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Forecasting share prices on The Johannesburg Stock Exchange using multivariate time series analysis

Introduction

The objective of this study is to determine whether share price fluctuations on The Johannesburg Stock Exchange are systematically related to economic activity and whether this relationship can be used to successfully forecast share prices. The Box-Jenkins transfer function model-building methodology will be used in determining whether any relationship exists and, if so, use the approach to generate forecasts of share prices.

Transfer function models

The general objective of the Box-Jenkins transfer function model-building technique is to estimate the dynamic response characteristics between a change in the level of one variable and a change in the level of another. If, for example, a series can be found which consistently leads share prices it might be possible to obtain accurate forecasts of share prices.

The transfer function model-building methodology used here is described in numerous publications, amongst others Box and Jenkins (1970), Makridakis, Wheelwright and McGee (1983) and Vandaele (1983). The approach will, therefore, only be briefly outlined.

The transfer function model is written in two general forms. The first form is:

$$Yt = V(B)\,Xt + Nt \qquad \dots 1$$

where
- Yt = the output series (eg share prices)
- Xt = the input series (eg interest rates)
- Nt = the combined effects of all other factors influencing Yt (referred to as the noise)
- $V(B) = V_o + V_1B + V_2B^2 + \dots + V_KB^K$ where K is the order of the transfer function and B is the backward shift operator.

The input and the output series must, if necessary, be appropriately transformed to take care of non-stationary variance and differenced to take care of non-stationary means. The order of the transfer function K can sometimes be large and for this reason the transfer function is also written as:

$$Yt = \frac{W(B)}{(B)} X_{t-b} + \frac{\theta(B)}{\phi(B)} A_t \qquad \dots 2$$

where
$$W(B) = W_o - W_1B - W_2B^2 \dots W_sB^s$$
$$\delta(B) = 1 - \delta_1B - \delta_2B^2 \dots \delta_rB^r$$
$$\theta(B) = 1 - \theta_1B - \theta_2B^2 \dots \theta_qB^q$$
$$\phi(B) = 1 - \phi_1B - \phi_2B^2 \dots \phi_pB^p$$

y_t = the transformed and differenced Yt value

x_t = the transformed and differenced Xt value.

At = A random noise value.

$\theta(B)$ and $\phi(B)$ are the moving average and autoregressive operators for the noise term.

r, s, p, q and b are constants to be determined using the Box-Jenkins approach.

The second form is considered to be more restrictive because the values of r, s, p and q are usually much smaller than the value of K in the first equation. In the second equation the subscript of x is t−b, which means that there is a delay of b periods before x begins to influence y. One of the purposes of transfer function modelling is to determine values for r, s, b, p and q.

The transfer function model-building methodology involves four steps:

- Identification
- Estimation
- Diagnostic testing
- Forecasting

The identification step attempts to determine the values of r, s, b, p and q. The principal analytical tools used are the estimated autocorrelation function of each series and the estimated cross-correlation function between them. It is first necessary to achieve stationarity of the input series, to prewhiten the series and then apply the same transformation to the output series so that the cross-correlation function between the two prewhitened series can be determined.

Apart from the input series there are usually in practical applications other factors causing changes in the output series. The combination of all these other factors is referred to as noise. The identification of a transfer function is complete when this residual noise is reduced to white noise. The residual noise series is modelled using the univariate approach. When the identification phase is completed the second phase begins, which involves estimating all the parameters by least squares fitting. Diagnostic checks are then performed to establish whether the model is adequate. This is done by examining the autocorrelation function of the residuals and the estimated cross-correlation function of the prewhitened input series and the residuals. Finally, the model is used to obtain forecasts of the required series.

Identifying leading indicators

Before a transfer function can be developed, leading indicators of share prices need to be identified. In this study the following candidates were considered to be possible leading indicators of share prices:

- The leading indicator of the South African economy published by the Reserve Bank as well as that of the Standard Bank.
- The yield on long-term RSA stock.
- The rate of growth in the money and near-money supply (M2).

The most likely candidate for a leading indicator is perhaps a leading indicator of the South African economy. Therefore, the leading indicator of the Reserve Bank and Standard Bank were chosen. The leading indicator of the Reserve Bank is published quarterly with a one quarter publication lag. That of the Standard Bank is published monthly with a one-month publication lag.

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The yield on long-term RSA stock was also chosen as a possible candidate as it is believed that an increase in the yield will lead to a decline in prices. Likewise a decrease in the yield on RSA stock will result in an increase in share prices. The yield on RSA stock is published monthly with no lag.

There is a widespread belief among economists that changes in the money supply have an important influence on the prices of common stocks. Empirical studies have confirmed a relationship between money supply and share prices. One of the first empirical studies to pay attention to the relationship between money and share prices was performed by Sprinkel (1964). Sprinkel found that changes in the money supply appear to lead changes in share prices and can thus be used to predict share prices. Hamburger and Rochen (1972) also concluded that monetary changes have a lagged effect on the share market. For this reason money and near-money was selected as a possible leading indicator of share prices. Information on the money supply is published monthly with a two month publication lag.

For each of the above indicators, 48 observations were obtained, one for the end of each calendar quarter beginning with the first quarter in 1970 and ending with the last quarter of 1981. The period 1970 to 1981 will be used to develop a transfer function model while the period 1982 to 1985 (3rd quarter) will be used to test the performance of the model.

Developing a transfer function model

In this section an attempt will be made to build a transfer function model that can be used to forecast quarterly share prices. The process will be illustrated using long-term interest rates as the input series. Results for the other two candidates will only be mentioned.

The first step in developing the transfer function model is to establish whether the input series is stationary in the mean and variance. A test suggested by Bendat and Piersol (1968) is used for this purpose. The test procedure will not be discussed in detail here. Only a brief outline will be given. The procedure involves dividing the sample record into a number of equal time intervals and then computing the mean and variance for each interval. The sequence of mean and variance values are then tested for the presence of underlying trends or variations other than those due to expected sampling variations. The results obtained using this procedure indicated that the input series was not stationary with regard to the mean but stationary with regard to the variance.

The autocorrelation function of the input series (see Figure 1) confirmed that the series was nonstationary in the mean. (The autocorrelation function cannot be used to test the stationarity of the variance.) First differences of the input series were taken in an attempt to achieve stationarity in the mean. The autocorrelation function of the first differences are shown in Figure 2 which appears to indicate that stationarity in the mean has been achieved.

Having achieved stationarity in the mean and variance it is now necessary to prewhiten the input series. The estimated autocorrelation function and the estimated partial autocorrelation function in Figure 2 has the appearance of an autoregressive – moving average process with autoregressive factor order one and a moving average factor of order one. A model of this form is shown in Figure 3:

$$(1 - \phi_1 B)\, Xt = (1 - \theta_1 B)\, \ell t \qquad 3$$

Figure 1: Sample autocorrelation function of the input series

```
          -1.0  -0.8  -0.6  -0.4  -0.2   0.0   0.2   0.4   0.6   0.8   1.0
Lag corr.
                                          I
 1 0.855                          +       IXXXXXX+XXXXXXXXXXXXXX
 2 0.687                       +          IXXXXXXXXXX+XXXXXX
 3 0.516                    +             IXXXXXXXXXXXXX
 4 0.384                  +               IXXXXXXXXXX  +
 5 0.279                +                 IXXXXXXX       +
 6 0.231                +                 IXXXXXX        +
 7 0.201                +                 IXXXXX         +
 8 0.177                +                 IXXXX          +
 9 0.174                +                 IXXXX          +
10 0.176                +                 IXXXX          +
11 0.208                +                 IXXXXX         +
12 0.204              +                   IXXXXX          +
13 0.205              +                   IXXXXX          +
14 0.185              +                   IXXXXX          +
15 0.151              +                   IXXXX           +
```

Figure 2: Sample autocorrelation and partial autocorrelation function of first differences of the input series

Autocorrelations

```
          -1.0  -0.8  -0.6  -0.4  -0.2   0.0   0.2   0.4   0.6   0.8   1.0
Lag corr.
                                          I
 1  0.254                        +        IXXXXXX+
 2  0.263                       +         IXXXXXXX+
 3  0.072                       +         IXX     +
 4  0.115                       +         IXXX    +
 5 -0.083                       +       XXI       +
 6 -0.064                       +       XXI       +
 7 -0.187                       +   XXXXXI        +
 8 -0.234                       +  XXXXXXI        +
 9 -0.213                     +     XXXXXI          +
10 -0.248                     +    XXXXXXI          +
11  0.012                     +           I         +
12 -0.258                     +    XXXXXXI          +
13 -0.078                    +          XXI          +
14 -0.073                    +          XXI          +
15  0.030                    +             IX        +
```

Partial autocorrelations

```
          -1.0  -0.8  -0.6  -0.4  -0.2   0.0   0.2   0.4   0.6   0.8   1.0
Lag corr.
                                          I
 1  0.254                          +      IXXXXXX+
 2  0.212                          +      IXXXXX +
 3 -0.039                          +     XI      +
 4  0.059                          +      IX     +
 5 -0.139                          +   XXXI      +
 6 -0.064                          +    XXI      +
 7 -0.129                          +   XXXI      +
 8 -0.172                          +  XXXXI      +
 9 -0.059                          +     XI      +
10 -0.136                          +   XXXI      +
11  0.191                          +      IXXXXX +
12 -0.263                          XXXXXXXI      +
13 -0.037                          +     XI      +
14  0.018                          +      I      +
15 -0.069                          +    XXI      +
```

where ϕ^1 = first order autoregressive parameter to be estimated

ℓt = the error term

θ^1 = first order moving average parameter to be estimated.

The parameters ϕ^1 and θ^1 are estimated using a least squares criteria. The estimated value of each parameter together with the t-values are:

Parameter	*Estimate*	*t-value*
θ^1	0,4663	1,96
ϕ^1	0,7607	3,50

A diagnostic check of the residuals of the model revealed that it represents a white noise, ie none of the calculated autocorrelations and partials of the residuals were significant. Furthermore, the quantity

$$Q = T\sum_{t=1}^{K} \gamma t(e)$$

where T = net number of observations

K = total number of autocorrelations estimate

$\gamma t(e)$ = estimated autocorrelation of residual e at lag t

is approximately distributed as X^2 with K-p-q degrees of freedom where p is the number of autoregressive parameters and q is the number of moving average parameters. The calculated value of Q is 12,72 for the first 15 autocorrelations. This is significant at the 0,05 level. The model estimated above was used to prewhiten the input series.

Having prewhitened the input series the next step is to prewhiten the output series (the industrial index in this example). The output series is prewhitened with the ARIMA model of the input series.

Once the input and output series have been prewhitened the cross-correlations for the input and output series can be determined. The cross-correlation function for the two series is shown in Figure 3. The cross-correlation function is used to determine the parameters b, r and s. The value of b is equal to the lag of the first significant cross-correlation. From Figure 3 one can, therefore, conclude that b = o. The interest rates on long-term government stock is, thus, not a leading indicator of share prices on The Johannesburg Stock Exchange. It is coincidental and this implies that interest rates would first have to be forecast one period ahead before share prices can be forecast. This obviously means that the forecast error will be bigger and that the transfer function model is less useful.

The same approach was used with the other leading indicators mentioned. It was found that all three of them do not lead the Industrial Index. As the two composite indexes as well as the money supply is published with a certain lag, use of these indicators would require a forecast of them of at least two periods ahead. This normally tends to generate larger forecast errors. For this reason their use in forecasting share prices was not further investigated.

The denominator order r is found by examining the cross-correlations for any pattern in them. This pattern, if it exists, will correspond to the autoregressive correlation function of an AR(r) model. If it does not exist then r = o. From Figure 3 it would appear that there is no pattern and therefore r = o.

The numerator order s is the number of periods that the AR(r) pattern is delayed. If there is no AR pattern in the cross-correlation function when S equals the number of non-zero cross-correlations less one. It would, therefore, appear that s = o.

The task of determining the parameters r and s are, however, not always crystal clear. It is rare to examine a cross-correlation diagram and have these two values (r, s) make themselves known unequivocally. It is usually necessary to try different values of r and s. The test of adequacy for the different models chosen is to estimate the parameters and then test the residuals for adequacy of it. Before the parameters can be estimated, however, the noise model XZt must be identified so that it can be estimated simultaneously with the transfer function.

Figure 3: The cross-correlations between the prewhitened input series and the prewhitened output series

```
             -1.0  -0.8  -0.6  -0.4  -0.2  0.0   0.2   0.4   0.6   0.8   1.0
Lag corr. ----------------------------------------------------------------
                                        I
 -15 -0.194                    +  XXXXXI       +
 -14  0.013                    +        I      +
 -13  0.168                    +        IXXXX  +
 -12 -0.064                     +     XXI     +
 -11 -0.142                     +   XXXXI     +
 -10  0.268                     +       IXXXXXXX+
 - 9 -0.141                     +   XXXXI     +
 - 8 -0.046                     +      XI     +
 - 7  0.120                     +       IXXX  +
 - 6  0.092                     +       IXX   +
 - 5  0.086                     +       IXX   +
 - 4 -0.038                     +      XI     +
 - 3  0.133                      +      IXXX +
 - 2  0.208                      +      IXXXXX +
 - 1  0.021                      +      IX   +
   0 -0.349                   XX+XXXXXXI     +
   1  0.106                      +      IXXX +
   2 -0.208                      + XXXXXI    +
   3  0.102                      +      IXXX +
   4 -0.010                     +       I     +
   5  0.103                     +       IXXX  +
   6 -0.207                     +  XXXXXI     +
   7  0.113                     +       IXXX  +
   8 -0.080                     +     XXI     +
   9  0.184                     +       IXXXXX +
  10 -0.136                     +    XXXI     +
  11 -0.120                     +    XXXI     +
  12  0.167                     +       IXXXX +
  13 -0.099                    +      XXI      +
  14  0.022                    +        IX     +
  15 -0.007                    +        I      +
```

In the above example the following model was ultimately selected:

$$Yt = \frac{(W_o - W_1B)\ Xt}{(1-\delta_1B)} + \frac{1}{(1-\phi_3B)}\ At \qquad (3)$$

The value of b is therefore o, that of r and s one and the noise model followed an "AR(3)" process.

The estimates of equation 3 and their t-values are:

Parameter	*Estimate*	*t-ratio*
W_o	−278,70	−2,79
W_1	− 72,80	−1,88
δ_1	−0,6408	−4,20
ϕ_3	0,5585	3,58

These estimates are made using only the first 48 observations of the sample data, covering the period 1970 to 1981.

Diagnostic checks

Once the transfer function has been identified and its parameters estimated it is necessary to verify that the model chosen adequately fits the data. This diagnostic checking is done by analysing the residuals. If the transfer function is properly specified then the residuals in the model should behave as white noise and be uncorrelated with the explanatory variables in the model. An examination of the residual autocorrelation function and of the cross-correlation function between the explanatory variable and the residuals (see Figure 4) reveal no significant correlations. It would, therefore, appear that the model is adequate.

Forecasting with the model

The model is used to forecast share prices one quarter ahead.

Figure 4: Cross-correlation between the prewhitened input series and the residuals and autocorrelation function of residuals

Cross-correlations

```
            -1.0  -0.8  -0.6  -0.4  -0.2   0.0  0.2   0.4   0.6  0.8  1.0
Lag corr.-----------------------------------------------------------------
 0 -0.043                        +     XI      +
 1 -0.172                       +   XXXXI       +
 2 -0.104                       +    XXXI       +
 3 -0.076                       +     XXI       +
 4  0.069                       +       IXX     +
 5  0.032                       +       IX      +
 6 -0.172                       +   XXXXI       +
 7 -0.045                       +      XI       +
 8 -0.091                       +     XXI       +
 9  0.213                       +       IXXXXXX +
10 -0.111                      +     XXXI        +
11 -0.083                      +      XXI        +
12  0.013                      +        I        +
13  0.110                      +        IXXX     +
14 -0.012                      +        I        +
15  0.044                      +        IX       +
```

Autocorrelations

```
            -1.0  -0.8  -0.6  -0.4  -0.2   0.0  0.2   0.4   0.6  0.8  1.0
Lag corr.-----------------------------------------------------------------
                                        I
 1  0.060                        +      IX     +
 2  0.006                        +      I      +
 3 -0.015                        +      I      +
 4 -0.018                       +       I       +
 5  0.148                       +       IXXXX   +
 6 -0.227                       + XXXXXXI       +
 7 -0.044                       +      XI       +
 8  0.127                       +       IXXX    +
 9 -0.002                       +       I       +
10  0.087                       +       IXX     +
11 -0.031                       +      XI       +
12 -0.075                       +     XXI       +
13  0.050                       +       IX      +
14 -0.088                       +     XXI       +
15 -0.055                       +      XI       +
```

In total 15 one quarter ahead forecasts are made, covering the period 1982 to 1985. As each new observation becomes available the whole process of identifying, estimating and diagnostic checking is repeated and then a forecast is made for the new quarter. The results are summarised in Table 1.

In order to perform the forecasting it is necessary to forecast interest rates for the next quarter. (Quarterly interest rates did not lead share prices but were coincident.) The model developed (ARIMA (1, 1, 1)) to prewhiten the input series will be used to generate one period ahead forecasts of the interest rate.

Table 1: Forecasting the industrial index

Period	Forecast		Actual	
	Interest rate	Index	Interest rate	Index
82Q1	13,55	663,85	14,35	548,1
Q2	13,55	542,21	14,55	481,6
Q3	13,88	500,57	12,15	650,7
Q4	12,50	666,42	10,88	705,9
83Q1	10,94	689,11	11,89	790,3
Q2	12,17	816,10	13,41	904,7
Q3	13,76	873,06	12,93	886,6
Q4	12,34	947,35	13,80	910,2
84Q1	14,17	973,72	14,00	974,5
Q2	14,58	943,55	14,65	940,2
Q3	14,98	951,21	16,26	790,1
Q4	16,22	809,59	16,44	846,7
85Q1	16,02	844,47	16,45	810,0
Q2	16,68	748,61	14,94	980,2
Q3	14,35	1 057,49	16,90	965,2

Testing the model

The model is tested by

(1) calculating the predicted R^2 over the period 1982-1985; and by

(2) simulating the results of three alternative portfolio strategies which could have been followed during the period 1982 to 1985 (third quarter).

Predicted R^2

The predicted R^2 is calculated between the series of 15 ex ante one period ahead forecasts of The Johannesburg Stock Exchange and the actual price changes that occurred. The R^2 for this model is 0,541 and is significant at between the 0,05 and 0,025 level.

Three alternative portfolio strategies

The second means of evaluating the model follows an approach adapted by Umstead (1975). This approach simulates the results of three alternative portfolio strategies which could have been followed during the test period. The test period in this particular example is the fifteen quarter period 1982 to 1985. It is assumed that each portfolio starts out with an initial wealth of R100. Portfolio number one buys and holds The Johannesburg Stock Exchange Industrial Index through the entire test period. Portfolio number two switches between 91-day Treasury bills and the Industrial Index depending on the forecast next quarter holding period return of each alternative. Portfolio number three buys 91-day Treasury bills each quarter.

For portfolio number 2, a decision has to be made every quarter whether to invest in equities or in Treasury bills. The decision is made by comparing the forecast quarterly holding period return for equities with the forecast (and actual) return for 91-day Treasury bills. If the forecast return for equities exceeds that of treasury bills then portfolio number two either stays invested in equities or switches into equities if it is currently in Treasury bills. If the forecast return for Treasury bills is greater than that for equities, portfolio two either stays in bills or switches into bills if it is currently in equities. The exact computational procedure will not be discussed here as it can be found in Umstead (1975). It is assumed that each time the portfolio is moved into or out of equities, transaction costs will equal 2 per cent. The round trip transaction costs are therefore 4 per cent.

The holding period (15 quarters) quarterly return (see Sharpe, 1978) calculated for each portfolio is:

Portfolio	*Holding period quarterly return*
1	4,1%
2	7,2%
3	3,9%

From the above it is clear that portfolio 2 attained the highest quarterly return, namely 7,2%. This is significantly higher than that obtained by the other two portfolios. It would, therefore, appear as if the use of the model would enable the investor to achieve higher returns.

Conclusion

In this paper the use of the Box-Jenkins transfer function model-building methodology was applied to The Johannesburg Stock Exchange. Although the model was tested on a limited data set of 15 quarters only, it seemed to indicate that the approach can achieve good results. The main drawback of the approach lies in the considerable effort involved in developing the model and then updating the model as new information becomes available. It is also possible that structural changes could occur in the economy or share market that will cause the identified relationship to become invalid. Furthermore, the model developed in this paper requires the user firstly to forecast the input series before a forecast of share prices can be obtained. This obviously means that less accurate forecasts are obtained.

The approach followed in this paper can be improved. It is theoretically possible to extend the methodology to include more than one input series. For example, variables such as the money supply can be included in the model. This might lead to even more accurate forecasts.

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