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# The valuation of call options on gilts and warrants in South Africa

## Introduction

In July 1984, three banks opened up Reuters' screens offering call and put options on South African gilts. This was a major step in the development of options markets in South Africa, and it is part of the world trend, which began over a decade ago in Chicago, towards more diverse options markets to serve the growing need for flexibility in managing investment portfolios.

Trading of options in Amsterdam and London may be traced back to the last two decades of the seventeenth century[1] and it is thought that the first options were traded in South Africa very soon after the founding of The Johannesburg Stock Exchange in 1887[2]. In South Africa there has been little development of the market, with most of the limited trading being in the over-the-counter market. In addition, although a number of warrants are traded on The Johannesburg Stock Exchange, the volume of trading is very small. However, interest in the formation of options markets of various types has grown and the Reserve Bank has recognised a need to help co-ordinate the actions of the interested parties. It is likely that the first institutional options markets will arise from these discussions.

The objectives of this paper are to describe the current state of the share and gilt options markets in South Africa and to test the applicability of the Black-Scholes options valuation model to selected warrants and gilt options.

## Data and methodology

Data for warrant prices over a number of years, together with the prices of the underlying shares, were obtained from The Johannesburg Stock Exchange. The warrants for which data were sought were AMIC options, East Daggafontein options, ERPM options, Frasers options, Lucem options, Omnia options, Sage options and Western Deep Levels options. Of these only AMIC, East Daggafontein, ERPM and Western Deep Levels were finally analysed because of the sparsity of the data and thinness of trading of the others.

Data on gilt and gilt option prices were obtained from a number of banks, while weekly data for long and short-term gilts and for Treasury bills, for use as surrogates for the risk free rate, were obtained from the Financial Mail.

Black and Scholes[3] developed a model for the valuation of options and this model has formed the basis for most subsequent work in the field of options. The model may be expressed as follows[4]:

$$c = S.N(d1) - Xe^{(-r_f t)}.N(d2)$$

where

c = value of a European call option

$$d1 = \frac{\ln(S/X) + r_f T + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d2 = d1 - \sigma\sqrt{T}$$

S = price of the underlying asset

X = the exercise price of the option

$\sigma^2$ = the instantaneous variance of the returns of the underlying asset

T = the time to maturity of the option

$r_f$ = the risk free rate

N( . ) = the cumulative probability for a unit normal variable

The price of a European put option, P, can be found by the principle of put-call parity, viz

$$c - P = S - Xe^{(-r_f t)}$$

It should be noted that the basic Black-Scholes equation does not apply to securities paying dividends. To take account of dividends a simple, though non-exact, adjustment may be applied[5]. The present value of all dividends expected during the life of the option are deducted from the price of the underlying security before entering that value into the equation. So S in the above formula is replaced by:

$$S - \Sigma e^{(-r_f t_i)}.D_{ti}$$

where

$D_{ti}$ = dividend after a period of time ti.

For ease of use in the analysis of the warrants this formula was extended to the hypothetical case where dividends are paid continuously. In that form, the equation requires only that a single dividend payment and an assumed dividend growth rate be entered into the model[6].

Certain problems arise when trying to apply the Black-Scholes formula to the valuation of warrants. Firstly, the life of a warrant is usually years rather than months, and the variance of the return on the share is likely to change substantially during that period. Secondly, warrants are usually issued on dividend-paying shares and hence the dividends expected to be paid during the life of the warrant must be estimated, especially for long-term options. Thirdly, the exercise price of a warrant sometimes changes on specified dates so that it may pay to exercise a warrant just before its exercise price changes. Another problem is that the exercise of a large number of warrants may result in a significant increase in the number of shares issued. An adjustment for this dilution of the equity is therefore necessary[7]. For the special case where the warrants are issued at no price as an accompaniment to another security, as is the case with all the warrants analysed in this paper, the value of the warrant as calculated by the direct application of the Black-Scholes equation must be multiplied by the ratio of the number of shares in issue before exercise of the warrants to obtain the true value of the warrant.

Gilt options are also options on "dividend"-paying securities, although the dividends in this case are certain. The discrete form of the dividend correction was applied in the analysis of these options.

The gilt cum price includes all accumulated interest on the gilt, whereas the clean price reflects no interest. As a result, the cum price follows a saw-tooth path with respect to the clean price of the gilt. To divorce the option from this saw-tooth effect, gilt options in South Africa are traded on the clean price of the underlying gilt or on the gilt spot rate.

When a gilt call option is exercised, the seller of the gilt receives the full cum price for the gilt based on the clean exercise price previously agreed. Thus, the seller of the option effectively receives a dividend which must be accounted for in the option price valuation. If, for example, the option is bought on the day after coupon payment and exercised on the day before the next coupon payment, the exerciser of the call option will have to pay the seller the full accumulated interest (the cum price). This situation is described in Figure 1 below. Similar logic will apply to an option period straddling a coupon payment period.

Volatility (the $\sigma$ term) plays an important role in the Black-Scholes equation. It is, theoretically, the annualised standard deviation of the instantaneous rates of return of the underlying security. In practice, it is calculated from periodic data either as the annualised standard deviation of the period-on-period returns of a security or as the annualised standard deviation of the continuously compounded returns of the security where:

period-on-period return $= \frac{S_i}{S_{i-1}} - 1$;

continuously compounded return $= \ln \frac{S_i}{S_{i-1}}$ ;

and $S_i$ = security price in period i.

The standard deviations of the returns are annualised by applying a factor to the standard deviations calculated directly from the return data as follows:

$\sigma^2$annual $= f.\sigma^2$

where f = 365,25 for daily data;
f = 260,9 for daily data excluding weekends;
f = 52,2 for weekly data; and
f = 12 for monthly data.

The option price calculated using the Black-Scholes equation is very sensitive to the volatility used, especially if the option is out of the money, and it is, therefore, important to estimate volatility accurately. In practice, the volatility of a share is not constant over time and it may not be appropriate to use historical volatility as an estimate of future volatility. In the case of a long-term option like a warrant, this is particularly so. After preliminary testing of the data for this study, it was considered that a 26 week volatility was appropriate for warrants (using weekly data) and a thirteen-week volatility was appropriate for gilt options (using daily data).

It is worth noting that every actual option price has implicit in it a volatility which will satisfy the Black-Scholes formula for given values of the other variables. This implicit volatility is a useful measure as it can be looked at over time and used to help predict the volatility to use in the formula when valuing options.

During discussions with local market participants, it was noted that the risk free rate to be used in the Black-Scholes equation is a matter of some debate, with the suggestion being made that an opportunity cost rather than a true risk free rate should be used. However, the theory is clear that it is the risk free rate that should be used as this is the rate that will enable a perfect hedge to be achieved by selling options on shares held[3, 6]. This rate will also be a universal rate devoid of any business specific risk. Hence, the risk free rate that should be used is the interest rate on a very low risk note that matures at the time the option expires[8].

The effect of the interest rate on the calculated value of an option is relatively larger on a long-term than on a short-term option. Thus, in general, warrants are quite sensitive to the risk free rate used while gilt options are not. In this study, the short-term bond rate was used for warrants and the Treasury bill rate for gilt options.

**Results for South African warrants**

The objective of this study was to determine the extent to which warrant prices quoted on The Johannesburg Stock Exchange over a period of time may be explained by the Black-Scholes option valuation model suitably adjusted for dividends and dilution of equity. AMIC options, East Daggafontein options, ERPM options and Western Deep Levels options were selected for study.

Data were analysed using a computer program developed for the purpose. A weekly closing share price over the period of study was used and matched against a weekly average warrant price, no weekly closing warrant prices being available. The historical risk free rate relevant in each week was used for the required risk free rate in the formula. In addition, the calculated option value, the volatility and implicit volatility corresponding to each data point were calculated. The means and standard deviations of the errors between the actual warrant price and calculated value, and between implicit volatility and actual volatility, were calculated and subjected to a t-test to determine whether the errors were significantly different from zero.

Table 1 summarises, for each of the options, the result that gives the closest agreement between calculated and actual option values over the period studied.

**Table 1: Summary of results**

| | AMIC | E Dagga | ERPM | W Deep |
|---|---|---|---|---|
| Option value corrected for | Divs | Nil | Nil | Dil+Div |
| Mean of % error in option price | 18% | 9% | 48% | −2% |
| SDev of % error in option price | 23% | 15% | 31% | 28% |
| t-test (option price) | 7,4 | 3,5 | 21,5 | 0,7 |
| Mean of % error in volatility | −18% | −26% | −74% | 7% |
| SDev of % error in volatility | 21% | 41% | 48% | 84% |
| t-test (volatility) | 8,0 | 3,5 | 21,3 | 0,8 |

**Note**

1 No dividends are expected from East Daggafontein and ERPM during the currency of the warrants.

2 Correction for dilution made no significant difference to the above values (see Figure 2).

Western Deep Levels options are the only warrant for which the error between actual and calculated value could not be said to be statistically different from zero.

Figures 2 and 3 show graphically the results of the analysis of AMIC and Western Deep Levels options. Figure 2, in particular, demonstrates the relatively small effect of a dilution correction and the relatively large effect of a dividend correction on option value.

A number of conclusions can be drawn from this study:

- The standard deviations of the errors in option price and volatility are large under all circumstances. This implies that the uncertainty in any individual calculated option value is high even though the model may on average give good results.

- The correction for dilution is usually small and has a relatively small effect on the valuation of the option. For three of the four warrants studied, a slightly better result was obtained by not correcting for dilution than by correcting for it. This is almost certainly due to the fact that the number of warrants in issue is usually very small relative to the number of shares in issue.
- For warrants on shares paying dividends it is essential to include a dividend correction, otherwise the warrant will be considerably overvalued by the model.

The long period of currency of a warrant leads to a number of problems in its valuation. The valuation becomes very sensitive to the risk free rate used and also to the assumption made about future dividends to be paid during the currency of the option. It is, therefore, hard to determine a definite reason for the discrepancy between market price and calculated value. The reason may not lie wholly in the problems of applying the Black-Scholes equation. Warrants are thinly traded as a rule and it is possible that the market itself is valuing them incorrectly. The large standard deviation of the error in option price seen above supports this contention. If this is the case, there is an opportunity to make abnormal profits by buying or selling incorrectly valued warrants. As the exercise date approaches, and the intrinsic value starts to predominate over time and volatility considerations, the market valuation is likely to be more accurate, and it may then be possible to realise profits. It is also likely that option valuation methods will become more widely known in the near future, and the market may then start to reflect values obtained through use of these techniques, in a self-fulfilling way.

**Results for gilt options**

Gilt options appear to be the most heavily traded options in South Africa at present with total daily turnover of underlying gilt stock estimated at between R40 million and R150 million.

A set of 85 actual call option contracts were studied. Unfortunately, no report can be made on put options as insufficient data were available to form a representative sample. The data were evaluated using the Black-Scholes option valuation formula to determine a "model" price and implied volatility and these results are compared with actual prices and volatilities in figures 4, 5, 6 and 7 below. These tests were performed using volatility rates based on 26 weeks, 13 weeks and 6 weeks of daily data as well as an exponentially smoothed 6 week volatility rate.

The results of these tests indicate a 0,98 correlation between calculated prices and actual option prices using a 13 week volatility rate. The correlation for the 26 week and 6 week volatility rates is 0,97 and 0,95 respectively.

Generally, the 13 week (or similar) volatility rate is a better predictor of the actual option price. The average percentage error between calculated and actual option prices is 0,38% with a standard deviation of 25%. (Six-week volatility rate figures are −2,58% and 34% respectively.) The above results are further supported by using a paired difference Student t-test on the results. The low resultant t-values indicate that there is no statistical difference between the actual and calculated option price even at the 20% significance level.

From the above it is suggested that the Black-Scholes option valuation model is a valid model to be used in the pricing of gilt options in South Africa. This result may well have been expected as the present market makers use this model as a basis for their option pricing. However, the prices offered on the Reuters' screens often vary significantly from the Black-Scholes valuation, whereas actual prices are well represented by this valuation. This indicates that most, if not all, traders use the Black-Scholes (or similar) valuation model to calculate option prices.

**Conclusion**

The current interest in options is unprecedented and it seems likely that development of options markets will be rapid. Gilts have taken the lead at present under the sponsorship of the banks.

Major questions about the application of the Black-Scholes equation revolve about the volatility and risk free rate to use, and in the case of gilts about the use of clean or cum values and the treatment of coupon payments.

The results of this study indicate that while the effects of dilution on warrant prices is usually small, it is essential to include a dividend correction otherwise the warrant is considerably overvalued by the model. It was found further that the most reliable volatility measurement to use for warrants was based on 26 weeks of weekly share price data. The risk free rate to be used is the interest rate on a very low risk note that matures at the same time the option expires.

Based on the above assumptions, the model gives good results on average. Large discrepancies do exist, however. These may be caused by the thin trading of the warrants or it is possible the market itself is valuing them incorrectly.

Gilt options are generally well valued by the Black-Scholes model. Tests indicate that a volatility rate based on 13 weeks (or similar) of daily data gives good results. The risk free rate to be used is the Treasury bill rate. All calculations should be based on the clean gilt price and the effect of the coupon payment as a "dividend" must be accounted for in the option price calculation.

Options have a place in the range of investments available to the investor. They can be used for speculative and hedging purposes and so give him a great deal of flexibility in managing his portfolio. However, to be effectively utilised, they must be properly valued. This paper has shown that, provided the necessary adjustments are made, the Black-Scholes model can be used to value both warrants and gilt options.

**References**

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**Figure 1: Gilt prices**

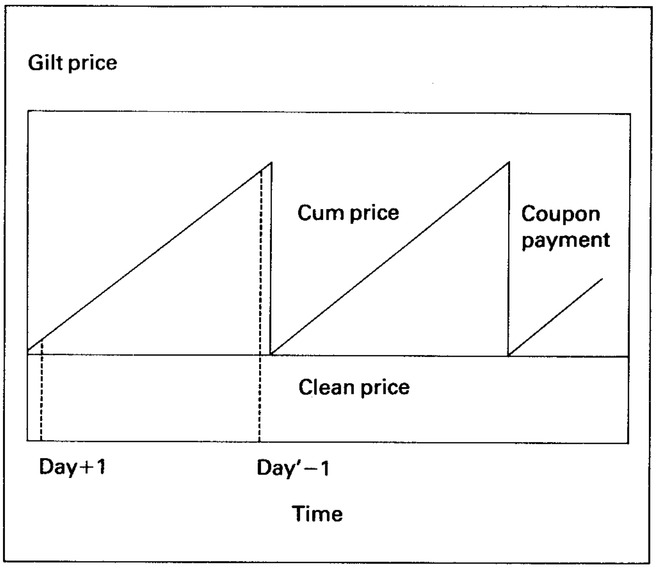


**Figure 3: Western Deep Levels options**

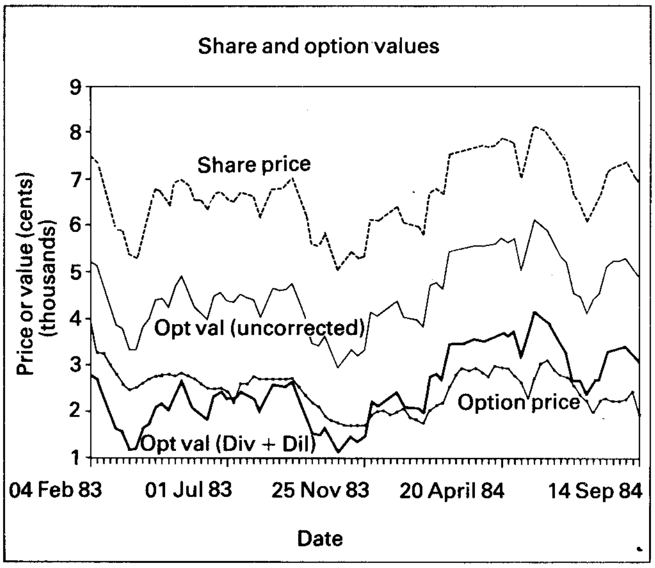


**Figure 2: AMIC options**

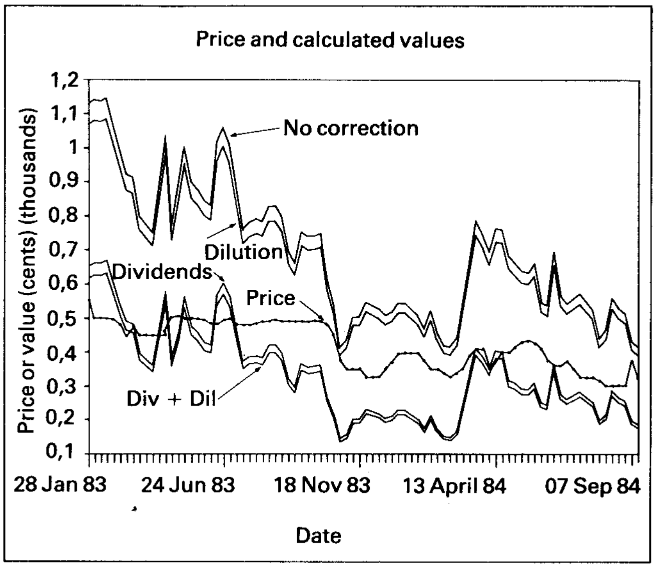


**Figure 4: Gilt option prices**

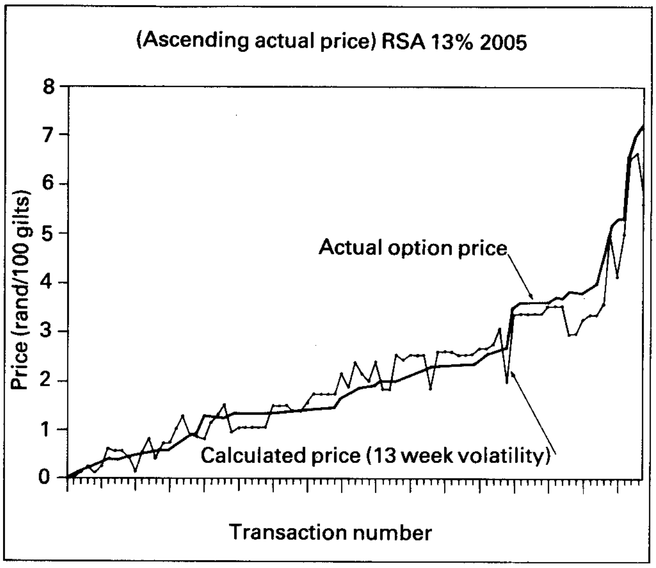

**Figure 5: Gilt option prices**

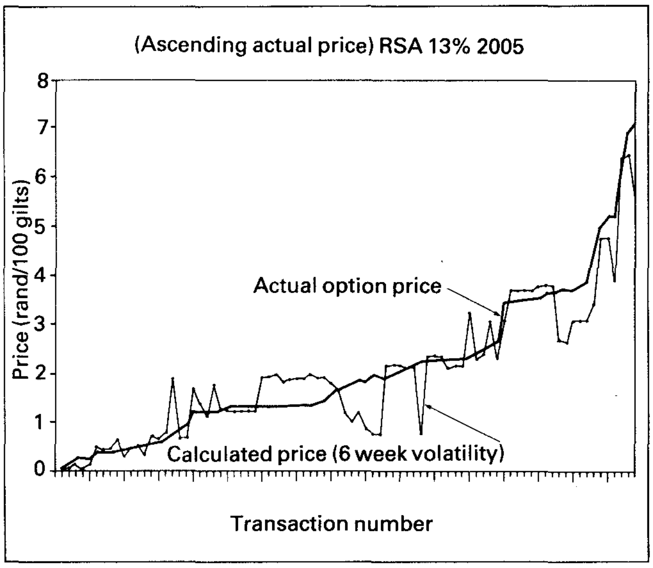


**Figure 7: Gilt options**

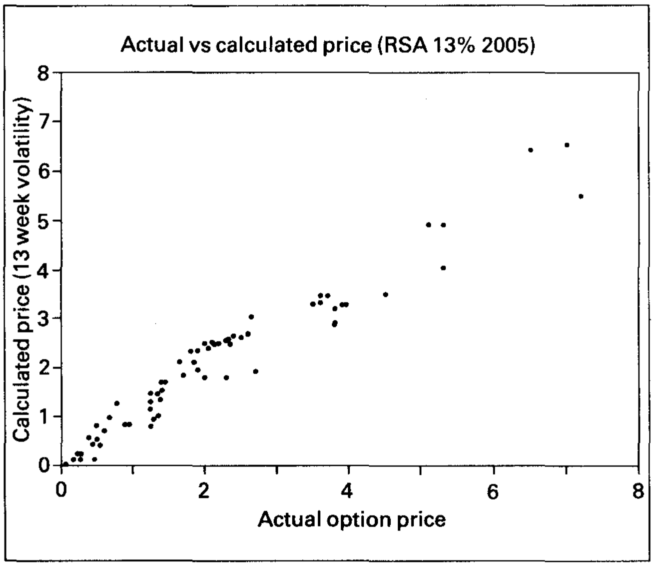


**Figure 6: Gilt option frequency graph**