

Notes on the characteristics and performance of some South African mutual funds

INTRODUCTION

The performances of South African mutual funds are usually compared by assuming that equal amounts were invested in each fund on a certain date. The values of these investments (usually assuming reinvestment of dividends) are then compared at a later date to determine which fund performed best over the particular period.

Figure 1 (a) and figure 1 (b) show typical examples of such a comparison:

- (1) If the initial amounts were invested on 30/12/69 (figure 1 (a)) and the comparisons made over the period 5/1/73 to 8/3/74 then Guardbank clearly outperformed SAGE.
- (2) However, if the initial investments were made on 30/12/71 (figure 1 (b)) then the performance of SAGE improves dramatically and generally appears to be better than that of Guardbank — except on

8/10/73 when Guardbank again seems to be the better of the two.

The results of these comparisons therefore depend on the period over which they are made and the outcome can indeed be confusing if more periods and funds with less extreme characteristics are compared.

These graphs also show the variability of the fund values over a period of time and this immediately raises the question whether these variations are indicative of fundamental changes in the investment policies of the fund managements, whether they are simply random fluctuations, or perhaps co-movements with the general movement of the market as a whole. The movement of the Rand Daily Mail 100 Industrial Index suggests the latter, but one would like to test this assertion statistically and to identify situations where changes in performance do indicate fundamental changes in investment policy.

Lastly, this kind of comparison ignores the risk associated with each fund. Superior investment returns are, on the average, correlated with higher risks and any comparison between the performances of funds, that does not control for risk, is invalid and misleading.

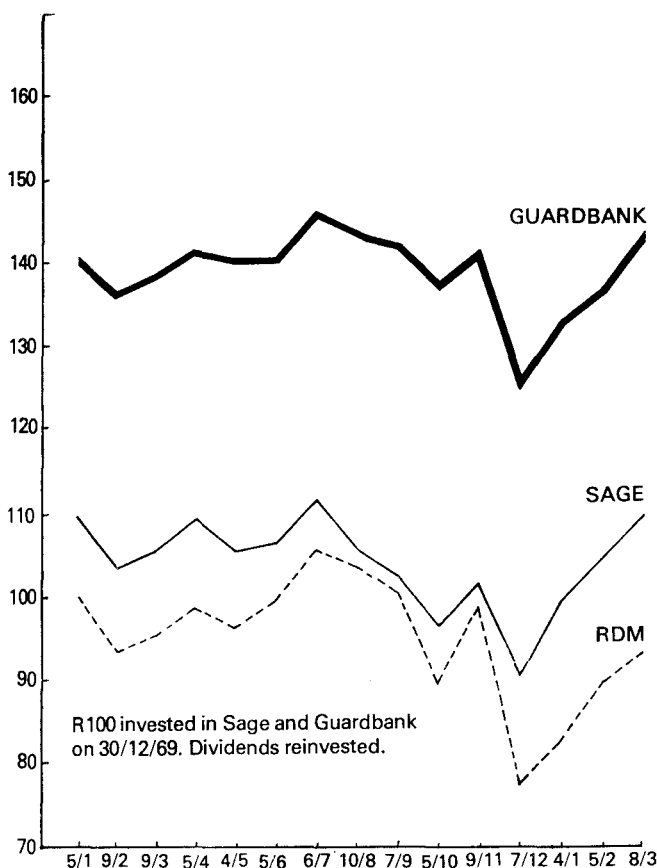


Figure 1(a)
Value of an investment on different dates between 5/1/73 and 8/3/74

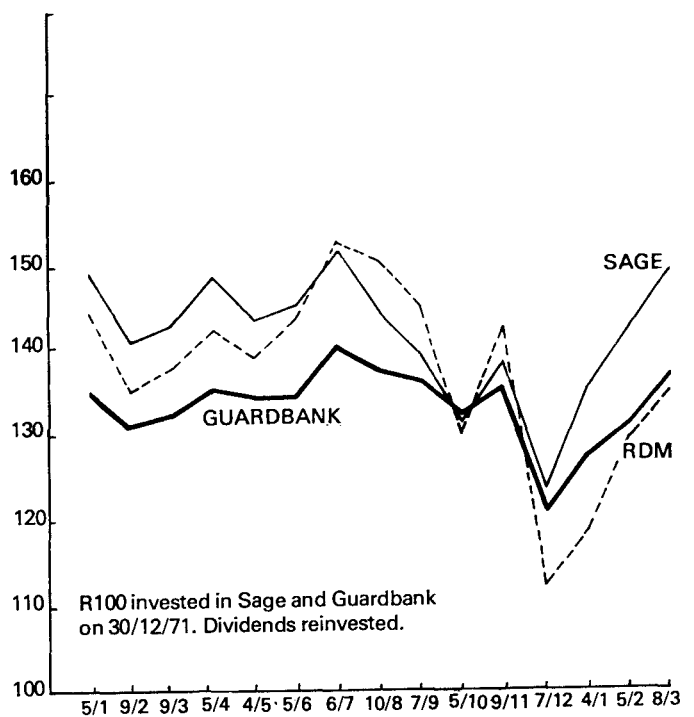


Figure 1(b)
Value of an investment on different dates between 5/1/73 and 8/3/74

Techniques that are relatively free of most of these shortcomings have, however, been developed and are extensively used to evaluate the performance of American mutual funds. We will briefly review these techniques and apply them to some South African funds. The use of control charts to detect fundamental changes in investment policy will also be illustrated.

MEASURES OF PERFORMANCE AND RISK

The standard deviation of return as a measure of risk

The generally accepted ex ante measure of investment performance is expected return¹ over some relevant future period. Investment risk can then be considered as the extent to which the actual return is likely to deviate from the expected return. An exact evaluation of this measure of risk requires knowledge of the probability distribution of future returns. However, since it is difficult to determine this distribution the standard deviation of future returns, which does indicate the spread of likely returns, (and is usually easier to estimate than the complete probability dis-

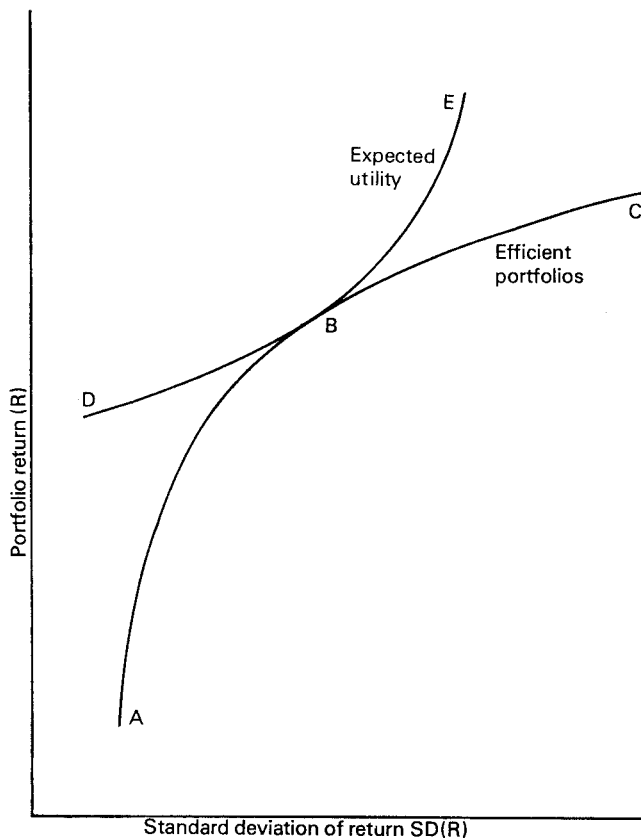


Figure 2
Efficient portfolios

tribution) seems to be a more practical measure of risk. Markowitz², in his pioneering article which laid the foundations for modern portfolio theory, first used expected returns and standard deviation of return as criteria (and the only criteria) for choosing and judging the performance of portfolios of risky assets. Markowitz assumed that investors are risk-averse, i.e. for a desired return they prefer a portfolio with minimum standard deviation of return or maximum return for a desired standard deviation of return. Portfolios with these characteristics are called efficient portfolios and fall on curve ABC in figure 2. From such a set of efficient portfolios the investor will choose that one which, for him, has maximum utility — where expected utility is now also specified in terms of expected return and standard deviation of return. Curve DBE in figure 2 is such a utility function and in this particular instance portfolio B would be the preferred portfolio.

Expected return and standard deviation of return also form the basis of the techniques for evaluating mutual funds which will be discussed here and the important implications of the assumption that these statistics can be used as sole criteria for choosing and evaluating portfolios, should be noted³:

- (1) Selecting a preferred portfolio from the efficient set in the (R, SD(R)) — plane which maximises utility implies, as noted above, that expected utility is also only a function of R and SD(R). The only known function satisfying this condition, as well as conforming to the notion that investors are risk-aversers, is a quadratic function

$$Utility (U) = a + bR + cR^2$$

so that

$$E(U) = a + bE(R) - c(E^2(R) + VAR(R))$$

The function U can only be a valid utility function in the restricted range (0, b/2c). It is unlikely that such a function will meet the requirements of all investors, as the work of Arditti⁴ and Friedman and Savage⁵ shows. Furthermore, it can also be shown that, under certain circumstances, the quadratic utility function implies seemingly irrational behaviour, i.e. if returns on assets are generally increased investors with a quadratic utility function will select portfolios with less associated risk.

- (2) These implied shortcomings of the standard deviation as a measure of risk are mostly of a conceptual nature. The results of extensive empirical investigations of the behaviour of price changes on the New York Stock Exchange by, among others, Mandelbrot⁶ and Fama⁷ may be of greater practical significance. Their investigations suggest that price changes conform to stable Paretian distributions with characteristic exponents less

1 Return on asset i in period t will generally be denoted by R_{it} . The expected value, variance and standard deviation of this return by $E(R_{it})$, $Var(R_{it})$ and $SD(R_{it})$ respectively. Subscripts may be omitted where they are superfluous. Unless otherwise stated return will be measured as the ratio of the value of the investment at time t to the value at time (t-1). Dividends received are assumed reinvested at time t.

2 Markowitz, H.: 'Portfolio selection', *Journal of Finance*, March 1952, pp. 77-91

3 For a more comprehensive discussion the reader is referred to the work of Sharpe, W. F.: 'Portfolio theory and capital markets',

McGraw-Hill, New York, 1970, pp. 187-201

4 Arditti, F. D.: 'Risk and the required rate of return', *Journal of Finance*, March 1967, pp. 19-21

5 Friedman, M., and Savage, L.: 'The utility analysis of choices involving risk', *Journal of Political Economy*, August 1948, pp. 279-304

6 Mandelbrot, B., 'The variation of certain speculative prices', *Journal of Business*, Vol. 36, no. 4 (1963)

7 Fama, E. F.: 'The behaviour of stock prices', *Journal of Business*, Vol. 38, no. 1 (1965)

than two. These distributions have, by definition, infinite standard deviations and, if this is the case, the Markowitz measure of risk appears to be meaningless.

However, as noted above, a quadratic utility function does reflect some of our notions about the behaviour of investors. Standard deviations can always be calculated from sample data so that even the shortcomings revealed by the work of Mandelbrot and Fama may be more apparent than real (of course tests of significance based on the standard deviation must be interpreted with care under these circumstances). Lastly, the real test of the validity of a theory is its ability to explain observable phenomena and make useful predictions, and, according to this criterion, the standard deviation of expected return seems to be an acceptable measure of investment risk⁸.

Volatility (or β) as a measure of risk

A more revealing (and easier understood) measure of risk which is, under certain conditions, equivalent to the standard deviation of return can be derived from the so-called Market Model. This model, first formulated by Markowitz⁹ and later extended by Sharpe¹⁰, states that the return on an individual asset is a linear function of a market factor (usually assumed to be some comprehensive market index) common to all assets as well as independent factors unique to the particular asset.

Symbolically the statement of the Market Model is:

$$R_{it} = \alpha_i + \beta_i M_t + e_{it} \dots \dots \dots 1(a)$$

where M_t represents the market factor and

$$E(e_{it}) = E(e_{it}, e_{jt}) = E(e_{it}, M_t) = 0 \dots 1(b)$$

Sharpe¹¹ and Lintner¹², using the Market Model to derive a theory of equilibrium in capital markets, showed that in equilibrium the β -coefficient appearing in the Market Model is the appropriate measure of risk for investors minimising the standard deviation of their expected investment returns.

The following more direct justification of the use of β as a measure of risk is given by Blume¹³:

If equal amounts are invested in n shares then the return on the portfolio in period t is

$$\begin{aligned} R_p &= \sum_{i=1}^n (1/n) R_{it} \\ &= \sum_{i=1}^n (1/n) (\alpha_i + \beta_i M_t + e_{it}) \end{aligned}$$

Taking the variance and using 1(b) gives

$$\begin{aligned} \text{Var}(R_p) &= \left(\sum_{i=1}^n (1/n) \beta_i \right)^2 \text{Var}(M_t) \\ &+ \sum_{i=1}^n (1/n)^2 \text{Var}(e_{it}) \\ &= \bar{\beta}^2 \text{Var}(M_t) + \overline{\text{Var}(e_{it})}/n \dots 2 \end{aligned}$$

where the bar in the last expression indicates an average.

If n is increased (through diversification) the last term in the expression above will decrease and the standard deviation of the portfolio return will be approximately equal to $\bar{\beta} \text{SD}(M_t)$. Since the standard deviation of the market return is common to all portfolios, $\bar{\beta}$ determines the standard deviation of any particular portfolio's return and is therefore the relevant measure of risk and thus β_i , since it contributes to $\bar{\beta}$, is also the relevant measure of risk for an individual share.

Again the validity of this argument rests on the validity of the Market Model. This model has been extensively tested for returns on the New York Stock Exchange and although the work of King¹⁴ shows that $E(e_{it}, e_{jt}) \neq 0$ (i.e. that the returns on shares, particularly in similar industries, are correlated) and the previously quoted investigations of Mandelbrot and Fama suggest that $\text{Var}(e_{it})$ is infinite, the model does appear to be sufficiently robust for the conclusions regarding the appropriateness of β as a measure of risk to be valid. For instance, Evans and Archer¹⁵ have shown empirically that, on the average, for portfolios consisting of as little as 15 to 20 shares, the variance of the portfolio returns is approximately equal to $\bar{\beta}^2 \text{Var}(M_t)$.

ESTIMATES OF RISK FOR SOME SOUTH AFRICAN MUTUAL FUNDS

Estimating a portfolio's risk

In the previous section the rationale for choosing the standard deviation of return, and under certain conditions the β -coefficient in the Market Model, as a measure of investment risk has been briefly outlined. Despite the known shortcomings of these measures they are extensively and successfully used to measure investment performance on especially the NYSE. We will therefore also use these measures, and especially β , to evaluate the performance of some South African mutual funds.

A portfolio's historic β -coefficient can be estimated by regressing the periodic return of the portfolio on the corresponding returns of a market index. Figure 3

8 Sharpe, op. cit., pp. 141-186
 9 Markowitz, H.: 'Portfolio selection: efficient diversification of investments', *Cowles Foundation Monograph*, no. 16, New York, John Wiley and Sons, 1959
 10 Sharpe, W. F.: 'A simplified model of portfolio analysis,' *Management Science*, January 1963, pp. 277-293
 11 Sharpe, W. F.: 'Capital asset prices: a theory of market equilibrium under conditions of risk', *Journal of Finance*, 1964, pp. 425-442
 12 Lintner, J.: 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *Review of Economics and Statistics*, 1965, pp. 13-37
 13 Blume, M. I.: 'On the assessment of risk', *Journal of Finance*, Vol. 26, no. 1, March 1971, pp. 1-10
 14 King, B. F.: 'Market and industry factors in stock price behaviour', *Journal of Business*, 1966, pp. 139-190
 15 Evans, J. L., and Archer, S. A.: 'Diversification and the reduction of dispersion: an empirical analysis', *Journal of Finance*, 1968, pp. 761-768

shows a plot of the monthly returns of the SAGE fund against the corresponding returns of the Rand Daily Mail 100 Industrial Index¹⁶ over the period 15/6/67 to 15/11/73.

The estimated regression line (which is usually called the portfolio's characteristic line) is

$$R_t = 0,3100 + 0,9506R_{mt}$$

and the historic risk of this fund as measured by β is 0,9506, the slope of the regression line. Another interpretation of this measure of risk is also now apparent: risk is the change in the portfolio's return relative to a change in the market return. In the case of a high-risk portfolio (i.e. β is large and the characteristic line is steep) a small change in the return on the market (either upward or downward) induces a relatively large corresponding change in the portfolio's return. A low-risk portfolio is less sensitive to changes in the market return. These conclusions, and indeed the use of β as a measure of risk, is only justified if i.a. the periodic returns are closely scattered around the characteristic line. This is the case for the SAGE fund and the squared correlation coefficient (r^2) is 0,8804.

The characteristic line for the Guardbank portfolio, estimated from monthly returns over the period 15/1/70 to 15/11/73, is

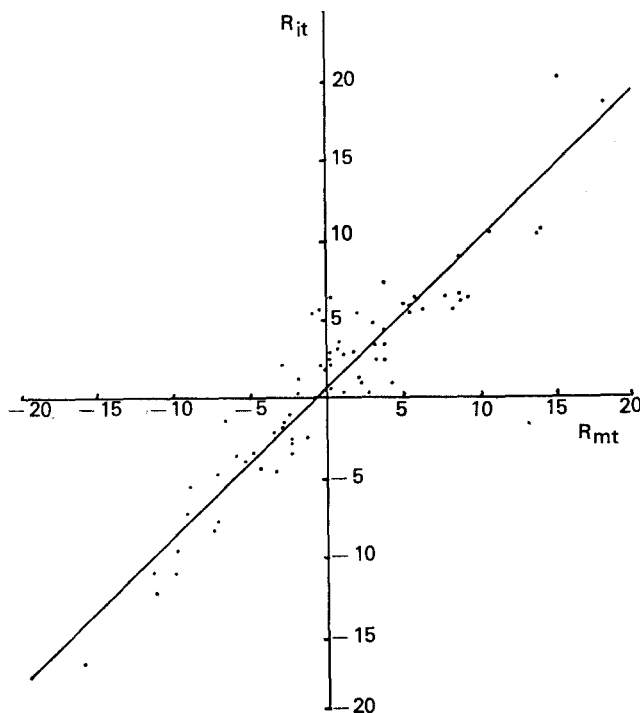


Figure 3
Characteristic line of Sage, fitted on monthly returns between 15/6/67 and 15/11/73

$$R_t = 0,7381 + 0,4700R_{mt}$$

with $r^2 = 0,7200$

Since the β -coefficient of Guardbank is less than SAGE's coefficient it is, according to our definition, a less risky fund than the latter.

These two characteristic lines also explain the apparent confusing nature of the relative performances of the funds as exhibited in figure 1, viz.:

- (a) The change in the monthly values of the funds are largely caused by the movement of the market and not by fundamental changes in their investment strategies.
- (b) Since SAGE has a steeper characteristic line than Guardbank it is likely to perform better than Guardbank in a rising market (as happened between 30/12/71 and 1973 — figure 1 (b)) and worse than Guardbank in a declining market, which was the case for the period 30/12/69 to 1973 — figure 1 (a).

The stability of a portfolio's estimated risk

The measures of risk calculated above will be useful and valid for comparing and predicting investment performance, provided that they remain stable. They can be unstable in at least two ways:

- (a) The β -values were estimated from monthly returns while investment performance over longer periods are usually of interest. The question is then whether the risk measured over short periods also applies to returns over longer periods¹⁷.
- (b) The β -values can change over a period of time, either because of a change in the nature of the market or as a result of a fundamental change in investment strategy.

These aspects of our estimates of risk have been tested to a limited extent and they appear to be remarkably stable.

Table 1 contains estimates of the characteristic line of the SAGE-fund estimated from monthly, 3-monthly and 6-monthly returns over the period 15/6/67 to 15/11/73. The β -coefficients increase slightly as the length of the period increases.

Table 1

The characteristic line of the SAGE-fund estimated from returns measured over periods of increasing duration between 15/6/67 and 15/11/73

Duration of periods over which returns were measured	Estimated characteristic line	r^2
1 month	$0,3100 + 0,9506R_{mt}$	0,8804
3 months	$0,7381 + 1,0158R_{mt}$	0,9113
6 months	$1,9431 + 1,0237R_{mt}$	0,9120

However, the highest estimate of β exceeds the lowest by only 7,6%. The intercept of the characteristic

16 The monthly return of the SAGE fund is the ratio of the month-end repurchase price to the repurchase price at the beginning of the month. Dividends declared are added to the month-end repurchase price. Monthly changes in the RDM Industrial Index are used as a measure of the return on the market. This measure is deficient in at least two respects. Firstly, the index is not representative of the total market and, secondly, dividends are ignored. However, these

shortcomings are probably not too serious since periodic changes in different indices are highly correlated and price changes appear to be the dominant factor in determining market returns.

17 Knowledge of the relation between the risk measured over short and that measured over longer periods is also of interest because it will enable us to detect changes in the risk of long-term returns from short-term observations.

lines increases as the period over which return is measured increases.

Changes in the risk of a portfolio over a period of time can be investigated in various ways. Successive deviations around the fitted characteristic line can be tested for significant positive or negative runs. Such tests were carried out for the lines fitted to the monthly return figures for both SAGE and Guardbank. The Z-values obtained were -1,2606 and 0,1188 respectively which is not statistically significant.¹⁸ As a further test of the stability of β the characteristic lines were estimated over two non-overlapping sub-periods. The results, given in table 2, show that

- (a) for SAGE the three characteristic lines are virtually identical, while
- (b) in the case of Guardbank the line seems to have become slightly steeper in the last period. The difference between the β for the two sub-periods is however not statistically significant¹⁹, the value of the t-statistic being -1,0548.

Table 2

Estimates of the characteristic lines of SAGE and Guardbank over different periods of monthly returns

Fund	Period used for fitting the characteristic line	Estimated characteristic line
SAGE	15/6/67—15/9/70	$0,3058 + 0,9703R_{mt}$
	15/10/70—15/11/73	$0,3088 + 0,9296R_{mt}$
	15/6/67—15/11/73	$0,3100 + 0,9506R_{mt}$
	15/1/70—15/11/71	$0,8058 + 0,4465R_{mt}$
Guardbank	15/12/71—15/11/73	$0,4906 + 0,5589R_{mt}$
	15/1/70—15/11/73	$0,7381 + 0,4700R_{mt}$

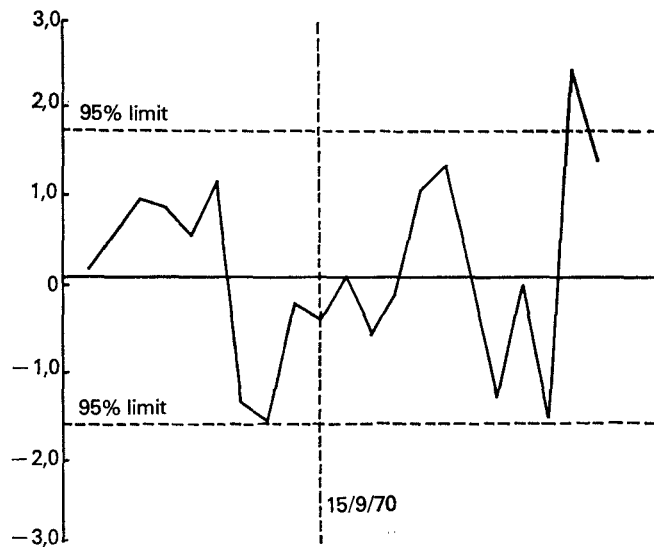


Figure 4
Control chart for the average deviation from the characteristic line of Sage (samples of four successive deviations)

The use of control charts to detect changes in portfolio risk

To detect a change in the risk of a portfolio the value of its β -coefficient should be monitored for statistically significant changes. However, β will only remain an acceptable measure of risk if the portfolio remains efficiently diversified, i.e. if periodic returns are scattered fairly closely around the characteristic line. Under these conditions expression (2), as well as the empirical evidence of figure 3, shows that the variation in portfolio return is largely accounted for by the variation in the market return and the portfolio's β -coefficient. A significant increase in the scatter around the characteristic line indicates less efficient diversification of the portfolio so that the standard deviation of portfolio returns then becomes the appropriate measure of risk.

Control charts for the average deviation and range of the deviations around the characteristic line of SAGE are given in figure 4 and figure 5. The 95% control limits for the average and the range of successive groups of four deviations are shown.

The control limits were calculated from deviations around the characteristic line fitted to monthly returns for the period 15/6/67 to 15/9/70. The trend of the average of the deviations again confirms the stability of SAGE's characteristic line. The last two plots of the average deviation, however, suggest a significant change in the β -coefficient of this fund. This is probably the result of the fund's investment in gold shares and the sharp increase in the value of these shares relative to the RDM 100 Industrial Index²⁰. The control chart for the range also shows that, probably for the same reason, the SAGE fund is becoming less efficiently diversified with respect to the RDM 100 Industrial Index. This seems to be the trend from about the beginning of 1971 and the control limit on this chart should be recalculated. The increase in the scatter of returns around the

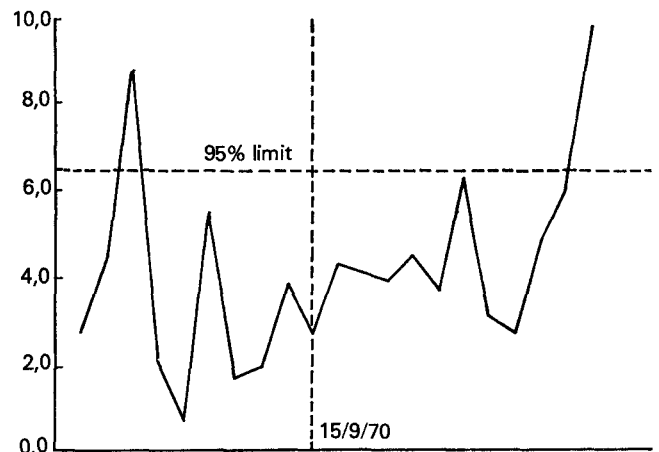


Figure 5
Control chart for the range of deviations from the characteristic line of Sage (samples of four successive deviations)

18 Siegel, S.: 'Non-parametric statistics for the behavioral sciences', McGraw-Hill, New York, pp. 52-60

19 Hald, A.: 'Statistical theory with engineering applications', John Wiley and Sons, New York, 1952, pp. 571-579

20 The percentage of the asset value of the SAGE fund represented by mining finance and gold shares increased from 9,12% on 30/6/73 to 16,94% on 31/12/73 and the percentage for gold shares alone increased from 2,71% to 8,33% over this period.

characteristic line is however not serious. For the line fitted to returns over the period 15/6/67 to 15/9/70 r^2 was 0,8998 while for the line fitted to returns over the period 15/10/70 to 15/11/73 r^2 only decreased to 0,8587.

THE COMPARISON OF MUTUAL FUNDS IN TERMS OF BOTH AVERAGE RETURN AND RISK

The expected return and risk of SAGE and Guardbank are plotted in figure 6. The expected returns were calculated from the characteristic lines, assuming a 1% monthly return on the market. An assumed riskless rate of return (for instance the return on treasury bills) of 0,4% is also shown, as well as lines connecting this riskless return to that of SAGE and Guardbank. The slopes of these lines, sometimes referred to as reward to volatility ratios, measure the return earned per unit of risk and form the basis of techniques developed and used by Treynor²¹ and Jensen²² for evaluating the performance of American mutual funds.

Although SAGE will provide a higher average return than Guardbank, this is achieved at a higher risk than that borne by Guardbank. If Guardbank bears the same risk as SAGE and manages to maintain its current reward to volatility ratio, then its expected return will be considerably higher than that of the SAGE fund. The 'risk-adjusted' performance of Guardbank is therefore considerably better than that of SAGE. This conclusion depends on, among other things, the assumed riskless rate of return. However, this return will have to increase to approximately 1,15% while all the other relevant factors remain stable, before our conclusion is reversed.

CONCLUSION

The β -coefficient, to the extent that it measures the fluctuations in the return of a well-diversified portfolio, has intuitive appeal as a measure of risk. We have furthermore shown that there is also some theoretical justification for using it to measure risk. The β -coefficient also measures the dependence of a portfolio's return on the return of the market and since our limited investigations have indicated that the β 's of South African mutual funds are likely to be stable, these coefficients will explain the historical movement in the value of a fund's units and can be used to predict future movements. If both average returns and the β -coefficients of funds are known, a risk-adjusted, and therefore valid, comparison of performance can be made. Outcomes of such comparisons will also only change as a result of significant changes in the fund's investment strategy or in the nature of the market. Lastly, if the complete characteristic line of a fund is known it is possible by using

control charts to monitor a fund's performance for such significant changes in the underlying investment strategy or nature of the market.

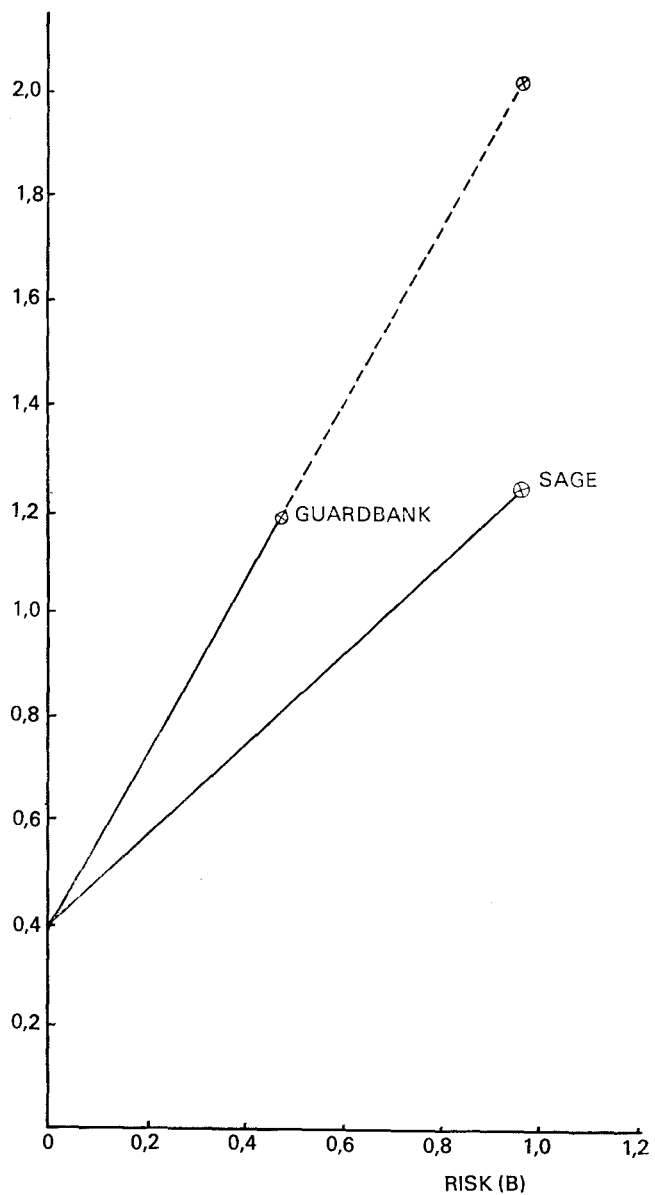


Figure 6
Risk-adjusted comparison between the performance of Guardbank and Sage

ACKNOWLEDGEMENT

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21 Treynor, J. L.: 'How to rate management of investment funds', *Harvard Business Review*, January and February 1965, pp. 63-75

22 Jensen, M. C.: 'The performance of mutual funds in the period 1945-1964', *Journal of Finance*, May 1968, pp. 389-416