

The gold price and the forward rate of interest

Introduction

Implicit in the term structure of interest rates are forward rates of interest. These forward rates can be regarded as forecasts of interest rates. The purpose of this paper is to explain the source of errors in the forecasts of short-term interest rates implied by the term structure of short-term interest rates in South Africa. The paper follows on from the work of M S Nurick, who tested the efficiency of such interest rate forecasts. (Nurick, 1982.)

The determination of forward rates

The work of Irving Fisher (1930), Friedrich Lutz (1940) and J R Hicks (1946) established that long-term interest rates over any period may be regarded as the average of the forward shorter term interest rates over the same term. Therefore, implicit in any term structure of interest rates are forward rates of interest. For example, implicit in rates quoted for a 6-month negotiable certificate of deposit (NCD) and a 3-month NCD on the same day is a rate of interest for a 3-month NCD in three months' time. The lender or borrower has a choice of lending or borrowing for six months at a fixed rate, or lending or borrowing for three months and then re-negotiating a further three-month transaction. The rate of interest for the second three-month period that would equalise the returns from the two transactions may be regarded as the implicit forward rate.

For example, consider a unit amount invested in two possible ways over a period of six months:

- (1) by buying a 6-month NCD;
- (2) by buying a 3-month NCD and then another 3-month certificate in three months' time.

If we let:

NCD6 be the (annual) rate for a 6-month NCD.

NCD3 be the (annual) rate for a 3-month NCD bill.

NCD3F be the (annual) rate for a 3-month NCD which will apply in three months' time.

In equilibrium, the accrued sum in both cases must be the same, thus:

$$(1 + \frac{NCD6}{2}) = (1 + \frac{NCD3}{4}) (1 + \frac{NCD3F}{4})$$

Solving the above equation for NCD3F yields the three months forward NCD3 rate.‡

Such a forward rate may be regarded as the market's expectation of rates. Clearly, if short-term rates were expected to be different from that implied by the formula, any economic actor who held a different view could hope to make superior returns. For example, if the implicit forward rate were regarded as too low, the obvious response would be to borrow for the longer period and lend consecutively for shorter periods. Such actions would of themselves raise the longer-term rates and reduce the shorter ones, so re-establishing the implicit forward rate according to expectations.

Nurick tested the forward rates implicit in 3-month and 6-month NCD yields in South Africa for their efficiency as forecasts of the 3-month NCD rate, using month-end data. It should be understood that such forecasts do not have to be accurate to be efficient. Inaccurate forecasts may be the result of additional information that only becomes available to participants in the market after the forecasts have been established. What is important for the efficiency of forecasts is that the errors in the forecasts do not reveal any systematic bias that could be exploited in advance by market participants.

Following the approach of Hamburger and Platt (1975), Nurick investigated the relationship between the errors in the forecasts and changes in the spot rate that arose over the period. Nurick indicated that:

"the forecast error is approximately equal to the actual change in spot rates which occurred during the forecast period, plus a constant, which may be viewed as an estimate of a constant liquidity premium. Since new information, which was unavailable at t , generates actual changes in the spot rate, three months hence, then if forecast errors coincide with these spot rate changes, it follows that forecast errors are due solely to the receipt of new information, or to unanticipated effects. This situation is easily recognisable as being consistent with optimal expectations formation, with NCD3F (adjusted) appearing thus far as a relatively weak, though optimal, forecaster of the expected future interest rate."

(Nurick (1982), p 104.)

Nor could Nurick refute the so-called semi-strong hypothesis about efficiency. He was not able to establish a forecasting model that would have done better than the yield curve in forecasting the 3-month NCD rate and their forecasting abilities. It should be emphasised that such alternative models have to be tested with realistic assumptions about the actual information available to market participants when they make their forecasts. It is, of course, possible to explain, with hindsight, why the spot rates (and the forward rates) moved. In South Africa, movements in short-term interest rates would be directly affected by changes in the price of gold. The state of the balance of payments or, more particularly, the level of foreign exchange reserves net of foreign liabilities held by the Reserve Bank, would clearly influence short-term interest rates and exchange rates. The balance of payments would, in turn, be very obviously affected by the dollar price of gold.

For these reasons, the links between the errors in the forecast of short-term interest rates implied by the structure of NCD rates and changes in gold price were examined.

The results of this examination are illustrated graphically for the period 1980 to 1985. Approximately month-end data were used. Figure 1 indicates the relationship

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‡For a more general formula and its derivation, see Nurick (1982), pp 96-97.

between the 3-month and 6-month NCD rates. Clearly, such movements are highly correlated. Figure 2 shows the relationship between the actual 3-month NCD rate and the 3-month NCD rate forecast for that point in time, as implied by the yield curve three months previously. In Figure 3, the relationship between differences in the actual and forecast NCD rates are graphed and are related to changes in the gold price. Changes in the gold price are regarded as the difference between the gold price ruling on the day the error was revealed and the gold price approximately three months before. In Figure 4, movements in the gold price and the rand/US dollar exchange rate are illustrated. As may be read in Figure 3, the forecast errors revealed no obvious cyclical pattern. Furthermore, it indicates that when the gold price in dollars falls, the interest rates implied by the yield curve are underestimates. When the gold price rises, the forecasts are overestimates. It was found that the relationship between the forecast errors and changes in the rand price of gold had little explanatory power.

The negative relationship between dollar gold price changes and unexpected movements in the NCD rate is also confirmed by regression analysis. The relationship between the forecast errors ($ERR = NCD3_t - NCD3F_t$) and gold price changes is statistically significant even though the variability in the forecast errors is high and the explanation due to the gold price movement only about 10%. One reason for this is apparent from Figure 3. It is seen that, although changes in the gold price (GDPL3) in one direction are generally associated with unexpected changes in NCD (ERR) in the opposite direction, the magnitude of such changes are not always the same.

The regression equation is as follows for the period January 1981 to May 1985.

$$NCD3_t - NCD3F_t = 1,980,971 - 0,00723 GDPL3 \quad (1)$$

(5,676) (-2,309)

$$R^2 = 0,095; R^{-2} = 0,077; DW = 0,906$$

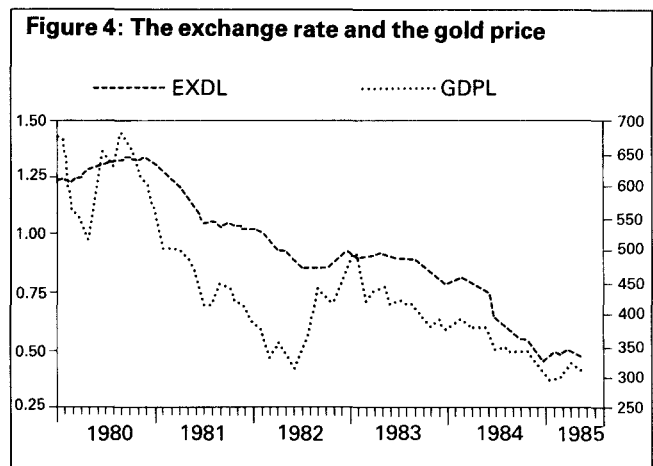
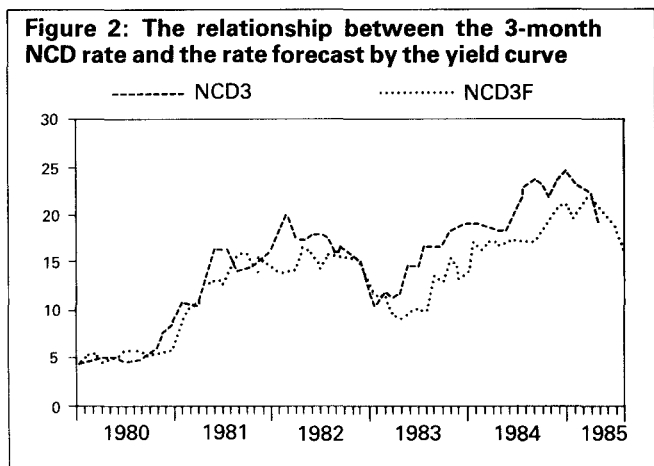
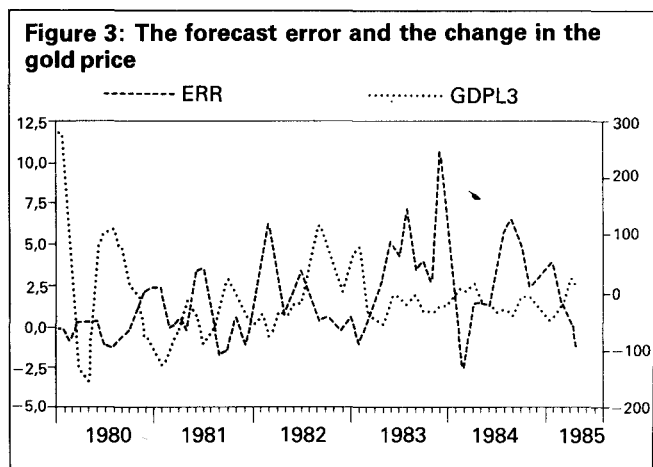
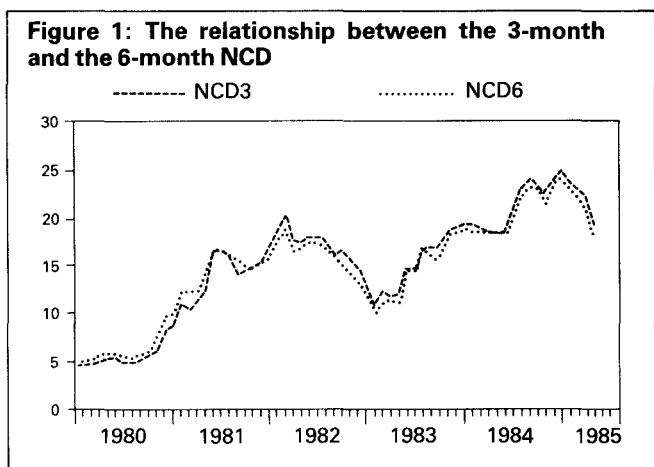
where $NCD3_t$ is the actual 3-month NCD rate, $NCD3F_t$ is the forward rate established three months previously and $GDPL3$ is the change in the gold price over that three-month period. t-Statistics are indicated in parentheses. Clearly, while the gold price effects are statistically significant, the regression fit itself is not a very close one.

The simple Pearson correlation coefficients between the forecast errors and changes in the gold price and lagged changes in the gold price is as follows:

i	CORR (ERR, GDPL3 (-i))
0	-0,3077
1	-0,2384
2	-0,0990
3	-0,0394

which, perhaps, is evidence of a stronger link between the variables than is indicated by a regression equation.

The DW statistic for equation (1) also indicates serial correlation in the error term. On investigation it was found that both the errors in forecasting interest rates and gold price changes exhibit a similar pattern over the first two lags as is indicated by the autocorrelations for both the forecast error and the gold price changes.



Autocorrelation function

LAG	ERR	GDPL3
1	0,5389	0,6932
2	0,2149	0,3292
3	-0,0604	0,1480
4	0,0799	0,1801
5	-0,0486	0,1851

As might be expected from the earlier discussion, movements in the 3-month NCD rate over three months and changes in the dollar gold price over the same period are also related in a statistically significant way. Again, the series is a rather variable one, producing rather a low R^2 , with evidence of serial correlations. The equation is as follows:

$$NCD_t - NCD_{t-3} = 0,74157 - 0,0133582 \text{ GDPL3} \\ (2,7938) \quad (-3,77)$$

$$R^2 = 0,184379 \quad R^{-2} = 0,171432 \text{ for 65 observations 1980 - 01 - 1980.05}$$

Where NCD_t represents today's rate and NCD_{t-3} represents the NCD 3-month rate three months previously, clearly, the gold price matters for short-term interest rate movements in South Africa even if it is not the only influence at work.

The implication of this analysis is that the success of the market's forecasts of short-term interest rates will be related to the success with which the market is able to forecast the price of gold in dollars.

As is indicated in Figure 4, there was much room for disappointment in the behaviour of the dollar gold price over this period. The results of this study, it is believed, add further support to the view that the market in short-term paper in South Africa is efficient. That is to say, the market can be beaten, as can almost any market in South Africa, but only with more accurate forecasts of the gold price.

References

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