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Investment basics XV
The role of share risk measurements in the management of investment portfolios

Introduction

Modern finance theory has provided new tools to assist portfolio managers in their investment decisions. The purpose of this article is to show how these tools might be used in the practical situation to quantify:

(a) the prospects of any particular portfolio;

(b) the trade-off between expected return and risk that has to be made when the composition of the portfolio is determined or changed.

These applications are illustrated using shares listed on The Johannesburg Stock Exchange.

Theory

Investment analysts and portfolio managers are concerned with the probable future performance of their investments. Since the future cannot be known with certainty, the possible future returns must be expressed in probabilistic terms. Following Markowitz (1, 2), it is now widely accepted that:

- the *expected return* on any given investment is the expected value of the probability distribution of possible outcomes, and
- the investment *risk* is the probability that a particular investment will yield less than the expected value.

Provided that the distributions of possible future returns are approximately Gaussian, it follows that investment risk can be identified with the standard deviation of the distribution of returns. (This parameter is usually denoted by the symbol σ.)

This *total investment risk* of a particular share may be split into two components, namely the *systematic risk* and the *unsystematic risk*. The former is closely related to the beta factor which specifies how a share responds to general market movements. Specifically, for share j:

$$(\textit{Systematic risk})_j = \beta_j.\sigma_m \qquad (1)$$

where β_j = "beta coefficient" of share j

σ_m = standard deviation of the distribution of returns on the overall market.

If over a given period the market generates an actual return of 10%, one would *expect* a share with a beta greater than one to achieve an even greater return – say 13% – and vice versa. This *expected* return can be precisely quantified via the Capital Asset Pricing Model. (See equation 5 below.)

In practice, however, share prices do not behave exactly in accordance with the beta factor; hence one would not be too suprised if, in the above example, the actual share return over the period was not 13% but 15% or 10% or perhaps even negative. This residual variation is independent of the overall market behaviour and arises from factors unique to the company in question, such as labour difficulties, the introduction of a new product, a change in the management structure and so on. This component of investment risk is called the *unsystematic risk.*

Consider now what happens when individual shares are combined into a portfolio. The expected return on the portfolio is then equal to the weighted average of the expected returns of the component securities, thus:

$$E(R_p) = \sum_i x_i.E(R_i) \qquad (2)$$

where E = expectations operator

R_p = return on the portfolio

x_i = proportion of portfolio invested in security i and

R_i = return associated with security i.

The beta coefficient of the portfolio is also determined quite simply as the weighted average of the component beta's, thus:

$$\beta_p = \sum_j x_j.\beta_j \qquad (3)$$

The *total risk* of such a portfolio is somewhat more complicated; while the prices of some shares in the portfolio will be rising, others might be falling; such "opposing movements" can materially reduce the standard deviation of the portfolio returns. (This is hardly new to professional portfolio managers who have long been aware of the benefits of diversification.) The variance of the portfolio's rate of return is then given by the expression:

$$\sigma^2 P = \sum_i \sum_j x_i.x_j.\ \rho ij.\ \sigma_i.\ \sigma_j \qquad (4)$$

where ρij = correlation coefficient between the returns of securities i and j and

σ_i = standard deviation of the distribution of returns of security i.

The *total risk* of the portfolio – ie the standard deviation of the portfolio returns – is then simply the square root of the variance σ_p^2. Equation (4) shows that this risk depends on the interrelationship between the security returns via the correlation coefficient ρ_{ij}. Of course, this total risk can still be decomposed into *systematic* (market related) and *unsystematic* components.

Unsystematic risk can be virtually eliminated by holding a well-diversified portfolio and modern investment theory holds that investors should therefore not *expect* to earn additional returns for bearing such risk. Hence only the *systematic* component of investment risk is taken into account in the pricing of capital assets. This principle is formalised in the well-known Capital Asset Pricing Model ("CAPM") which states that:

$$E(R_j) = R_F+\beta_j\left[E(R_m)-R_F\right] \quad (5)$$

Where $E(R_j)$ = Expected return on any share j

R_F = Return on a "risk-free" asset

$E(R_m)$ = Expected return on the "overall market"

β_j = "Beta coefficient" of the share j.

With some manipulation it is possible to re-write equation (5) in the form:

$$E(R_j/R_m) = R_F+\beta_j\left[R_m-R_F\right] \quad (6)$$

where $E(R_j/R_m)$ = the expected return on the share j conditional upon an actual market return R_m having been realised.

This expression allows the return actually realised on a security to be compared with the return that might have been expected given a particular return on the market.

We have now stated all of the relationships needed by portfolio managers to quantify the prospects of their portfolios. (A more extensive discussion with examples drawn from the JSE is given in reference 3.) In order to utilise these relationships, one needs only the risk-estimates for the individual shares; these are available from professional share risk measurement services for virtually every stock market in the world.

An illustrative example

We shall now illustrate the above concepts by means of simple examples. To do so we shall utilise the share risk measurements summarised in table 1 below; these parameters were taken from a 1979 issue of the *Risk-inform* service of the Graduate School of Business Administration of the University of the Witwatersrand. The shares were deliberately chosen to emphasise differences between two theoretical portfolios.

Table 1: Summary of share risk parameters and other data. (Measured against RDM-100 index)

Share	Code	Total Risk (% p.a.)	Specific Risk (% p.a.)	Beta	Price in cents on 790105	Price in cents on 790629
Placor	PLT	54,2	52,2	1,14	60	65
Utico	UTI	44,3	41,2	1,44	105	130
Amrel	AMR	44,5	36,4	1,97	160	260
Chloride	CLO	16,0	14,1	0,52	380	500
ICS	ICS	26,2	18,9	1,48	255	212
Asseng	ASE	19,7	17,1	0,65	345	270
RDM-100		12,50			274,7	309,1

Consider the properties of a portfolio *(portfolio 1)* comprised of an equal investment in each of the first three shares. The following parameters may be estimated for *portfolio 1*:

By equation (3)

$$\text{Portfolio } \beta_{P1} = (⅓ \times 1,14) + (⅓ \times 1,44) + (⅓ \times 1,97) = 1,52 \quad (7)$$

If the choice of shares is not restricted excessively to a limited number of market sectors (ie if we are considering generally well diversified portfolios) then the correlation coefficients between the "residual share returns" (ie the return remaining after removal of the *systematic* component) will generally be low. If we assume these coefficients to equal zero, then equation 4 reduces to:

$$\sigma^2(\rho) = \sum_1 x_i^2.\sigma_i^2. \quad (8)$$

Hence for *portfolio 1* we can write:

$$(\textit{Unsystematic risk})_{P1} = \left[\left(\frac{52,2}{3}\right)^2 + \left(\frac{41,2}{3}\right)^2 + \left(\frac{36,4}{3}\right)^2\right]^{½} = 25,3\% \text{ p.a.} \quad (9)$$

By equation (1) and (7) we have:

$$(\textit{Systematic risk})_{P1} = \beta_{P1}\times\sigma_m = 1,52\times12,5 = 19,0\% \text{ p.a.}$$

It follows now that the total risk of *portfolio 1* is equal to the sum of the *systematic* and *unsystematic* components, thus:

$$(\textit{Total risk})_{P1} = \left[(25,3)^2+(19,0)^2\right]^{½} = 31,6\% \text{ p.a.} \quad (10)$$

A perfectly diversified portfolio would have no remaining *unsystematic risk*, ie all of the risk associated with the portfolio would be *systematic*. Hence we may estimate the degree of diversification of the portfolio as the ratio of the *systematic risk* to the *total risk*, thus:

$$\text{Degree of diversification} = (19,0)^2/(31,6)^2 = 36\%. \quad (11)$$

Since *unsystematic risk* contributes 100%−36%=64% of the *total risk*, the portfolio is not very well diversified. (This is of course to be expected from a portfolio containing only 3 shares.) Whether such poor diversification is "good" or "bad" depends entirely on how well the portfolio manager is able to pick "underpriced" securities. If his record is not good or he believes the market to be efficient, then he should seek to increase the degree of diversification (probably to well above 95%).

His past performance may be instructive in making such decisions. Over the previous 6-month period he has in fact achieved the following capital gain:

Performance of *portfolio 1* (12)

$$1 = \left[⅓\times\left(\frac{65}{60}-1\right)\right] + \left[⅓\times\left(\frac{130}{105}-1\right)\right] + \left[⅓\times\left(\frac{260}{160}-1\right)\right] = 31,5\%$$

Over the same period, the market has achieved the following gain:

$$\text{Performance of Index} = (309,1)/(274,7)-1 = 12,5\% \quad (13)$$

We know from equation 6 that, given the market return of

(13), one would have expected from *portfolio 1* the following return:

$$\text{Benchmark return for } \textit{portfolio 1} = \frac{11\%^*}{2} + 1{,}52 \times \left[12{,}5\% - \frac{11\%^*}{2}\right]$$

$$= 16\% \qquad (14)$$

Thus the return of nearly 32% actually realised by *portfolio 1* is greatly in excess of the 16% that was expected. This would indicate that the portfolio manager might be good at picking securities and hence that his chosen degree of (low) diversification is justified. (Efficient market theorists would contend conversely that he had been lucky and shouldn't expect to achieve the same superior performance in the future.)

Provided that the portfolio manager can quantify his expectations about the likely performance of the market over the next one-year period, he will also be able to specify the *expected* return on *portfolio 1* via the CAPM of equation 5. Assume that he expects the market to rise by 20% overall. Then:

Expected return on *portfolio 1*
over next year = 11%+1,52 (20%−11%)
= 24,7%.

However, although he may *expect* this performance, he cannot be sure of achieving it. Therefore he will want to quantify the likelihood of not achieving this return, ie his "downside potential". Given that the portfolio returns have an approximately Gaussian distribution, there is a probability of nearly 16%† (ie about one chance in six) that the realised returns will be lower by one standard deviation than the expected value. Hence the portfolio manager should expect that in about 1 year out of 6 his portfolio:

- will "underperform" the expected value by the *total risk* parameter and further, with the same frequency,
- will "underperform" the market by the value of the *unsystematic risk* parameter.

Hence we may write the following estimates:

Total downside potential = 31,6% p.a.
Potential below index = 25,3% p.a.

(Similar computations may be made for the "upside potential". When computational precision is required it is preferable to promote Gaussian distributions by working with continuously compounded returns when estimating the share risk statistics.)

Share risk measurements thus allow the portfolio manager to estimate via simple calculations the probable performance of his portfolio, as well as the likelihood of him underperforming or overperforming his target. Such quantification is of assistance when risk-return trade-offs have to be made between alternative portfolio compositions. To illustrate this further, consider an alternative investment set, *portfolio 2*, which is comprised of investments in the last 3 shares of table 1 in the proportions ¼ to ½ to ¼ respectively. Table 2 below compares the various parameters for *portfolio 1* and *2.*

Table 2: Comparison of alternative portfolios

Parameter	Portfolio 1	Portfolio 2
Beta coefficient	1,52	1,03
Unsystematic risk	25,3% p.a.	10,9% p.a.
Systematic risk	19,0% p.a.	12,9% p.a.
Total risk	31,6% p.a.	16,9% p.a.
Degree of diversification	36%	58%
Past performance	31,5%	−6%
Benchmark return	16,1%	12,7%
Expected return next year	24,7%	20,3%
Total downside potential	31,6%	16,9%
Potential below index	25,3%	12,9%

The choice facing the portfolio manager has been clearly quantified: He can choose *portfolio 1* with an expected return that is a full 4,4% p.a. greater than that of *portfolio 2.* However, in so-doing his downside potential will be greatly increased, in fact, virtually doubled. His choice will depend on his attitude to risk, i.e. on how he makes the risk-return trade-off. Share risk measurements do not provide guidelines in this regard – they merely enable the decision to be made on a quantitative basis.

Conclusions

This note has outlined a simple methodology for quantifying portfolio alternatives in a risk-return framework. Examples drawn from JSE-listed shares were used to illustrate the technique. The final investment decision by the portfolio manager must ultimately depend on his personal attitude to risk, but the likely consequences of his decision can now be expressed in quite specific terms.

References

1. Markowitz H. M. Portfolio selection. *Journal of Finance,* Vol. 7, March 1952, pp 77–91.
2. Markowitz H. M. *Portfolio Selection: Efficient Diversification of Investments.* New York: John Wiley and Sons, 1959.
3. Gilbertson B. P. The pricing of industrial shares on The Johannesburg Stock Exchange. *The Investment Analysts Journal,* No. 14, September 1979, pp 21–36.

* In this equation we have used the yield on the debentures of AAA-rated companies as a measure of the risk-free rate. Division by a factor of 2 allows for the fact that we are considering a 6-month period only.

† Actually 15,8%; see any set of tables of the standardised Gaussian distribution.