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Investment basics XVIII
Risk and return – part I

Introduction

The primary concerns of the investment manager are the valuation of individual securities and the combination of individual securities into well-balanced portfolios. All securities offer a return, and the returns from most are subject to uncertainty or risk. Hence, the investment manager is concerned with constructing portfolios that are efficient in the sense that they maximise returns for a specified level of risk or minimise risk for a specified level of returns.

Modern finance theory has a great deal to say about the calculus of risk and return that is of specific, quantitative assistance to the investment manager in translating the abovementioned objectives into precise operational terms. The purpose of this article is to present a brief, informal account of those aspects of modern finance theory that bear most directly on these issues – namely portfolio and capital market theory. We shall address the following questions:

- How are the returns from individual securities and portfolios measured?
- How are the risks of holding individual securities and portfolios quantified?
- How does a portfolio – ie a diversified holding of securities – reduce risk?
- How does one construct efficient portfolios that optimise return with respect to risk?
- How, in efficient capital markets, are securities priced or valued?

The measurement of historical returns

The return on an investor's portfolio for any given period of time may be expressed as follows:

$$R_p = (V_1 - V_0 + D_1)/V_0 \quad (1)$$

where:

R_p = return on portfolio
V_1 = the portfolio market value at the end of the period
V_0 = the portfolio market value at the beginning of the period
D_1 = cash distribution to the investor during the period.

The calculation assumes that any interest or dividend income received on the portfolio and not distributed to the investor is reinvested in the portfolio and thus reflected in V_1. Furthermore, the calculation assumes that any distribution occurs at the end of the period. It also assumes that there are no capital inflows or outflows during the period, otherwise the calculation would have to be modified to reflect the changed asset base.

Now let us consider the returns to a portfolio over a number of such periods. There are three possible measures for multi-period returns, namely:

- the arithmetic average;
- the time-weighted or geometric average; and
- the money-weighted average.

Given the following consecutive monthly returns on a portfolio, as measured by equation (1), −10%, 20% and 5%, the arithmetic average return would be 5%, as shown below:

Period	Return
1	−10%
2	+20%
3	+ 5%
Total return	15%
Average return per period	5% = 15%/3

The general formula is:

$$R_a = (R_{p1} + R_{p2} + \ldots R_{pn})/n \quad (2)$$

where:

R_a = the arithmetic average return
R_{pk} = the portfolio return in period k, where k ranges from 1 to n
n = the number of periods for which returns are measured.

On the basis of the same data, however, the time-weighted or geometric return would be 4,3%.

Period	Return
1	(1−10/100)
2	× (1+20/100)
3	× (1+ 5/100)
Total return	1+13,4/100
Time-weighted return per period	$(1+13,4/100)^{1/3}-1=4,3\%$

The general formula for the time-weighted return per period is:

$$R_t = [(1+R_{p1})(1+R_{p2})\ldots(1+R_{pn})]^{1/n} - 1 \quad (3)$$

where:

R_t = the time-weighted rate of return per period
R_{pk} = the portfolio return for period k, where k ranges from 1 to n
n = the number of periods for which returns are measured.

The average and time-weighted rates of return are in general unequal. In the case of the former, the amount invested is assumed to be maintained, through additions or withdrawals, at its initial value. The latter, however, is the return on a portfolio that varies in size because it is assumed that all proceeds are reinvested. In fact, the average return will always exceed the time-weighted return unless the returns in each period are identical, in which case the two measures of return will be equal. As shown in a previous article in this series, the time-weighted or geometric average is usually the more representative measure of the returns from an investment because it more accurately reflects the consequences of the buy-and-hold policies often followed by investors[1].

The third measure – namely, the money-weighted rate of return – defines the average rate of growth of all funds invested in a portfolio during the relevant time span, ie the initial value plus any inflows less any outflows, and is therefore influenced by the timing and magnitude of inflows and outflows. It is equivalent to the well-known internal rate of return and is computed in exactly the same way as is the yield to maturity on a bond. The general formula is:

$$V_o = \frac{F_1}{(1+R_m)} + \frac{F_2}{(1+R_m)^2} + \ldots \frac{F_n}{(1+R_m)^n} + \frac{V_n}{(1+R_m)^n} \quad (4)$$

where:

V_o = the initial value of the portfolio

F_k = inflows (positive or negative) in period k, for k ranging from 1 to n

V_n = the terminal value of the portfolio at the end of n periods

n = the number of periods for which returns are measured

R_m = the money-weighted rate of return per period such that the sum of all terms on the right hand side of equation 4 equals the value of the initial portfolio, V_o.

The money-weighted return represents the internal rate of return on the portfolio for the given time span. Because, however, it is affected by inflows and outflows which are typically beyond the investment manager's control, it is not as satisfactory a measure of managerial performance as is the time-weighted or geometric rate of return.

Expected returns and risk

The historical return from holding a share or portfolio is a simple matter of fact. The future return from a share or portfolio, however, is conjectural as it depends on a variety of factors that cannot be forecast with certainty. Some of these factors are specific to the company, such as the efficiency of its management or plant. Others pertain to the industry in which the company operates. Other and still more general factors relate to the state of the domestic and international economies and the socio-political climate. How, against this uncertain background, is one to measure the expected return and risk associated with a security or portfolio?

Assume, for the moment, that we have the information set out in columns 1 to 3 of Table 1. By way of paraphrasing this information, we may say the share is expected to achieve a return of −10% if environmental state "a" occurs, which state has a probability of occurrence of 0,20; 5% if state "b" occurs, its probability being 0,35; and 15% if state "c" occurs, its probability being 0,45. Then the expected return on this share, E(R), is, as shown in column 4, simply the sum of the products of each possible return and the probability of that return. We may express this as:

$$E(R) = \sum_{s=1}^{s=n} p_s R_s \quad (5)$$

where:

E(R) = the probability – weighted average of returns

R_s = the return associated with state s

p_s = the probability of state s.

Table 1: Risk and return for a single security

(1) State	(2) Probability	(3) Return	(4) Probability × return	(5) Deviation from average return	(6) Deviation squared	(7) Probability × deviation squared
	p	R	pR	R−E(R)	(R−E(R))²	p(R−E(R))²
a	0,20	−10,0%	−2,00%	−16,50%	272,25%²	54,45%²
b	0,35	5,0%	1,75%	−1,50%	2,25%	0,79%
c	0,45	15,0%	6,75%	8,50%	72,25%	32,51%
Expected return	E(R) =		6,50%			
Variance					σ²	= 87,75%²
Standard deviation					σ	= 9,37%

In the given example, the probability-weighted or expected return is 6,5%. Now suppose that the return in state "a" is – 30% and in state "c" 23,9%, everything else remaining the same. The expected return in this new set of circumstances would still be 6,5%, but one would surely consider the share to be riskier than before because the difference between the worst and best outcomes has increased from 25% to 53,9%. In other words, the adverse and favourable outcomes are more widely spread about the average or expected return than before. This suggests that we may use the degree of dispersion of outcomes about the average as a measure of risk. The most convenient measure of dispersion is the standard deviation, which is calculated in the manner shown in columns 5 to 7 of Table 1. Column 5 lists the deviation of the expected return for a given state from the overall expected return. These deviations are squared in column 6 to remove negative signs. In column 7, the squared deviations are weighted by their respective state-probabilities. The sum of the squared deviations is known as the variance. The standard deviation is found by taking the square root of the variance and gets us back to our original dimensions or units. In other words, the variance in the example is 87,75% squared but the standard deviation is 9,37%, a straightforward percentage just like the percentage returns from which it is derived. In general notation:

$$\sigma = \left[\sum_{s=1}^{s=n} p_s (R_s - E(R))^2 \right]^{1/2} \quad (6)$$

where:

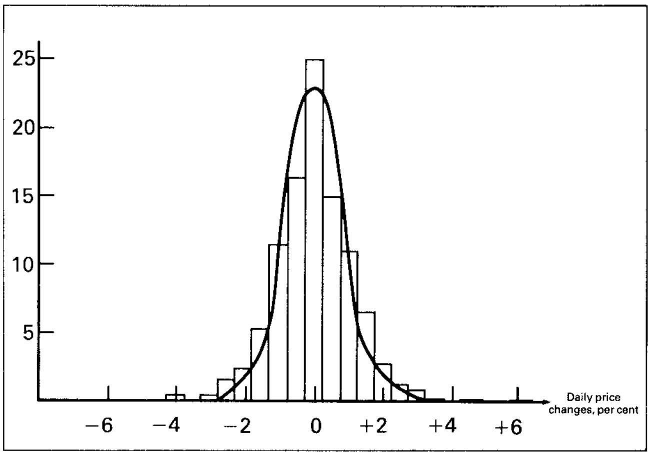
σ = standard deviation

R_s = return in state "s"

p_s = probability of state "s".

It should be remarked at this point that the standard deviation takes account of outcomes more favourable than the mean as well as those below it. In other words, it includes upside as well as downside potential. For most people, however, risk connotes downside potential only and this raises a doubt about the suitability of the standard deviation as a risk surrogate. If, however, it can be shown that rates of return are generally symmetrical about the mean, as illustrated in Figure 1 below, and that the symmetry has stability through time, the doubt would fall away. It is interesting that empirical studies of rates of return both overseas and in South Africa broadly confirm the symmetry of distribution and stability of returns, and so it does not matter that the standard deviation, which measures total variability, is twice as large as below-the-mean variability. What is important is that the standard deviation, which is a mathematically convenient concept, gives the same risk rankings for a group of portfolios as would some mathematically intractable below-the-mean measure of variability. This makes it an appropriate measure of risk.

Figure 1: Daily price changes of GM stock from 1976 to 1978 are approximately normally distributed[2]



In this article, we have defined the concepts of risk and return and have discovered how to measure the expected return and risk associated with a single security. Still to be covered are the following issues:

- the expected return and risk for a portfolio;
- the role of diversification in the reduction of portfolio risk;
- the meaning of efficient portfolios;
- the valuation of securities in efficient capital markets.

These will be dealt with in future articles in this series.

References

1 Firer, C. Investment basics XVII. *The Investment Analysts Journal,* No 26, November 1985.

2 Brealey, R and Myers, S. *Principles of Corporate Finance.* McGraw-Hill, 1981, p 135.