JU de Villiers*

Investment Basics XXXIX. The relationship between futures and spot prices*

1. FORWARD AND SPOT PRICES

In a perfect market, the forward price on a non-dividend paying security is determined by the cost of carry relationship. To own the security in the future, investors could either enter into a forward contract now to purchase the security at price $F_{0,t}$ in the future, or they could buy the security now at a price S_0 and keep it till the future date. If they buy the security now they have to finance the purchase price and carry the investment to the future. Paying S_0 now is equivalent to paying $S_0(1+r)^t$ in the future. When prices are in equilibrium, we therefore find that:

$$F_{0,t} = S_0(1+r)^t \qquad \text{... (1)}$$

A higher forward price would make risk free arbitrage possible. Investors can buy the underlying security for S_0 (borrowing money at a rate r to do so) and simultaneously sell a forward contract at $F_{0,t}$. At the expiry of the forward contract the investors then deliver the security to the buyer, repay their loan [amounting to $S_0(1+r)^t$ by now] and show a riskless profit [since the proceeds $F_{0,t}$ exceed the amount they owe $S_0(1+r)^t$].

If the forward price is lower than that of the cost of carry relationship risk free arbitrage is also possible (at least in a perfect market, which we are assuming for the time being). Investors can sell the security short at S_0 (investing the proceeds at a rate r) and simultaneously buy the forward contract at $F_{0,t}$. At the expiry of the forward contract they can accept delivery of the security in terms of the forward contract, use the security to honour the short sale commitment, pay $F_{0,t}$. for the security from their investment [amounting to $S_0(1+r)^t$ by now] and show a riskless profit [since their investment $S_0(1+r)^t$ now exceeds the amount they have to pay $F_{0,t}$].

2. FUTURES AND FORWARD PRICES

Futures contracts are similar to forward contracts with one important difference. Futures contracts are marked to market daily. The net proceeds of a futures contract depends upon the interest on margin payments which in turn depends upon the interest rate and the timing of the margin payments. Cox, Ingersoll and Ross (1981) have shown that the futures price would equal the forward price if short term interest rates were known with certainty. Futures and forward prices need not be equal if short term rates are not known in advance. Cornell and Reinganum (1981) investigated the differences between futures and forward prices in foreign exchange markets, and found few meaningful differences. Park and Ho (1985) looked at agricultural and precious metal markets and found that futures prices were significantly higher than the corresponding forward prices. In any market where the underlying commodity is positively correlated with interest rates, the buyer of a futures contract would prefer a futures contract to a forward contract [see Kolb (1997: 86) or Bodie Kane and Marcus (1999: 707-708) for a discussion].

In practice, the distinction between futures and forward prices is not believed to be economically meaningful. Valuation procedures that apply to forward contracts are commonly used for the valuation of futures contracts as well [see, for instance Brealey and Meyers (1996: 714), Reilly and Brown (1997: 834) Kolb (1997:87) or Bodie Kane and Marcus (1999: 708)]. The cost of carry relationship outlined above for perfect markets is therefore amended to allow for the effect of storage costs, dividend payments, convenience yields and market imperfections on futures prices.

3. STORAGE COSTS

The cost-of-carry relationship in (1) applies to a security that can be stored at no cost. The only cost of carrying this security into the future is the cost of financing its purchase. When carrying commodities such as maize or beef there are also the additional costs of storage, insurance and transportation. Investors who buy the commodity have to incur these costs, and investors selling the commodity save themselves the storage costs. Suppose that the storage costs are s per year (expressed as a percentage of the cost of the commodity). This has to be added to the financing costs in (1), so that the cost-of-carry relationship for a commodity with storage costs becomes:

$$F_{0,t} = S_0(1+r+s)^t \qquad \text{... (2)}$$

4. DIVIDENDS AND CONVENIENCE YIELDS

If a security (or the group of securities that make up a share index) pays a dividend, this benefit accrues to the buyer of the asset and is forfeited by the seller of the asset. Investors arbitraging between spot and futures prices have to take the benefit received or foregone into account. The exact answer is obtained by calculating the present value of all dividends to be paid over the duration of the futures contract and

*Department of Business Management, University of Stellenbosch, Private Bag X1, Matieland 7602, Republic of South Africa. Email: judv@maties.sun.ac.za

subtracting this from the spot price. For share index futures a fair approximation is obtained by assuming that the shares in the index pay dividends in a constant and continuous stream over the year. Someone buying a portfolio representing the index would receive a constant dividend income of d percent per year (where d is the dividend yield of the index). Someone selling a portfolio of shares representing the index would forfeit this income. The cost of carrying this asset is therefore reduced by the dividend yield. For a share or share index with no storage cost but paying a dividend the cost of carry relationship therefore becomes:

$$F_{0,t} = S_0(1+r-d)^t \quad \ldots (3)$$

For shares or share indices, the dividend yield (d) is usually smaller than the interest rate (r). The term within brackets (1+r-d) is then greater than one, and the futures price higher than the spot price.

Commodities do not pay dividends, but ownership of the commodity may infer a similar benefit that could affect the relationship between spot and futures prices. A food producer owning a ton of maize has the same exposure to the price of maize as another producer who has bought a futures contract for a ton of maize. Producers would prefer the ton of maize to the futures contract, because the futures contract cannot be used as raw material if they suddenly need it. It is therefore more convenient to have maize in storage than to have a futures contract on maize. This benefit is called the "convenience yield" of the commodity. It is difficult to quantify. It varies over time and between commodities. It is high for commodities with an uncertain demand or supply. It is seasonal for commodities with a seasonal demand or seasonal supply. For arbitrage, it is also important to realise that the manufacturer using the commodity is the only party that can benefit from the convenience yield. A high convenience yield therefore restricts the ease with which arbitrage transactions can be undertaken.

The benefit obtained from the convenience yield can be incorporated in the cost of carry relationship just like the dividend yield. If the convenience yield is d percent per year, and the commodity has a storage cost of s percent per year, then the cost of carry relationship becomes:

$$F_{0,t} = S_0(1+r+s-d)^t \quad \ldots (4)$$

We have seen above that the futures price of a share index will usually be higher than the spot price. This will also be the case for most commodity futures. It is possible for a commodity to have such a high convenience yield (d) that this exceeds the sum of the financing cost (r) and the storage cost (s). The term within brackets (1+r+s-d) will then be less than 1, and the futures price will be lower than the spot price.

5. MARKET IMPERFECTIONS

The relationships outlined above are based on the assumption that investors can make riskless profits whenever the price deviates from the arbitrage condition. In practice, market imperfections (transaction costs, differential borrowing and lending rates, restrictions on short selling) make it impossible to profit from arbitraging small deviations from the arbitrage conditions.

Suppose that the cost of transacting in the spot market is T percent of the transaction price. Investors will only be able to make arbitrage profits if the deviation from the cost of carry relationship is sufficient to offset the transactions cost. Arbitrage boundaries therefore develop on both sides of the cost of carry price. The futures price can vary within these boundaries since any potential arbitrage profits would be more than offset by the transaction cost incurred to exploit the arbitrage opportunity. The arbitrage boundaries are:

$$S_0(1-T)(1+r+s-d)^t \leq F_{0,t} \leq S_0(1+T)(1+r+s-d)^t \quad \ldots (5)$$

In practice the interest rate investors have to pay when borrowing is higher than the rate they receive when lending funds. When the futures price is lower than the lower arbitrage boundary, investors buy the futures contract, sell the shares and invest the proceeds in the capital market in order to make an arbitrage profit. The relevant interest rate for the lower boundary in (5) is therefore the rate for lending money (r_L). For the higher boundary in (5) the arbitrage transaction involves selling the future, buying the shares and borrowing money to finance the purchase. The relevant interest rate is then the rate for borrowing money (r_B). Since r_B is greater than r_L this widens the arbitrage boundaries to become:

$$S_0(1-T)(1+r_L+s-d)^t \leq F_{0,t} \leq S_0(1+T)(1+r_B+s-d)^t \quad \ldots (6)$$

The arbitrage transaction in the lower bound in equation (6) involves selling the asset short and investing the proceeds at the rate r_L. In practice, there are various restrictions on short selling, and the full proceeds of a short sale are not immediately available to the seller. Suppose that only a fraction f of the proceeds is available to be lent, then the effective interest rate on the transaction price is reduced to ($f.r_L$). Applying this to the arbitrage boundaries in (6):

$$S_0(1-T)(1+f.r_L+s-d)^t \leq F_{0,t} \leq S_0(1+T)(1+r_B+s-d)^t \quad \ldots (7)$$

Large institutions can trade relatively cheaply, can borrow or lend at close to the same rate and can sell short easily. The arbitrage boundaries would therefore be narrower for a large institution than for the ordinary private investor. The institution would exploit prices that are outside its arbitrage boundaries and within the arbitrage boundaries of other investors. The effect would be to drive the market price of futures contracts to within the arbitrage boundaries of the large institutions.

6. CONCLUSION

In a perfect market the relationship between futures and spot prices depends on interest rates, storage costs, dividend payments and convenience yields. Any deviation from the theoretical relationship will afford investors a riskless profit opportunity and will quickly be arbitraged away. In practice there are various market imperfections (transaction costs, differential borrowing and lending rates, restrictions on short selling) that prevent complete arbitrage. This allows the futures price to vary within arbitrage boundaries around the theoretical price.

LIST OF SYMBOLS USED

d	dividend yield of a share or share index, or the convenience yield of a commodity
$F_{0,t}$	current price of a futures or forward contract with maturity at time t
f	fraction of the amount of a short sale immediately available to the seller
r	risk free interest rate
r_B	interest rate when borrowing
r_L	interest rate when lending
S_0	current price of a share or a commodity, or current level of a share index
s	rate of storage cost
T	transactions cost as a percentage of the transaction amount
t	maturity of futures or forward contract

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