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# Investment basics XVII
# What is the return on your investment?

*This paper addresses the issue of what the appropriate methodology is for calculating holding period returns on risky investments in order to correctly specify the return to the investor and permit of inter-investment comparison especially when holding periods of different lengths are involved.*

*The arithmetic and geometric means are presented as alternative specifications for the returns. The geometric mean is shown to be the correct method of calculating average holding period returns. An example of the degree of misspecification which can occur when using the arithmetic mean is drawn from the returns of companies on the JSE.*

### Introduction

Of great interest to any investor is the return achieved on his or her investment. However, the specification of this return is a matter of some controversy.

In the case of fixed interest securities such as bonds, where both the periodic payments (interest) and the final repayment of capital are certain, given the current price of the bond an investor can with relative ease calculate the return on the proposed investment using the concept of the yield-to-maturity (Sharpe, 1985). The yield-to-maturity is the discount rate that equates the present value of the known future cash flows with the current price of the security.

Of course, if there is any risk that the promised payments will not materialise either on the expected dates or in the anticipated amounts, the expected yield-to-maturity will be different from that calculated. Indeed, risky bonds promise higher yields so that their expected yields will fall into line with those of less risky bonds.

Yield-to-maturity calculations do not, however, take into consideration any changes in the market value of the bond prior to maturity. It is, thus, implied that the owner intends to hold the instrument to maturity.

For other types of securities, such as ordinary shares, there is no maturity period, and thus the concept of yield-to-maturity is not applicable.

The question, therefore, arises as to what measure of return will have general applicability and will permit of the intercomparison between various asset classes?

### Single period returns

The holding period return (HPR) is a measure that can be used for any instrument in order to calculate the return it provides over the interval from the beginning to the end of an investor's planning period.

In order to calculate a holding period return for a given time period, the following formula may be used:

$$HPR = \frac{P_1 - P_0 + D_1}{P_0} \qquad (1)$$

where $P_0$ is the price of the security at the beginning of the period, $P_1$ is its price at the end of the period, and $D_1$ is the payment received by the security holder at the end of the period. The return, thus, occurs in two forms: $P_1 - P_0$ is the capital appreciation or depreciation of the asset and $D_1$ represents some form of cash or perhaps non-cash distribution. ($D_1$ would generally be an interest or dividend payment, but could also be, for example, the market value of a rights issue.) The formula presents no problems in calculation if the time period under consideration is a single holding period – say one year.

When the holding period extends over a number of years, the performance of the investment can be measured by comparing its value at the end of the holding period with the value at the beginning (Sharpe, 1985). The ratio between these two values is known as the holding period value relative (HPVR), and is related to the holding period return as follows:

$$HPVR = 1 + HPR = \frac{\text{(value at end of holding period)}}{\text{(value at beginning of holding period)}} \qquad (2)$$

(In order to evaluate holding period returns under these conditions it is necessary to assume that any payments received during the period are reinvested, usually in more units of the security at the then current market price.)

Investors are accustomed to the quotation of returns on an annual basis, and the quotation of a return over say three years does not easily lend itself to comparisons with other investment returns over differing holding periods. Holding period returns should, therefore, be converted to their equivalent returns per period for comparison purposes.

### Annualised holding period returns

There are two different methods which can be used to achieve this goal. One can simply calculate annual returns for the security for each of the years in the holding period and then form the arithmetic average of the annual holding period returns. Alternatively, one can form the geometric mean of the annual returns.

The geometric mean of a series of N numbers a1, a2, . . . aN can be defined (Stevenson and Jennings, 1984) as:

$$\sqrt[N]{\pi_{i=1}^{N} a_i;} = \sqrt[N]{(a1).(a2)\ldots(aN)} \qquad (3)$$

Geometric means for a series of annual price and dividend data can be calculated using the following steps:

1 Annual holding period value relatives are calculated for each period using the formula
$$HPVR_t = (P_t + D_t)/P_{t-1} \qquad (4)$$
2 The product of all holding period value relatives is formed.
3 The Nth root of this product is taken. N is the number of periods over which the mean is being calculated.
4 The geometric mean is obtained by subtracting one from the Nth root.

The question, thus, arises as to whether the two methods provide the same answers, and if not, which is the more "correct" result.

The table below shows the annual returns over a 12-year period for three shares listed on The Johannesburg Stock Exchange together with the returns on the JSE Actuaries All-Market Index. Two of the shares chosen (Loraine and Toyota) are very volatile (ie, have high systematic risk) and the third (OK Bazaars) is very much less volatile.

| Year ending December | Loraine | Toyota | OK Bazaars | JSE Actuaries All-Market Index |
|---|---|---|---|---|
| 1973 | 179,5% | −37,1% | 11,6% | 6,3% |
| 1974 | 120,0% | −11,8% | 6,0% | 15,9% |
| 1975 | −72,3% | 36,8% | 26,8% | −10,8% |
| 1976 | −40,0% | −24,6% | −19,8% | −2,3% |
| 1977 | 59,8% | −33,3% | 3,9% | 27,9% |
| 1978 | −27,8% | 64,0% | 37,7% | 34,4% |
| 1979 | 188,9% | 49,0% | 66,7% | 87,0% |
| 1980 | 197,4% | 197,5% | 52,2% | 39,2% |
| 1981 | −66,4% | 94,4% | 31,2% | 0,6% |
| 1982 | 87,2% | 3,1% | −0,4% | 32,5% |
| 1983 | −14,9% | 152,0% | 7,6% | 13,8% |
| 1984 | −4,1% | 53,0% | −22,5% | 7,0% |
| Arithmetic mean return 1973–1984 | 50,6% | 45,3% | 16,8% | 20,9% |
| Geometric mean return 1973–1984 | 14,7% | 29,1% | 13,9% | 18,7% |

Although in the cases of the overall index and OK Bazaars the arithmetic and geometric means were not very different, for the two volatile shares, very much higher values were obtained for their arithmetic means.

To see why a simple arithmetic averaging of the holding period returns does not provide a satisfactory measure of return on investment, consider a non-dividend paying stock the share price of which was observed to have the following values at the end of each of three successive years:

| Year | Share price | Annual return |
|---|---|---|
| 0 | 100 | |
| 1 | 200 | 100% |
| 2 | 300 | 50% |
| 3 | 100 | −67% |

The arithmetic mean of the three annual returns is 27,7%. However, any investor will appreciate that if the cost of an investment was 100, no cash interim payments were received, and the value of the investment at the end of the holding period was 100, then a zero return was achieved.

The geometric mean annual holding period return ($R_g$) in the above example can be calculated as follows:

$$1+R_g = \sqrt[3]{(1+1{,}00)\,.\,(1+0{,}50)\,.\,(1-0{,}67)} = 1$$

Therefore $R_g = 0\%$

In this simplified example it is obvious that the geometric mean gives the best representation of the "average" annual return earned by the investor on the investment.

In general, the more extreme the values encountered in the range of annual returns being subjected to the averaging procedure, the more divergent will be the results obtained from the two techniques. This is highlighted by an example quoted by Krige and Austin (1980). They calculated the means of 235 monthly returns on the JSE All Gold Index over the period 1960–1979. (This period included a month with a return of over 8 000% pa) The arithmetic mean was 101% pa whereas the geometric mean was only 8%!

In order to convert a holding period return into an annualised return, the appropriate procedure (Sharpe, 1985) is to find the value of the equivalent period return $R_g$ that satisfies the relationship:

$$(1+R_g)^N = 1+HPR \qquad (5)$$

where the right hand side of the equation is the HPVR.

Now the HPVR is given by $V_N/V_O$ (equation 2)

$$\text{Therefore HPVR} = (V_N/V_{N-1})\,.\,(V_{N-1}/V_{N-2})\ldots(V_1/V_0) \qquad (6)$$

$$= (1+R_N)\,.\,(1+R_{N-1})\ldots(1+R_1) \qquad (7)$$

$$\text{Hence } (1+R_g)^N = (1+R_N)\,.\,(1+R_{N-1})\ldots(1+R_1) \qquad (8)$$

$$\text{And thus } R_g = \sqrt[N]{\prod_{i=1}^{N}(1+R_i)}-1 \qquad (9)$$

Thus, the equivalent return per period is the geometric mean of the individual period returns.

$R_g$ can therefore be calculated by using the equation

$$1+R_g = \sqrt[N]{HPVR} \qquad (10)$$

$$= \sqrt[N]{(V_N/V_O)} \qquad (11)$$

Equation 11 can be used to demonstrate that the geometric mean incorporates the concept of compound returns. A sum of money (PV) invested at i% per annum compounded for t years will grow to a future amount (FV) according to the standard compound interest formula:

$$FV = PV(1+i)^t \qquad (12)$$

Making i the subject of the formula gives

$$i = \sqrt[t]{FV/PV} - 1 \qquad (13)$$

Note that FV/PV is the holding period value relative of the investment, and that equations 11 and 13 are identical.

The value calculated for the compound interest rate from the above formula is in fact the compound growth rate of the investment over the holding period.

**Estimating compound growth rates using HPVRs**

The computation of annual growth rates over a number of periods is, therefore, dependent only on the number of periods and the opening and closing values of the variable (equation 11). If either of these values is atypical of the data set being studied then erroneous estimates of the compound growth achieved will be made.

This is particularly important in areas such as the use of historical dividend growth rates in order to forecast future dividend growth rates for use in financial models.

For example, the constant growth dividend model (or Gordon model) is often used as the basis for determining the cost of equity capital of firms with relatively stable growth rates (Van Horne, 1983).

The model can be stated as:

$$\text{Cost of equity capital} = D_1/P_0 + g \qquad (14)$$

where g is an estimate of the future constant growth rate in dividends. This figure is usually forecast on the basis of the historical dividend growth rate, sometimes modified to take into account future expectations.

Injudicious selection of the start of the historical dividend time series can cause a bias in the estimate made for the compound growth rate, and thus distort the estimate of cost of capital. Similarly, problems will occur if the growth in dividends in the last period reviewed is very different from that occurring in the preceding periods. Since the cost of capital is the basis for the setting of corporate hurdle rates for capital budgeting investment decisions, the consequences can be very far reaching indeed.

### Conclusion

One can conclude by asking which of the two means is the appropriate measure of the returns an investor has actually achieved over time? In fact, each is appropriate under particular circumstances.

The geometric mean measures changes in wealth over more than one period on a buy and hold (with dividends reinvested) strategy. If, however, the investor sold his shares at the end of the first period and reinvested an amount similar to his original capital for the next period, and repeated this process from period to period, (in other words pocketed any capital gains and made up any capital losses at the end of each period) the geometric mean would not be a correct representation of his portfolio's performance over time. In such circumstances, the arithmetic mean would provide a better measure of the typical investment performance over a single historical period.

Since, perhaps, the buy and hold strategy is more common, one can conclude that in general geometric (or compounded) rates of return are better representations of annualised multiperiod returns on investment.

### References

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