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Investment basics XIX
Risk and return – part 2

Introduction

Definitions of risk and return for a single security were given in part 1 of this article. It will be recalled that the historical return for a given period of time is:

$R = (V_1 - V_o + D_1)/V_o$

where

R = return on the security

V_1 = market value of the security at the end of the period

V_o = market value of the security at the beginning of the period

D_1 = cash distribution to the investor at the end of the period.

We could readily calculate the future return on a security by means of the above formula if we knew the future values of V_1 and D_1. But in an uncertain world we cannot forecast V_1 with any accuracy and will often be unsure of D_1. The best that we can do in such circumstances is to form estimates of V_1 and D_1. These estimates will depend upon our perceptions of the environment, especially its political, economic and business dimensions, in addition to our perceptions of internal factors such as management, product quality and so on. In general, our estimates for V_1 and D_1 will be higher in relation to a favourable expected environment than they would be for an unfavourable expected environment. Accordingly, we deduce that the overall expected return for a security is the sum of the returns associated with each possible environmental state multiplied by the probability of occurrence of such state.

As regards risk, it was suggested that an appropriate and convenient surrogate is the standard deviation of the overall expected return. The standard deviation of the return for a single security was shown to be calculated thus:

1. For each environmental state, calculate the square of the deviation between the return for that state and the overall return.

2. Multiply the squared deviations by the respective state-probabilities and sum these products, thus obtaining the variance of returns.

3. Take the square root of the variance, thus obtaining the standard deviation.

Portfolio risk and return

The case of risk and return for a single security having been dealt with, the next step is to consider the question of risk and return on a portfolio consisting of two securities, X and Y. The conclusions reached in examining this minimum portfolio will be found to be broadly applicable to portfolios comprising any number of securities.

Assume that the portfolio consists of 60% of security X and 40% of security Y where X and Y have the state-dependent returns given in columns 3 and 4 of Table 2. The overall expected returns on X, Y and the portfolio are calculated in columns 6, 7 and 8 of the table.

Table 2: Return on a portfolio of securities X and Y

(1) State	(2) Probability P	(3) Return on security X R_x	(4) Return on security Y R_y	(5) Return on portfolio R_p	(6) = (2) × (3) pR_x	(7) = (2) × (4) pR_y	(8) = (2) × (5) pR_p
a	0,10	5,0%	–1,0%	2,6%	0,50%	–0,10%	0,26%
b	0,40	7,0%	6,0%	6,6%	2,80%	2,40%	2,64%
c	0,30	–4,0%	2,0%	–1,6%	–1,20%	0,60%	–0,48%
d	0,20	15,0%	20,0%	17,0%	3,00%	4,00%	3,40%
Expected returns							
	$E(R_x)$				5,10%		
	$E(R_y)$					6,90%	
	$E(R_p)$						5,82%

It may be seen from Table 2 that, as we would intuitively suppose, the expected return on a portfolio is simply the weighted average of the expected returns on the individual securities of which it is composed. In notation, where W_x and W_y represent the weightings of X and Y:

$$E(R_p) = W_x E(R_x) + W_y E(R_y) \quad (7)$$

$$5{,}82\% = 0{,}6 \times 5{,}10\% + 0{,}4 \times 6{,}90\%$$

Let us now proceed to measure the risk inherent in our two-security portfolio. We know that risk for a single security is equivalent to the standard deviation of its returns. Portfolio risk is calculated in the same way, as is shown in Table 3:

Table 3: Risk on a portfolio of securities X and Y

(1) State	(2) Probability P	(3) Probability x squared deviation for security X $p(R_x-E[R_x])^2$	(4) Probability x squared deviation for security Y $p(R_y-E[R_y])^2$	(5) Probability x squared deviation for portfolio $p(R_p-E[R_p])^2$
a	0,10	0,001%^{2}	6,241%^{2}	1,037%^{2}
b	0,40	1,444%	0,324%	0,243%
c	0,30	24,843%	7,203%	16,517%
d	0,20	19,602%	34,322%	24,998%
Variances		σ_x^2 = 45,890%^{2}	σ_y^2 = 48,090%^{2}	σ_p^2 = 42,795%^{2}
Standard deviations		σ_x = 6,774%	σ_y = 6,935%	σ_p = 6,542%

By way of illustration, the fourth figure in column 3 is arrived at as follows:

$19{,}602 = 0{,}2 \times (15{,}0 - 5{,}10)^2$

Tables 2 and 3 illustrate a fact of profound and far-reaching importance. Whereas the expected return on a portfolio is simply the weighted average of the expected returns of its constituent securities, the risk of a portfolio, σ_p, is in general not equivalent to the weighted average of the standard deviations of its constituent securities, σ_x and σ_y. As may be seen from Table 3, σ_p equals 6,54%, which is less than either σ_x (6,77%) or σ_y (6,93%). This result, which runs counter to one's intuition, has, nevertheless, a simple explanation, the appreciation of which is basic to an understanding of the conditions under which a portfolio can reduce risk. The risk of a portfolio depends not only on the risks that are particular to each of its constituent securities but also on the extent to which the returns on each security are jointly affected by underlying events, ie different environmental states. If, across the spectrum of possible future environmental states, high returns on X coincide with low returns on Y, the risk in a portfolio of X and Y would be relatively low because the individual returns have a tendency to offset each other. On the other hand, if high returns on X coincide with high returns on Y and low returns on X with low returns on Y, portfolio risk will be high owing to the relative absence of compensating effects. The degree to which the returns of X and Y vary together is measured by a statistical quantity known as the co-variance. The covariance is defined by multiplying the standard deviations of the returns on X and Y by another statistical quantity, namely the correlation coefficient between the returns on X and Y. The calculations pertinent to our example are set out below:

Table 4: Correlation and covariance of securities X and Y

(1) State	(2) Probability	(3) Deviation of returns for security X $(R_x - E[R_x])$	(4) Deviation of returns for security Y $(R_y - E[R_y])$	(5) Probability – weighted product of deviations $p(R_x - E[R_x])(R_y - E[R_y])$
a	0,10	−0,10%	−7,90%	0,079%^{2}
b	0,40	1,90%	−0,90%	−0,684%
c	0,30	−9,10%	−4,90%	13,377%
d	0,20	9,90%	13,10%	25,938%
Covariance (COV_{xy})				= 38,710%^{2}

$$\text{Correlation } (COR_{xy}) = \frac{COV_{xy}}{\sigma_x\sigma_y} = \frac{38{,}710\%^2}{6{,}77\% \times 6{,}93\%} = 0{,}825\%$$

It can be shown in the case of the two-security portfolio, that portfolio risk is related to the risks of its constituents in terms of the formula:

$$\sigma_p = [W^2_x\sigma_x^2 + (1 - W_x)^2\sigma_y^2 + 2W_x(1-W_x)COR_{xy}\sigma_x\sigma_y]^{1/2} \quad (8)$$

which, in terms of the example in Table 3, gives the following result:

$$\sigma_p = [(0{,}6^2 \times 6{,}77^2 + 0{,}4^2 \times 6{,}93^2 + 2 \times 0{,}6 \times 0{,}4 \times 0{,}825 \times 6{,}77 \times 6{,}93)]^{1/2} = 6{,}54\%$$

As is intuitively obvious, the correlation coefficient, COR_{xy}, has a major effect on portfolio risk, other things being equal. An illustration of this effect is provided in Table 5.

Table 5: Effect of correlation of returns on portfolio risk

	Weights	Return	Risk with correlation equal to −1	0	1
Security X	0,5	10%	6%	6%	6%
Security Y	0,5	12%	10%	10%	10%
Portfolio	1,0	11%	2%	5,8%	8,0%

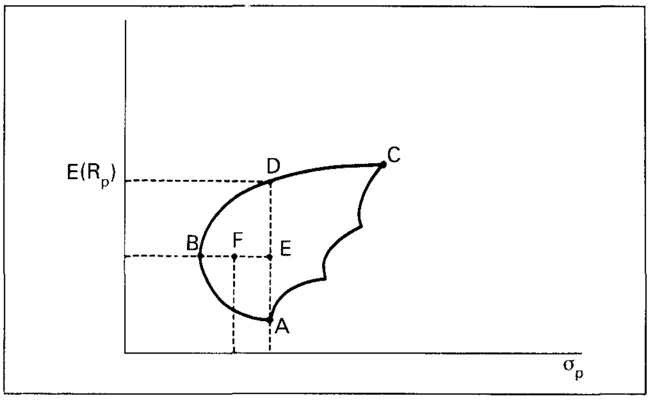
As can be seen from Table 5, and verified from equation (8), portfolio risk varies inversely with the correlation of returns. With perfect correlation of returns ($COR_{xy} = 1$) portfolio risk is simply the weighted average of the risks of the individual securities ($8\% = 0{,}5 \times 6\% + 0{,}5 \times 10\%$). The less the degree of correlation, the greater the risk-reducing benefits of a portfolio in the sense that portfolio risk is less than the weighted average of the individual security risks. Indeed, with perfect negative correlation ($COR_{xy} = -1$), it would be possible to choose weights for X and Y such that portfolio risk would be zero.

Efficient portfolios

In general, it is possible to reduce the risk of a portfolio only at the expense of reducing its return. But some portfolios are more efficient than others in the sense that their composition is such that they provide the maximum possible return for a given level of risk or the minimum level of risk for a given return.

The efficient portfolio concept is illustrated in Figure 2:

Figure 2: Investment portfolio opportunities



Given the individual returns for a group of securities, it is possible to combine them into innumerable portfolios each having a return, $E(R_p)$, and a risk, σ_p. These various risk-return combinations would be bounded on the left and from above as indicated by the curve ABC in Figure 2. It will be obvious that any portfolio lying on the boundary segment BC is efficient in that it provides an optimum risk-return combination compared with that provided by any point lying below it or to its right. In Figure 2, portfolio E is superior to portfolio A because at the same level of risk it offers a greater return than A does. However, portfolio E is clearly inferior to portfolio D. Portfolio D is an optimal risk-return combination because no other combination of securities at the given risk level offers a better return. And the same applies in the horizontal dimension. Portfolio F is superior to portfolio E because its risk is less for a given level of return but portfolio B dominates both F and E.

The boundary segment BC constitutes what is known as the efficient frontier. Every point on it is optimal and none is objectively more so than any other. Portfolio C's return, for example, is higher than portfolio B's but so is its risk. Investors with low risk-aversion would prefer portfolio C but those with high risk-aversion would prefer portfolio B and so the only criterion for choosing between them, or any other pair of portfolios lying on the efficient frontier, is the subjective one of the investor's attitude towards risk.

Thus far, we have been concerned with defining the concepts of risk and return for a security and a portfolio of securities. We have seen that the risk of a portfolio is less than the weighted average of the risks of its constituents when the correlation of their returns is less than unity and we have briefly explored the concept of an efficient frontier of portfolios. In the next article of this series, we shall consider the risk-reducing properties of portfolios a little more deeply. We shall find that the risk that can be potentially eliminated by diversification is that which is peculiar or unique to the individual company. All companies, however, are to varying degrees similarly affected by the operation of political and economic forces, and the pervasive effect of these forces on company earnings, dividends and share prices constitutes a systematic or market risk that cannot be eliminated by diversification. Thus, the risk of a well-diversified portfolio is the market risk of its constituent securities. These considerations will naturally lead us to a more precise definition of market risk and to a description of the capital asset pricing model in relation to the valuation of securities.