 0,02459
σ_{MI} = 0,03063
σ_{IN} = 0,02096
σ_{EDR} = 0,05959
σ_{AAC} = 0,03993
σ_{BAR} = 0,03678
$\rho_{MI,IN}$ = 0,44499
R_{AS} = Return on Actuaries All Shares Index
R_{MI} = Return on Actuaries All Mining Index
R_{IN} = Return on Actuaries Industrial Index

TABLE 2
Beta-coefficients of major JSE Sectors

Index	Beta estimation period			
	1964 to 1968		1974 to 1978	
	β	SE(β)	β	SE(β)
Industrial	0,977	0,121	0,532	0,079
Gold	0,820	0,126	1,411	0,089

Source: Campbell G. (Ref. 1).

3. SHARE RISK MEASUREMENTS BASED ON TWO FACTORS

It is, fortunately, a relatively simple matter to generalise the "conventional market model" of equation (1) to a situation involving a multiplicity of factors (see for example Sharpe[2]). For simplicity, we restrict our attention to only the two factors identified in Section 2 above and hence we may reformulate the market model as:

$$R_j = \alpha_j + \beta_{MI,j} . R_{MI} + \beta_{IN,j} . R_{IN} + e_j \qquad (3)$$

where

R_{MI}, R_{IN} are the returns generated by the JSE Actuaries Mining and Industrial Indexes respectively

and

$\beta_{MI,j}$, $\beta_{IN,j}$ are constants relating to share j

The desired risk parameters α_j, $\beta_{MI,j}$, $\beta_{IN,j}$ and $\sigma^2(e_j)$ are conveniently estimated via multiple regression analysis, which analysis is quite similar in concept to the simple linear regression model described in Section 1 above.

Table 1B summarises the results of such a multiple regression analysis on the three shares previously analysed in the conventional manner. The results of this new analysis are intuitively appealing. The returns on the gold mining share EDR depend mainly on the returns generated by the "underlying mining factor", R_{MI}; the contribution from R_{IN} is not statistically significant*. The converse is true of the (mainly) industrial share BAR. Finally, the analysis of AAC reveals its important dependence on both mining and industrial factors, and both of these factors are statistically significant determinents of its share returns.

Generalisations of the usual portfolio relationships may be used to quantify the risk profile of the resulting portfolios, namely

$$E(R_p) = \sum_i x_i . E(R_i) \qquad (4)$$

$$\alpha_p = \sum_i x_i . \alpha_i \qquad (5)$$

$$\beta_{MI} = \sum_i x_i . \beta_{MI,i} \qquad (6)$$

$$\beta_{IN} = \sum_i x_i . \beta_{IN,i} \qquad (7)$$

$$\sigma_p = \sum_i \sum_j x_i . x_j . \rho_{ij} . \sigma_i . \sigma_j \qquad (8)$$

where x_j is the proportion of the portfolio funds that is invested in asset i, and ρ_{ij} denotes the correlation between the expected returns on securities i and j.

If it can be assumed that

- the correlation between the residual terms e_i and e_j is zero for every pair of securities i and j (frequently a good approximation),
- the correlation between $R_{MI,j}$ and e_j is zero and between $R_{IN,j}$ and e_j is zero (frequently a good approximation), and
- the correlation between R_{MI} and R_{IN} is zero,

then we may write equation (8) as

$$\sigma_p^2 = \beta^2_{MI} . \sigma^2_{MI} + \beta^2_{IN} . \sigma^2_{IN} + \sum_j x_j^2 . \sigma^2(e_j) \qquad (9)$$

For a portfolio comprised of investments of 20%, 30% and 50% in EDR, AAC and BAR respectively, we may thus compute

$\alpha_p = -0{,}001$
$\beta_{MI} = 0{,}590$
$\beta_{IN} = 0{,}669$

and thus we would *expect* the return on such a portfolio to be determined as follows:

$$E(R_p) = -0{,}001 + 0{,}590\ E(R_{MI}) + 0{,}669\ E(R_{IN}) \qquad (10)$$

where E denotes the expectations operator.

Quantification of the expected return from this portfolio thus requires the portfolio manager to take a view on the outlook for each of R_{MI} and R_{IN} individually, rather than a single estimate for the composite index R_{AS} as would be the case if the conventional "single factor" market model were used. This is probably a more realistic procedure.

*For a large sample size, /t/ >1,96 indicates statistical significance at the 95% confidence level.

In addition to the expected return, the portfolio manager will wish also to determine the level of risk. From equation (9) – thus making the implied assumptions** – we estimate the total risk of portfolio as follows:

$\sigma^2_p \quad = (5{,}232 \times 10^{-4}) + (3{,}098 \times 10^{-4})$

and so the specific risk of the portfolio,

$\sigma^2(e_p) \quad = 3{,}098 \times 10^{-4}$

or

$\sigma(e_p) \quad = 1{,}76\%$

Hence the portfolio manager should recognise that, even given particular returns, R_{MI} and R_{IN}, on the two indexes, there is one chance in six*** that his portfolio return might be 1,76% or more lower than the return derived via equation (10).

4. CONCLUSION

The mining and industrial sectors each comprise a significant proportion of the total market capitalisation of the JSE. In addition, it seems that the fortunes of the two sectors can at times be influenced by quite different factors. It therefore appears desirable that "market model" analyses of share price behaviour should take at least "two factors" into account. This is quite easily done via multiple regression analysis. Although portfolio analysis based on the resulting share risk parameters is slightly more complicated than that conventionally performed, its results are intuitively acceptable and probably of greater value in the practical situation.

**In fact, in this particular analysis, the correlation coefficient between R_{MI} and R_{IN} was found to be not zero but 0,445 (see Table 1) so that an additional term should be added into the computation of equation (9). When this is done it is found that $\sigma^2_p = 9{,}46 \times 10^{-4}$ and not $8{,}33 \times 10^{-4}$ as estimated below.

***From the properties of the standardised Gaussian distribution, the probability of an outcome more than one standard deviation below the mean is 0,1587, or approximately one-sixth.

REFERENCES

1. Campbell, G. Risk and Return on The Johannesburg Stock Exchange. Unpublished MBA dissertation, Graduate School of Business Administration, University of the Witwatersrand, 1979.
2. Sharpe, W. F. Portfolio Theory and Capital Markets. McGraw-Hill, 1970, page 122ff.