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Investment basics XX
Risk and return – Part 3

Introduction

It is appropriate at this stage to recall some of the key points that were dealt with in Parts 1 and 2 of this series:

(a) Investors prefer higher to lower returns.

(b) Investors prefer lower to higher risk, risk being measured in terms of the standard deviation of returns.

(c) Diversification can, and normally will, reduce the risk of a portfolio. For example, if the rates of return of two securities have the same expected value and are independent, it can be shown that a portfolio of both securities in suitable proportions will have a lower risk than a portfolio consisting solely of one or other of the securities.

(d) The returns of individual securities are, in general, positively but not perfectly correlated, owing to the operation of broad market forces that affect them similarly to a greater or lesser extent.

(e) The total risk of a security may be broken down into market risk, which reflects the way in which the security's returns move in sympathy with those of other securities, and specific risk, which reflects the influence of factors specific to the security.

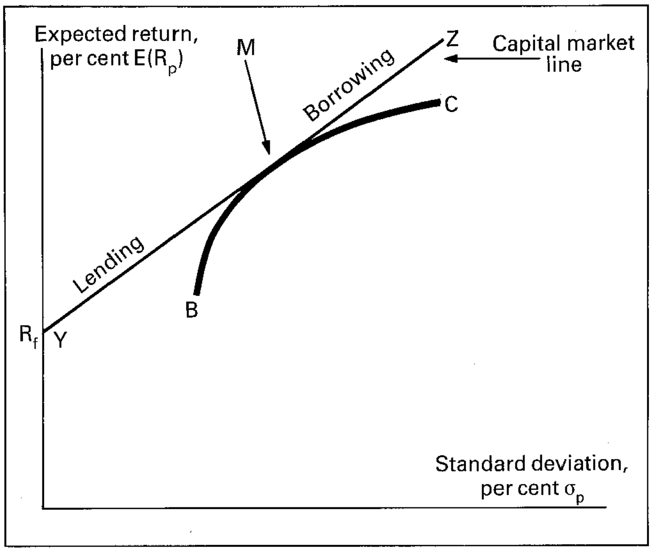
(f) The risk-reducing properties of diversification have to do with the reduction of specific risk. A portfolio of 15 to 20 securities will normally reduce specific risk to negligible proportions. Clearly, however, diversification cannot reduce market risk since this arises from the fact that the securities are subject to a process of simultaneous valuation in a market characterised by continually changing expectations.

The capital market line

The foregoing considerations imply, as is intuitively obvious, that there must be a trade-off between risk and return in any rational process of asset valuation. Investors will clearly require a higher return from a risky portfolio of equities than they would from a risk-free government security, such as a treasury bill. This trade-off is illustrated in Figure 3. In Figure 3, the curve BMC represents the efficient frontier of portfolios discussed in Part 2 of this series, in which it was stated that no point along the curve was superior to any other. In the presence of a risk-free security, that conclusion no longer holds, as may be seen from the line YZ which passes through the risk-free rate and is tangential to the curve BMC at M. The line YZ, in fact, represents the new efficient frontier. At point M, it gives the same combination of risk and return as equity portfolio M, but all other points on the line YZ plainly yield a higher return for any level of risk than is obtainable along the curve BMC.

What can be said about the nature of portfolio M? In efficient markets the dissemination of information is swift and no one can be consistently, or even usually, in the position of having superior access to valuable information. If, however, each investor normally has only the same information as everybody else, equilibrium

Figure 3: The capital market line



requires that the tangential portfolio, M, is a portfolio in which all equities are held according to their market value weights. Thus, equilibrium is not reached until the portfolio M is the market portfolio.

Any portfolio along the efficient frontier YZ consists of some combination of the risk-free asset and the market portfolio M. Assume that the proportion invested in M is w, the proportion invested in the risk-free asset therefore being $1 - w$. Let $E(R_m)$ be the expected return from R1 invested in the market portfolio and σ_m the standard deviation of expected returns on this investment. Let R_f be the risk-free rate, $E(R_p)$ the expected return on any portfolio along YZ and σ_p the standard deviation of expected returns for that portfolio. Then:

$$E(R_p) = (1 - w)\,R_f + wE(R_m) \qquad (9)$$

$$= R_f + w[E(R_m) - R_f] \qquad (10)$$

and

$$\sigma_p = w\sigma_m \qquad (11)$$

Solving equation (11) for w and substituting in equation (10), gives the equation of the capital market line, namely:

$$E(R_p) = \frac{R_f + [E(R_m) - R_f]\sigma_p}{\sigma_m} \qquad (12)$$

If the investor were to set w equal to 0, his entire portfolio would consist of risk-free assets having a zero standard deviation. In this case $E(R_p)$ would be equal to R_f and σ_p to zero, as represented by point Y in Figure 3. If the investor were to set w equal to 1, his entire portfolio would consist of the market portfolio: hence $E(R_p)$ would equal $E(R_m)$ and σ_p would equal σ_m, as represented by point M in Figure 3. Any value of w between 0 and 1 would give rise to a risk-return combination along YM. Points to the right of M along MZ correspond to values of w greater than 1. To reach such points the investor would have to borrow at the rate. His choice of any particular portfolio along YZ will depend upon his particular risk preference.

The security market line

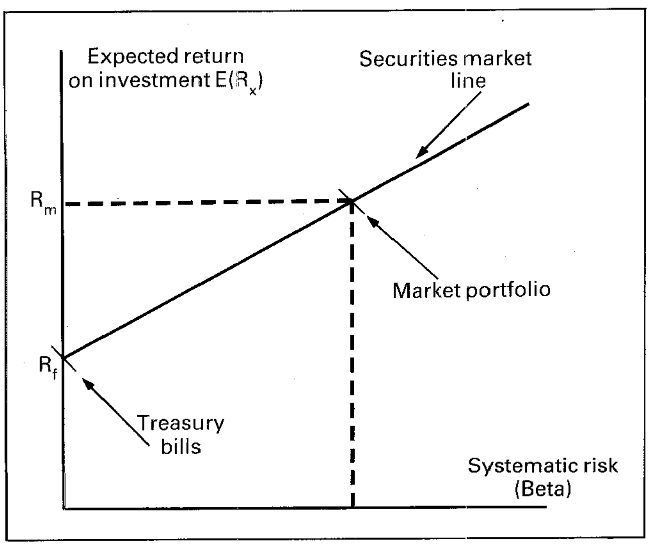
The capital market line represents the relationship between the expected return and risk that investors could achieve by varying the proportions of the riskless asset and market portfolio in their overall portfolios. Investors may attempt, however, to achieve a better combination of risk and return by departing from the market portfolio, ie, by holding more or less of a particular equity than is represented in the market portfolio. If it were possible to obtain a superior risk-return relationship by this means, the market would not be in equilibrium as investors would seek to exploit the perceived opportunity. This brings us to the question of the expected return for a particular risky security and to the basis on which individual risky securities are valued.

In the general context of risk-return equilibrium, the valuation of the individual risky security is necessarily concerned with its expected return and its risk. For such a security, however, the relevant risk is not the standard deviation of the security's returns, ie its total risk, but the marginal effect the security has on the standard deviation of an efficiently diversified portfolio, such as the market portfolio. This marginal effect is measured only by the systematic or market risk of the security, because the other component of the security's total risk, namely its unsystematic or unique risk, tends to become negligibly small in well-diversified portfolios. The difference between the expected return on the market and the risk-free rate may be termed the market risk premium. The risk premium for the individual risky security, X, is therefore some factor, β or beta, times the market risk premium, $E(R_m) - R_f$, and the total expected return on the security is the sum of that product and the risk-free rate:

$$E(R_x) = R_f + \beta[E(R_m) - R_f] \quad (13)$$

This concept is illustrated in Figure 4. Note that the beta of the market portfolio is unity, as may be seen by writing $E(R_m)$ for $E(R_x)$ in the above equation. Note also that all securities must plot along the sloping line, known as the security market line, if the market is to be in equilibrium. Were this not the case, certain risky securities would offer superior or inferior risk-return combinations and their prices would be adjusted by investors until equilibrium had been restored.

Figure 4: The securities market line



Next we need to define beta, which is best done by means of a simple numerical example. It should be noted that the example relates to historical returns on security X and on the market portfolio. This is because, although we are interested in expected returns, we must perforce use historical data as an imperfect surrogate for future risk-return relationships.

Table 5: The calculation of beta

Period	(1) (R_m-R_f)	(2) (R_x-R_f)	(3) Col 1 average of Col 1	(4) Col 3^2	(5) Col 2 average of Col 2	(6) Col 3 × Col 5
1	– 0,03	0,02	– 0,08	0,006	– 0,04	0,0032
2	0,24	0,29	0,19	0,036	0,23	0,0437
3	– 0,11	– 0,16	– 0,16	0,026	– 0,22	0,0352
4	0,20	0,16	0,15	0,023	0,10	0,0150
5	0,13	0,18	0,08	0,006	0,12	0,0096
6	0,09	0,14	0,04	0,002	0,08	0,0032
7	– 0,14	– 0,11	– 0,19	0,036	– 0,17	0,0323
8	0,20	0,18	0,15	0,023	0,12	0,0180
9	0,06	0,11	0,01	0,000	0,05	0,0005
10	– 0,14	– 0,20	– 0,19	0,036	– 0,26	0,0494
Averages	0,050	0,061				

Variance $(R_m - R_f) = 0{,}0194$

Covariance $(R_m - R_f, R_x - R_f) = 0{,}0210$

Given the covariance and the variance for excess market returns, security X's beta is:

$$\beta_x = \mathrm{Cov}(R_m - R_f, R_x - R_f)/\mathrm{Var}(R_m - R_f) \quad (14)$$
$$= 0{,}0210/0{,}0914$$
$$= 1{,}083$$

If $\beta_x = 1$, security X would have exactly the same risk as the market portfolio. In fact, $\beta_x = 1{,}083$, which indicates that security X is somewhat riskier than the market portfolio.

Some applications of portfolio and capital market theory

In bringing this series to a conclusion, we make brief reference of some applications of portfolio and capital market theory. The central theme of these theories is that of the relationship or trade-off between risk and return. Portfolio and capital market theory confirm the common-sense notion that under equilibrium conditions risk and return are directly related, implying that if one desires a greater return one must court greater risk. Their special significance, however, resides in the fact that they provide the tools for quantifying this intuitively obvious relationship. The numerical examples in this article are intended to show how the necessary calculations should be carried out.

A question of major and continuing importance in portfolio management is that of the investment performance of some portfolio, for example, a pension or mutual fund, compared with the performance of other funds or of the market as a whole. It should be clear by now that a comparison of investment performances is meaningless unless proper account is taken of the possibly quite disparate risk profiles of the portfolios being compared. Portfolio theory enables the requisite adjustment for risk to be made in objective terms, as is shown simply and concisely in the numerical examples in Gilbertson[1].

Perhaps the most important question facing financial managers and investors, however, is that of estimating a company's cost of equity capital. In the absence of reliable estimates of the cost of equity capital, financial managers lack a suitable basis for the evaluation of projects and investors a suitable basis for the evaluation of shares. The capital asset pricing model is a rational and objective method for quantifying the cost of equity capital and thus promoting the efficient allocation of resources within both the firm and the capital market as a whole. In terms of the example given in the preceding section, if the risk-free rate is 7%, the market price of risk is 5% and security X's beta is 1,083, then, from equation (13):

$$\text{X's cost of equity capital} = 7\% + 1{,}083\,(12\% - 7\%) = 12{,}4\%$$

Here, however, it is important to bear in mind two caveats. First, the estimation of the market risk premium and of beta is based on historical surrogates for the future. Secondly, the cost of equity capital derived above would apply strictly only to new projects that, in terms of pure business risk and financial structure, are replicas of the existing firm. The difficulties posed by the first caveat, namely that historical relationships may not apply strictly enough in the future, is inescapable but there are ways of allowing for those posed by the second. To consider these issues further, however, would take us too far afield in an introductory and informal survey. What must be stressed in conclusion is that portfolio and capital asset pricing theory are powerful techniques for the analysis of the central problems of finance and investment.

References

1. Gilbertson, B P (1984). The Role of Share Risk Measurements in the Management of Investment Portfolios. *The Investment Analysts Journal,* No 24, pp 43–46.