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Measuring volatility using bilinear GARCH models

1. INTRODUCTION

Although there is now an extensive literature on modelling financial volatility using the ARCH class of models (Engel, 1982; Bollerslev, 1986; Bollerslev, Chou and Kroner, 1992) among others, little attention has been paid to the covariance structure between the lagged values of independent variables. This structure is always assumed to have no impact in the modelling of an ARCH process. It is, however, clear that if two or more variables are related then their covariance structure might be important in any modelling process. The bilinear model takes into account variations within the independent variables as well as covariations between the variables. This is very important in the study of financial market data where the covariance between independent variables may play a significant role in determining market volatility. For the ARMA and GARCH models, these lags combinations are always ignored. De Gooijer (1989) provides evidence of the importance of bilinear models in the modelling of stock returns series. The residual variance of his study using the bilinear models were, however, marginally smaller than those from alternative linear models.

The remainder of the paper is divided into five sections. A simple bilinear generalised autoregressive conditional heteroscedasticity (BGARCH) is formulated in section two. A simple test for the inclusion of the parameters in the GARCH and BGARCH models is derived in section three in the framework of Lagrange multiplier technique. Section four goes through a theoretical parameter estimation procedure in the maximum likelihood function framework of the Newton-Raphson method. The empirical evidence, using the BGARCH model, is discussed in section five. Section six concludes the paper.

2. BILINEAR GARCH (BGARCH) MODEL

This article looks at a simple bilinear GARCH model as a tool for modelling returns using the FTSE 100 index series. The model is an extension of the GARCH model. The bilinear models are a non-linear class of models. There is extensive literature on this class of models (see Granger and Andersen, 1978a and 1978b). The simple vector bilinear GARCH model was developed and tested by Biekpe (1996). For the purpose of this study, we examine a simple scalar second order bilinear GARCH model of the form

$$\sigma_t^2 = \sum_{i=1}^{q} \alpha_i \sigma^2_{t-1} + \sum_{i=1}^{q}\sum_{j=1}^{p} \delta_{ij}\sigma_{i-1}\varepsilon_{j-1} + \sum_{j=0}^{p} \varepsilon^2_{j-1} \quad \ldots(2.1)$$

where σ_{t-i} and ε_{t-j} are respectively the lag values of the variance and residuals of the FTSE 100 index. For $i \neq j$ $E(\sigma_{t-i}\varepsilon_{j-1}) = 0$ where E(.) is the expectation operator.

For the statistical derivations of the asymptotic stationarity, see second-order stationarity and invertibility conditions of a generalised bilinear model in vector form from Biekpe (1996). The primary aim in the study is to present the BGARCH models as another useful tool for modelling volatility. The question as to whether the covariance structure of the BGARCH model may be relevant in volatility measure is very central in this study.

3. TESTING FOR BGARCH

The procedure for testing the BGARCH model is similar to that of the GARCH (Bollerslev, 1986). Here we set

$$\sigma^2_t = z_{ij}^T W z_{ij} \quad (i = 1,..,q)\ (j = 1,...,p) \quad \ldots(3.1)$$

where W is a qxp matrix of coefficients, w_{ij} represents the individual coefficients in W and σ^2_t the variance at time t. Using the Lagrange multiplier (LM) test, the main aim is to test the hypothesis

$H_0: w_{ij} = 0$ versus $H_1: w_{ij} \neq 0$

The test statistic (Engle, 1982) is given by:

$$LM = \frac{1}{2} f_o^T z_o (z_o^T z_o)^{-1} z_o^T f_o \quad \ldots(3.2)$$

$$\text{where } f_0 = \left[\frac{\varepsilon_1^2}{\sigma^2_1} - 1 \ldots\ldots \frac{\varepsilon_T^2}{\sigma^2_T} - 1\right]$$

$\sigma_1^2 \frac{\partial \sigma_1^2}{\partial w} \ldots\ldots \sigma_T^2 \frac{\partial \sigma_T^2}{\partial w}$ and ε^2_t represents square of the error component.

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If H_0 is true, then the LM will be asymptotically chi-square with r = pq degrees of freedom, where r is the total number of elements in w_{ij} for $i \neq j$. It is well established (Engle, 1982) that by normality, an asymptotically equivalent test statistic for (3.2) is

$$LM = Nf_o^T z(z^T z)^{-1} \frac{z^T f_o}{f_o^T f_o} = NR^2 \qquad \text{...(3.3)}$$

where R^2 is the squared multiple coefficient between f_0 and z_0 and N is the number of observations.

4. PARAMETER ESTIMATION

Consider a simple bilinear GARCH model given by

$$\sigma_t^2 = \sum_{i=1}^{q} \alpha_i \sigma^2_{t-1} + \sum_{i=1}^{q}\sum_{j=1}^{p} \delta_{ij}\sigma_{i-1}\varepsilon_{j-1} + \sum_{j=0}^{p} \varepsilon^2_{j-1} \qquad \text{...(4.1)}$$

where ε_t is independent and normally distributed with mean zero and variance σ_ε^2. The general approach of estimation is by maximum likelihood estimation method. Maximising the likelihood function is the same as minimising the function,

$$L(\Theta) = \sum_t \varepsilon_t^2 \qquad \text{...(4.2)}$$

where $\Theta = (\alpha_0^2, \alpha_1, ...\alpha_q, \beta_1, ...\beta_p, \delta_{11}, ...\delta_{pq})$

For convenience, let

$$\theta_0 = \sigma_0^2, \theta_1 = \alpha_1, ...\theta_p + 1 = \beta_1 ......\theta_{p+q} = \beta_p, \theta_{p+q+1} = \delta_{11} ....\theta_{p+q+pq} = \delta_{pq}$$

Set n = q+p+qp. The first and second partial derivatives of $L(\theta)$ are respectively given by,

$$\frac{\partial L_\theta}{\partial \theta_i} = 2\sum_{t=1}^{N} \varepsilon_t \frac{\partial \varepsilon_t}{\partial \theta_i} \qquad (i = 1,2,...,n)$$

$$\frac{\partial^2 L_\theta}{\partial \theta_i^2} = 2\sum_{t=1}^{N} \frac{\partial \varepsilon_t}{\partial \theta_i}\frac{\partial \varepsilon_t}{\partial \theta_j} + 2\sum_{t=1}^{N} \varepsilon_t \frac{\partial^2 \varepsilon_t}{\partial \theta_i \partial \theta_j}$$

Now, assume that $\varepsilon_t = \frac{\partial \varepsilon_t}{\partial \theta_i} = \frac{\partial^2 \varepsilon_t}{\partial \theta_i \partial \theta_j} = 0$ for all $t < 1$.

Also let $G^T_\theta = (\frac{\partial L}{\partial \theta_1} .... \frac{\partial L}{\partial \theta_n})$ and

$J(\theta) = [\frac{\partial^2 L_\theta}{\partial \theta_i \partial \theta_j}]$, where $J(\Theta)$ is a matrix of second order derivative.

Expanding $G(\theta_e)$ near $\theta_e = \theta$ in a Taylor series produces

$$[G(\theta_e)]_{\theta_e = \theta = 0} = G(\theta) + J(\theta)(\theta_e - \theta),$$

where θ_e is the estimate of θ. Rewriting this equation generates $\theta_e - \theta = -J^{-1}(\theta)G(\theta)$ the Newton-Raphson equation,

$\theta^{(k+1)} = \theta^{(k)} - J^{-1}(\theta^{(k)})G(\theta^{(k)})$, is obtained.

5. EMPIRICAL EVIDENCE: GARCH(1,1) versus BGARCH(1,1,1)

The daily FTSE 100 index data from Datastream was used. The period covered by the empirical work is from January 1980 to December 1991. The initial step was to address the case for dependence and non-linearity. The data used in this study has been differenced (first order). The Augumented Dicky-Fuller (ADF) test for the differenced series was 2,38 which is below the ADF critical value of 3,46 at the 5% level of significance. This, therefore, indicates that the difference series was stationary.

5.1 Dependence

In order to find out whether the data was not independent the autocorrelations of the series were calculated and it was noticed that the autocorrelations were significant for shorter lags and in particular lag one. A significant first lag autocorrelation also implied a rejection of the idea that the residuals of returns follow a white noise process. The autocorrelations of the squared returns and squared residuals were also significant even at long lags. Since the calculations are not based on any stringent distributional assumptions, they show clearly that the residuals (and hence the returns) exhibit high levels of intertemporal dependence. This implies that the residuals of returns are not likely to be a realisation of a strict white noise process, which implies that any assumption of normality is questionable.

Since the presence of significant autocorrelation in squared returns and squared residuals account for the thick tails and peakedness of the empirical distributions of most financial data, further research in the skewness and kurtosis of returns will be very valuable in the understanding of the distributional properties of stocks returns.

5.2 Non-linearity

The ARCH class of models are applied on the basis that the past errors contain some useful information which can be used for predicting future volatility. The ARCH class of models are highly non-linear. Also, as discussed earlier, the normality and independent assumptions in these models are not conclusive and therefore can not be relied upon. Despite these weaknesses, it is useful to compare the GARCH and bilinear GARCH models as tools for modelling volatility. To estimate the parameters for the GARCH and BGARCH models, different lag lengths of the two models were selected for testing using the Berndt, Hall, Hall and Hausman (BHHH) (1974) algorithm. It was found that the GARCH(1,1) and BGARCH(1,1,1) yielded better results. The regression output is shown below. The results include both the returns and variance regression equations for the GARCH (1,1) and the BGARCH (1,1,1)

GARCH(1,1)

$$R_t = \underset{[4{,}383]}{0{,}074} + \underset{[6{,}872]}{0{,}159}R_t + \varepsilon_t$$

$$\sigma^2_t = \underset{[3{,}551]}{0{,}035} + \underset{[6{,}399]}{0{,}094}\,\varepsilon^2_{t-1} + \underset{[32{,}953]}{0{,}850}\,\sigma^2_{t-1}$$

BGARCH(1,1,1)

$$R_t = \underset{[3{,}955]}{0{,}067} + \underset{[6{,}674]}{0{,}161}R_{t-1} + \varepsilon_t$$

$$\sigma^2_t = \underset{[3{,}666]}{0{,}040} + \underset{[5{,}738]}{0{,}096}\,\varepsilon^2_{t-1} + \underset{[29{,}240]}{0{,}841}\,\sigma^2_{t-1} \underset{[-2{,}116]}{-0{,}046}\,\varepsilon_{t-1}\sigma_{t-1}$$

where R(t) is expected return at time t. The numbers in parenthesis, are the T-statistics. The Lagrange multiplier test (see equation 3.3) for the inclusion of the parameter in the conditional variance equation of the BGARCH(1,1,1) is 6,92 compared to 6,81 for the GARCH(1,1). These two values are both significant at the 5% level. This implies that the coefficient of the covariance structure of the bilinear model is also significant. The implication is that both models are significant at the 5% level of significance. The centred R^2 between the estimates of $\varepsilon^2(t)$ and estimates of $\sigma^2(t)$ were also computed. For the BGARCH(1,1,1), R^2 was 0,038 and for GARCH(1,1), R^2 was 0,022. These values are rather small but comparable to other previous findings in the measure of market volatility.

6. CONCLUSION

The importance of co-variance structure in finance is well established in the financial research literature. After all, the correlation coefficient plays a central role in the choice of, for instance, a portfolio of assets. It is, therefore, felt that the covariance structure of a bilinear functional form of a GARCH model may account for more of the non-linear behaviour in market returns compared to the 'ordinary' GARCH model. The bilinear GARCH model may, generally, represent a better volatility measure compared to the ordinary GARCH model. While the view that there is no adequate substitute for examining the data for what might be an adequate model is retained, it is not the intention here to universally argue for one model over another. Rather, the aim is to provide a framework in which the issues involved in this choice can be understood, and within which, other models of this type can be researched. The most important objective of this paper was to underline the usefulness of bilinear GARCH models in modelling volatility. Further research into both non-stationarity, non-linearity and, hence, the chaotic nature of financial markets may provide better understanding in the study of stock returns volatility. Distributions such as the Pearson Type IV, other than the normal distributions, could be fed into the variance equation since the normality assumption, in this context, is highly questionable.

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