R Thomson*

A re-evaluation of the South African Bond Exchange-Actuaries Indices

1. INTRODUCTION

The JSE-Actuaries Fixed Interest Index was launched in January 1983 with historical data from 1980 onwards. There were two series—one for RSA government bonds and one for Eskom and SATS bonds—subdivided into four categories by term to redemption, viz. up to three years, three to seven years, seven to twelve years, and over twelve years. All bonds with fixed coupons and redemption dates were included. For each series and category a "clean" index, a "cum" (i.e. all-in) index and an "interest paid schedule" (i.e. ex-coupon adjustment) were published. The ex-coupon adjustment, which reflected coupons paid since the start of the calendar year on the bonds in the index, was reset to zero at the beginning of each year. Equal nominal weightings were used for all bonds included in the index (Liddle & McLeod, 1987: 203, 205).

Because of difficulties with the timely provision of accurate index values and a general lack of understanding of the way in which the indices were compiled, they were not widely used. In 1986 the JSE-Actuaries Index Subcommittee undertook a revision of the bond indices. Market participants were consulted by a working group of the Subcommittee comprising the late Mr Chris Liddle and Professor Heather McLeod, who recommended the introduction of a new JSE-Actuaries Fixed Interest Performance Index. The new series was to fulfil the function of a benchmark to facilitate investment performance measurement, rather than that of an indicator of the level of the market. The latter function was met by a new JSE-Actuaries yield curve. It was envisaged that the new series would be introduced with effect from 1 January 1988, with historical values from 1 January 1986 (*ibid.*: 206, 208-9, 212, 215-6). In the end, publication commenced in October 1988, but historical values were produced as planned. Gilts and semi-gilts were combined in the new indices, because it was found that marketability was more important than the name of the issuer, but the four categories by term to redemption were retained (including an all-bond index combining all four). The equal nominal weighting was replaced by market capitalisation weighting, so as to reflect the portfolio of a passive manager.

The new series was based on clean prices only, and an "interest yield" (i.e. a running yield) was published for each category. The calculation of an all-in index and of ex-coupon adjustments was discontinued. A formula for the calculation of returns on the indices for the purpose of investment performance measurement was proposed by Liddle and McLeod (*op. cit.*) and has been widely used. The reasons for this proposal were as follows (*ibid.*: 209-11, 233):

- The proposed method would facilitate the analysis of investment performance into its income and capital components.
- It was thought to be consistent with the treatment of equities in the JSE-Actuaries All-Share Index.
- The clean price exhibits relatively minor discontinuities when bonds go ex-coupon.
- The use of a clean index, an all-in index and an ex-coupon adjustment was regarded as unnecessarily complex.
- Tax could be more easily handled.
- Interest earned was considered to be more relevant than interest received.

The indices were duly introduced as proposed.

Glansbeek and Conway (1994) point out that the formula proposed by the working group contained a flaw in that it implied twelve coupon payments on each bond each year, instead of two, thus incorrectly implying an acceleration of coupon payments. They proposed an alternative formula, as explained below.

Yet another formula has until recently been in use by the consulting actuaries participating in the annual Consulting Actuaries Fund Survey (Alexander Forbes, unpublished) and was made available to users by a widely-used on-line information provider; this formula is also discussed below.

Furthermore, Paterson (1995: 1-2) points out that a change in the clean price of a bond does not reflect a proportionate capital gain. This is because the clean price incorporates an adjustment for accrued interest. He proposed that a new index series be used, based on all-in prices.

The purposes for which an index may be designed are (Institute and Faculty Education, 1999):

(1) to provide a measure of short-term market movements;
(2) to provide a history of market movements and levels;

*Visiting Professor, Department of Statistics and Actuarial Science, University of the Witwatersrand, Private Bag 3, Wits 2050, Republic of South Africa. The author acknowledges with thanks the advice of Prof A Asher with regard to certain aspects of this paper, as well as the criticism of the referees.
Email: rthomson@icon.co.za

(3) as a tool for estimating future movements in the market based on past trends;
(4) as a benchmark against which to assess the investment performance of portfolios;
(5) for valuing a notional portfolio;
(6) for analysing sub-sectors of the market;
(7) as a basis for "index funds" designed to track the particular market; and
(8) to provide a basis for the creation of derivative instruments.

For bonds, purposes (1) to (3) may be at least partly satisfied by models of the term structure of interest rates. The most important purposes of bond price indices are arguably (4), (7) and (8). For each of those purposes it is important that the performance of the indices should be replicable by investment in the assets comprising the index.

In this article the approach of Liddle and McLeod is described and criticised and other alternative approaches are discussed. Numerical comparisons of the various methods are made in Section 3.

2. ALTERNATIVE APPROACHES

2.1 Liddle and McLeod's approach (LM)

Formula for the return on the index

The formula proposed by Liddle and McLeod for the determination of the return on the index (*op. cit.*: 222-3) is as follows:

$$y_t^{LM} = \frac{W_t - V_0 - (K_t - J_t)}{V_0 + \frac{1}{2}(K_t - J_t)} \qquad \dots (1)$$

where:

y_t^{LM} represents the return on the index from time 0 to time t;

V_t represents the all-in value of the portfolio at time t;

K_t represents the all-in price paid for purchases from time 0 to time t (after adding any brokerage etc. and deducting any underwriting commission), less the net (all-in) realisation proceeds of sales from that sector over the period; i.e. the net new money invested over the period, which may be negative;

W_t represents the value of the portfolio at time t had one invested V_0 at time 0 and K_t at time $\frac{t}{2}$ in the index

$= \frac{I_t^C}{I_0^C} V_0 + \frac{I_t^C}{I_{\frac{t}{2}}^C} K_t$;

I_r^C is the clean price index at time r as published;

J_t is the "interest that would have been earned" over the period, based on the interest yield on the index

$= (V_0 + \frac{1}{2}K_t)(tg_{\frac{t}{2}})$;

g_r is the "interest yield" on the price index at time r as published, i.e. the "total income expected to accrue over a one-year period from holding the stocks in the index, expressed as a percentage of the sum of the clean prices." (Liddle and McLeod, 1987: 210)

This formula follows Milburn-Pyle (1979: 12); the notation has been changed to conform to the notation used in this paper. It is justified on the basis that it constitutes the solution to the equation:

$$W_t = V_0(1 + y_t^{LM}) + (K_t - J_t)(1 + \tfrac{1}{2} y_t^{LM});$$

the first term of which requires the return to apply over the whole period and the second requires it to apply over the latter half. It should be noted, however, that, whereas Milburn-Pyle defines J_t as "income received", Liddle and McLeod define it as "interest . . . earned", by which they mean "accrued".

The error in Liddle and McLeod's approach

In the appendix it is shown that the error in Liddle and McLeod's approach is approximately:

$$\Delta = -\tfrac{1}{4}df + e(1+e) - d(1 + \tfrac{3}{4}d) \qquad \dots (2)$$

where:

$\Delta \approx y_1^{LM} - y$;

y = the return earned over a year by an investment in the index at the start of the year;

$d = \frac{c}{V_0}$;

$e = \frac{c}{V_{\frac{1}{2}}}$; and

$f = \frac{V_1 - V_0}{V_0}$.

For this purpose, the index was taken as a single bond with an annual coupon of c per unit of the nominal value and a period of s to the next coupon payment.

This result was tested numerically over a reasonable range of values and it was found that the values of Δ were of the same order of magnitude as those of $y_1^{LM} - y$.

It may be noted that the approximate error is independent of the period to the next coupon payment.

Furthermore, under the simplifying assumptions that e = d and f = 0 (which they may be expected to be on average over a period), then:
$\Delta = \frac{1}{4}d^2$.

Thus, for example, if d = 10%, then Δ = 0,25%, which means that, over an extended period, Liddle and McLeod's approach would overstate the return by ¼%. This reflects the fact that interest is paid on average three months (i.e. one-half of the interval between payments) after it accrues; the overstatement constitutes interest on coupon payments for a three-month period.

It may also be seen from Equation (2) that:

- if f is positive (as when the yield drops or the bond is at a discount) the tendency for y^{LM} to overstate the yield may be offset or reversed, and vice versa;
- if e > d, i.e. when $V_{\frac{1}{2}} < V_0$ (as when the yield rises or the bond is at a premium) the tendency for y^{LM} to overstate the yield is enhanced and vice versa.

Thus, when prices are rising, the error in y^{LM} is generally reduced or reversed, whereas in falling markets it is generally enhanced. This means that it is more difficult for investment managers to match the index in falling markets than in rising markets.

Justification for the use of clean prices

Liddle and McLeod (1987: 210, 233) argued that the proposed method would facilitate the analysis of investment performance into its income and capital components. In modern investment performance analysis (e.g. Bodie, Kane and Marcus, 1996: 773-800) no distinction is made between income and capital; these are merely accounting distinctions, which have no significance for investment performance.

Liddle and McLeod thought that their method was consistent with the treatment of equities in the JSE-Actuaries All-Share Index (*ibid.*: 209, 211, 233). In fact the price of an equity is an all-in price. There is no such thing as a clean price in the equity market. Dividends do not accrue until they are declared. Prices remain cum-dividend until the ex-dividend date, when they drop to reflect the dividend becoming payable.

The clean price does not exhibit discontinuities when bonds go ex-coupon (*ibid.*: 211, 233). Discontinuities will not create problems unless the index is used as a barometer of the market. The authors explicitly stated that this was not intended. The need for a barometer of the market was to be met by a yield curve. The purpose of the index was to facilitate investment performance measurement.

The use of a clean index, an all-in index and an ex-coupon adjustment was regarded as unnecessarily complex (*ibid.*: 209). This may be true, but if so, on the strength of the above arguments, it is the clean index that should be dropped.

The authors argued that, under the proposed method, tax could be more easily handled (*ibid.*: 233). Tax could have been more accurately handled by means of accrued interest indices such as those published with the FT-Actuaries Indices in the United Kingdom. That alternative does not appear to have been considered. The way in which bonds are taxed in South Africa has subsequently changed, so that this argument now falls away. The matter of tax is considered in a later section.

2.2 Glansbeek and Conway (GC)

In order to avoid exaggeration, Glansbeek and Conway proposed that the formula for J_t be replaced by:

$$J_t = \left(1 + \tfrac{1}{2} g_{\frac{t}{2}}\right)^{2t} - 1.$$

They motivated this proposal on the grounds that the interest yield is effectively compounded half-yearly. However, if this value is used in the simplifying example used above, the yield over the year is:

$$y_1^{GC} = \frac{g\left(1 + \frac{1}{4}g\right)}{1 - \frac{1}{2}g\left(1 + \frac{1}{4}g\right)} > \frac{g}{1 - \frac{1}{2}g} = y_1^{LM}.$$

Thus, under the simplifying assumptions, Glansbeek and Conway's return is even greater than Liddle and McLeod's. It appears that, in their illustrative calculations, Glansbeek and Conway did not apply their formula in this way. On the basis of their argument it would have been appropriate to determine the yield as:

$$y_t^{GC} = \left\{\frac{I_t^C}{I_0^C} - 1\right\} + \left\{\left(1 + \tfrac{1}{2}g\right)^{2t} - 1\right\},$$

where the first curly bracket represents capital gain and the second represents interest, including interest on reinvested interest. Yet it appears that they did not apply it in this way either. Attempts to establish what they did in fact do were unsuccessful.

In any case, Glansbeek and Conway's approach does not accommodate Paterson's (*op. cit.*: 1-2) argument that a change in the clean price of a bond does not reflect a proportionate capital gain. Nor does it allow for capital gain on the reinvested coupon.

2.3 Consulting actuaries (CA)

The formula in use by the Consulting Actuaries Fund Survey (Alexander Forbes, unpublished) is:

$$y_t^{CA} = \frac{ct\left\{\dfrac{I_t^C}{\frac{1}{2}\left(I_0^C + I_t^C\right)}\right\} + I_t^C}{I_0^C} - 1\,;$$

where:

$$c = \tfrac{1}{2}\left(g_0 I_0^C + g_t I_t^C\right).$$

The expression c approximates the average coupon during the period (0, t), and the expression

$$ct\left\{\frac{I_t^C}{\frac{1}{2}\left(I_0^C + I_t^C\right)}\right\}$$

represents the coupon increased by the increase in the index from halfway through the period to the end of the period.

There are two problems with this formula:

- it does not allow for the accrual of interest on the reinvested coupon; and
- it does not allow for the fact that a change in the clean price of a bond does not reflect a proportionate capital gain.

If this formula is applied to the case considered in the appendix with e = d and f = 0, the yield over the year is:

$$y_1^{CA} = g\,,$$

which understates the yield by $\frac{1}{4}g^2$.

2.4 Clean price index with an improved return formula (CI)

In order to develop an improved formula based on the clean price index we consider the interest on bond interest payments during the period (0, t) for $0 < t \leq 1$. Assuming (without loss of generality) that there is no accrued interest at time 0, bond interest accruing during the period per unit commencing market value will amount to:

$$\frac{1}{I_0^C}\int_0^t I_r^C g_r\,dr \approx t\tilde{g}$$

where:

$$\tilde{g} = \frac{I_0^C g_0 + I_t^C g_t}{2I_0^C}$$

represents the average running yield during the period.

This interest will accrue on average at time $\frac{1}{2}t$. It will be payable on average three months after it accrues, that is at time $\frac{1}{2}t + \frac{1}{4}$. Interest at the rate of $\tilde{g}$ on those payments during the period $\left(\frac{1}{2}t + \frac{1}{4}, t\right)$ will therefore amount to:

$$t\tilde{g}\left\{t - \left(\tfrac{1}{2}t + \tfrac{1}{4}\right)\right\}\tilde{g} = \tfrac{1}{2}t\tilde{g}^2\left(t - \tfrac{1}{2}\right).$$

The total amount of interest will therefore be:

$$t\tilde{g}\left\{1 + \tfrac{1}{2}\left(t - \tfrac{1}{2}\right)\tilde{g}\right\}.$$

Now the annualised rate of capital appreciation over the period (0, t) may be approximated by means of the formula

$$f = \frac{I_t^C - I_0^C}{tI_0^C}.$$

Allowance must also be made for capital appreciation on the reinvested interest. By analogy with the discussion of interest on interest, this will amount to:

$$\tfrac{1}{2}tf\tilde{g}\left(t - \tfrac{1}{2}\right);$$

so that the total return over the period per unit commencing market value is:

$$y_t^{CI} = t\left[f + \tilde{g}\left\{1 + \tfrac{1}{2}\left(t - \tfrac{1}{2}\right)\left(\tilde{g} + f\right)\right\}\right];$$

where:

$$\tilde{g} = \frac{I_0^C g_0 + I_t^C g_t}{2I_0^C}\,;\text{ and}$$

$$f = \frac{I_t^C}{tI_0^C} - 1.$$

2.5 All-in index with ex-coupon adjustment (A)

The FT-Actuaries Bond Indices in the United Kingdom comprise an all-in index with an ex-coupon adjustment (Dobbie and Wilkie, 1978: 19, 21-2). The ex-coupon adjustment is calculated by determining, on each day, the total amount of interest becoming payable on bonds going ex-coupon on that day. This amount is weighted by the same factor as the all-in prices on that day and the result is added to the previous day's ex-coupon adjustment. The total is reset to zero at the start of each year. The ex-coupon adjustment at any date thus represents the total interest payments to which the holder of the constituent bonds with an all-in value equal to the index would have become entitled since the start of the year. The interest payments to which such an investor would have become entitled over a portion of a year may be found by subtraction. Thus the return is:

$$y_t^A = \frac{I_t^A - I_0^A + X_t^1 - X_0^1}{I_0^A};$$

where:

I_r^A represents the all-in index at time r; and

X_r^1 represents the ex-coupon adjustment at time r.

2.6 Total return index with coupon reinvested on ex-coupon date (T1)

An index could be calculated (on an all-in basis) such that the value of the index is increased by the coupons on the index (calculated as above) whenever a bond goes ex-coupon. On this basis the return on the index will be:

$$y_t^{T1} = \frac{I_t^{T1}}{I_0^{T1}} - 1;$$

where:

I_r^{T1} is the total return index at time r as described in this section.

2.7 Total return index with coupon reinvested on payment date (T2)

An index could be calculated (on an all-in basis) such that the value of the index is increased by the coupons on the index (calculated as above) whenever a coupon payment is made. While the bond is ex-coupon an adjustment will be made to the index equal to the discounted value of the coupon due. The discounted value will be determined from the yield curve according to the outstanding period to the coupon payment date. On this basis the return on the index will be:

$$y_t^{T2} = \frac{I_t^{T2}}{I_0^{T2}} - 1;$$

where:

I_r^{T2} is the total return index at time *r* as described in this section.

3. NUMERICAL COMPARISONS

3.1 Standard parameters

In this section the alternatives outlined in the previous section are discussed and compared with Liddle and McLeod's approach.

For the sake of simplicity and ease of analysis, the comparison was made with indices based on a single bond and the parameters of the bond were varied so as to reflect the sensitivity of the various methods to changes in the parameters. A standard set of parameters was adopted for illustrative purposes and each parameter was varied to show the effect on the calculated returns over a one-calendar-year period. The standard set of parameters was as follows:

Term to redemption at start of year	20 years
Coupon	15%
Yield to redemption:	
at start of year	15%
at end of year	15%
Month of first coupon payment:	June
Yield on coupons outstanding	15%

All yields are defined as half-yearly compound. It is assumed that the bond goes ex-coupon one month before each coupon payment. Yields to redemption for month-ends during the year are interpolated linearly between those at the start and end of the year. For the standard set of parameters, this means that they are constant throughout the year. To test the robustness of the results against changes to the standard parameters, the parameters were varied as explained in Subsection 3.3.

3.2 Comparison on standard parameters

For the purposes of comparison, indices were constructed from the returns by means of the formula:

$$I_t^X = 1 + y_t^X,$$

where X represents the method under consideration. The ratio of that index to I_t^{T2}, which by definition is accurate, was calculated for $t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, 1$. The ratios on the various methods are shown in Figure 1.

It may be seen from Figure 1 that Liddle and McLeod's index (*LM*) immediately drifts upward, reflecting the bias discussed above.

The all-in index with an ex-coupon adjustment (*A*) increases at the end of the fifth month because the all-in price decreases by the discounted value of the coupon when the bond goes ex-coupon, whereas the ex-coupon adjustment reflects the full amount of the coupon. At the end of the sixth month the adjustment is zero and the index is correct. Thereafter the index drifts downward, reflecting the fact that the coupon has not been reinvested. The Consulting Actuaries index (CA) follows a similar path.

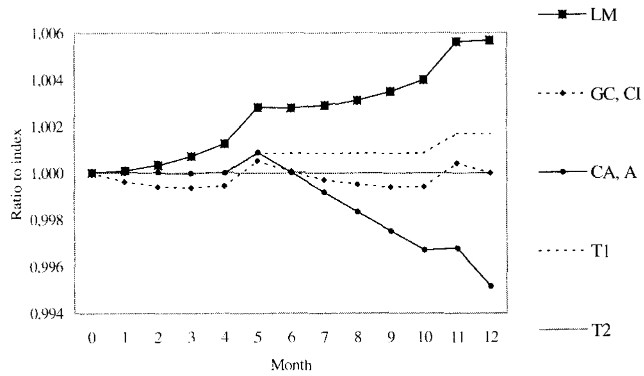

Figure 1: Annual rebalancing

Glansbeek and Conway's index (GC) drifts downwards initially, following the downward drift in the clean price, which is caused by the use of simple interest in the calculation of accrued interest. This effect gradually reduces over the six-month period. As for the all-in index, there is an up-tick when the bond goes ex-coupon, but this is rectified the following month, after which the cycle is repeated. The improved clean price index (CI) follows a similar path.

The total return index with coupons reinvested on ex-coupon date (T1) increases at the end of the fifth month for the same reason as A, but does not drift downward thereafter, because the coupon has been reinvested on the ex-coupon date. In general, in investment performance measurement, returns on the indices are compared with linked internal rates of return determined on a quarterly or monthly basis. Figures 2 and 3 compare the indices on those bases respectively.

LM follows approximately the same course as before. On the quarterly basis, A now reverts to the true value at the end of the sixth and twelfth months because the coupon is (fortuitously) reinvested when it is paid. On the monthly basis, the coupon is invested when the bond goes ex-coupon. A therefore follows T1, which follows the same course as before. As the frequency of rebalancing increases, the error in CA becomes increasingly positive, tending towards LM. This is because it effectively allows for the reinvestment of accrued interest on the rebalancing dates. GC and CI are relatively unaffected by the increase in frequency.

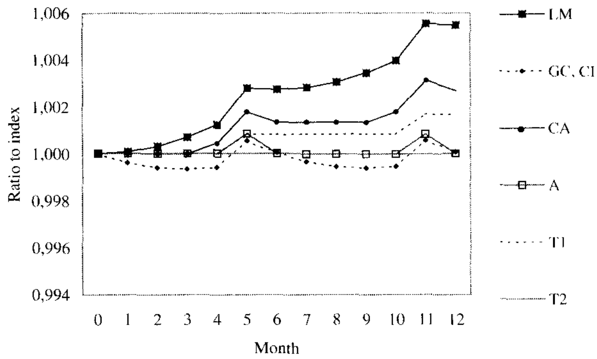

Figure 2: Quarterly rebalancing

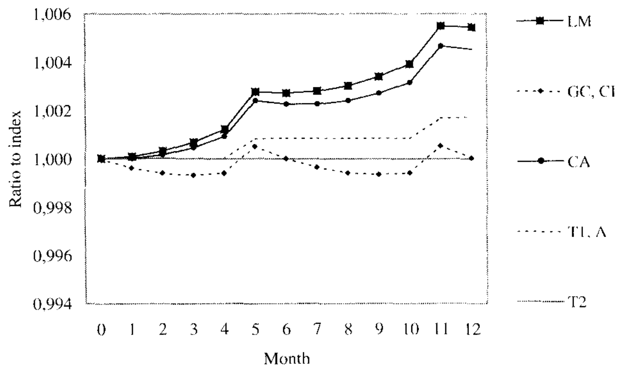

Figure 3: Monthly rebalancing

3.3 Sensitivity tests

Table 1 compares the minimum, average and maximum errors for various combinations of parameters over a one-year period, with next coupon dates ranging from January to June. The errors, which are shown as percentages, are:

$$100\left(\frac{I_t^X}{I_t^{T2}}-1\right).$$

From that table it may be seen that CI produces the lowest errors of all the indices based on clean prices. T1 produces lower errors than any of the other approximations, but even in that case the errors are sometimes unacceptably large. In the case of LM and CA the error is unacceptably large for most combinations. In the case of GC, CI and T1 it is unacceptably large if the yield to redemption changes substantially over the year. In the case of A the error is particularly large for the standard case. On the grounds of accuracy it is evident from the above analysis that a total return index with coupons reinvested on payment dates is most desirable.

4. PRACTICAL CONSIDERATIONS

From a practical point of view it must be borne in mind that the calculation of *T2* is relatively complicated as it requires the use of short-term yields. Not only does this necessitate the determination of a yield curve before the indices can be calculated, it also begs the question whether the yields themselves are reliable. On the other hand, once the index has been determined, it is simple to determine an accurate rate of return on the index over any period.

Table 1: Sensitivity of indices to variations in the parameters

Term	Cpn	Yield to redemption		Rebal	Error per cent of T2																	
					LM			GC			CA			CI			A			T1		
yrs	%	%		mths	min	aver	max	min	aver	max	min	aver	max	min	aver	max	min	aver	max	min	aver	max
20	15	15	15	12	0,0	0,3	0,6	-0,1	0,0	0,1	-0,6	-0,1	0,2	-0,1	0,0	0,1	-1,1	-0,2	0,1	0,0	0,1	0,2
20	15	15	15	3	0,0	0,3	0,6	-0,1	0,0	0,1	0,0	0,1	0,3	-0,1	0,0	0,1	-0,2	0,0	0,2	0,0	0,1	0,2
20	15	15	15	1	0,0	0,3	0,5	-0,1	0,0	0,1	0,0	0,2	0,5	-0,1	0,0	0,1	0,0	0,1	0,2	0,0	0,1	0,2
10	15	15	15	3	0,0	0,3	0,6	-0,1	0,0	0,1	0,0	0,1	0,3	-0,1	0,0	0,1	-0,2	0,0	0,2	0,0	0,1	0,2
20	10	15	15	3	0,0	0,3	0,5	-0,1	0,0	0,1	0,0	0,1	0,3	-0,1	0,0	0,1	-0,2	0,0	0,2	0,0	0,1	0,2
20	15	20	15	3	0,0	1,3	3,1	-0,2	0,5	1,7	0,0	0,8	1,5	-0,5	-0,1	0,4	-0,6	0,0	0,6	0,0	0,3	0,6
20	15	10	15	3	-1,9	-0,6	0,0	-1,7	-0,5	0,1	-0,9	-0,4	0,0	-0,7	0,0	0,4	-0,3	0,0	0,2	-0,3	-0,1	0,0
20	15	15	20	3	-1,7	-0,5	0,1	-1,8	-0,6	0,1	-0,9	-0,4	0,0	-0,8	0,0	0,4	-0,3	0,0	0,2	-0,3	-0,1	0,1
20	15	15	10	3	0,0	1,0	2,6	-0,2	0,5	1,5	0,0	0,6	1,3	-0,5	-0,1	0,4	-0,5	0,0	0,5	0,0	0,2	0,5

The determination of the rate at which the outstanding coupon should be valued is debatable. Ideally, the rate should be the short-term interest rate determined from the yield curve according to the outstanding period to the coupon payment date for each bond. This necessitates first calculating the yield curve. Another possible method would be to use the yield to redemption for each bond whose coupon is being valued. If the yield to redemption does not change when the bond goes ex-coupon, it implies that the yield at which the coupon is valued is equal to the yield at which the remaining payments to redemption are valued. The error in using this approximation is likely to be small, being of the order of up to one month's interest at the difference between the short and long rates on half a year's coupon on ex-coupon bonds. Thus, for example, if yields are 10% a year at the short end and 15% at the long end, if coupons are 15% a year, if bonds are uniformly distributed over the maturity spectrum and if one-sixth of the bonds are ex-coupon, the error is only 0,002%. This is likely to be less than the rounding accuracy of the index. Furthermore, the error will be transitory. The decision therefore hinges on whether the use of the yield curve would create timing or programming problems in the calculation of the bond indices.

It may be questioned why it is considered unnecessary to publish all-in price indices, ex-coupon adjustments and accrued interest indices, when in the United Kingdom it is considered necessary to do so. The main reason for the publication of these series in the United Kingdom is to facilitate the calculation of the effect of tax on passive portfolios so as to permit comparisons with linked internal rates of return net of tax. In South Africa the taxation of interest on bonds in the hands of financial institutions is based on yields to redemption at historic dates or on interest and capital appreciation. In the former case it is not possible to devise a simple method of calculating tax on the index. In the latter case, tax can be allowed for on increases in the total return index. The Fixed-Interest Working Group of the Institute and Faculty of Actuaries (Feldman *et al*, 1997: 5) acknowledged that there was a demand for total return indices in the United Kingdom. In any case, in investment performance calculations, no allowance is generally made for tax; comparisons are invariably made gross of tax.

5. CONCLUSION

It is apparent that the interests of users of the South African bond indices would be best served by the discontinuance of the current series of Bond Exchange-Actuaries indices and running yields and their replacement with a new series of total return indices allowing for the reinvestment of each coupon on each constituent bond on the payment date, with an adjustment to allow for unpaid coupons on bonds ex-coupon from time to time. The adjustment would not have to be separately published; all that would be required would be one total return (including the adjustment) for each term category and for all bonds.

REFERENCES

Alexander Forbes (unpublished). Index performance using only published index data. Correspondence with the author dated 12 January 1999.

Bodie Z, Kane A and Marcus,AJ. 1996. *Investments.* Chicago: Irwin.

Dobbie GM and Wilkie AD. 1978. The FT-Actuaries fixed interest indices. *Journal of the Institute of Actuaries,* 105(1):15-26.

Feldman KS, Bergman B, Cairns AJG, Gwilt GD, Lockyer PR and Turley FB. 1997. *Report of the Fixed-Interest Working Group.* London: Faculty and Institute of Actuaries.

Glansbeek and Conway. 1994. Revision of index performance calculations. *The Investment Analysts Journal,* 40:52-55.

Institute and Faculty Education. 1999. *Core Reading: subject 301.* Oxford.

Liddle CER and McLeod HD. 1987. The JSE-actuaries fixed interest index. *Transactions of the Actuarial Society of South Africa,* 7(1):203-236.

Milburn-Pyle P. 1979. Measuring the investment performance of pension funds in South Africa. *The Investment Analysts Journal,* 14:9-20.

Paterson AA. 1995. *The JSE-Actuaries All Bond Indices - two flaws.* Stellenbosch: Alexander Paterson Faure.

Appendix
Determination of the Error in Liddle and McLeod's Approach

Suppose, without loss of generality, that the index consists of one unit of a single bond with an annual coupon of c and a period of s to the next coupon payment date. Let:

$$d = \frac{c}{V_0};$$

$$e = \frac{c}{V_{\frac{1}{2}}};$$

$$f = \frac{V_1 - V_0}{V_0}; \text{ and}$$

$$K_1 = 0.$$

Then:

$$g_{\frac{1}{2}} = \frac{c}{V_{\frac{1}{2}} - c(\frac{1}{2} - s)};$$

$$J_1 = V_0 e;$$

$$I_r^C = V_r - c(\tfrac{1}{2} - s) \text{ for } r = 0, \tfrac{1}{2} \text{ or } 1; \text{ whence}$$

$$W_1 = \frac{V_1 - c(\frac{1}{2} - s)}{V_0 - c(\frac{1}{2} - s)} V_0;$$

and thus, from equation (1):

$$y_1^{LM} = \frac{\dfrac{V_1 - c(\frac{1}{2} - s)}{V_0 - c(\frac{1}{2} - s)} V_0 - V_0 + V_0 e}{V_0 - \frac{1}{2} V_0 e}.$$

This reduces to:

$$y_1^{LM} = \frac{V_1 - V_0}{V_0 - c(\frac{1}{2} - s)} + \frac{c}{V_{\frac{1}{2}} - c(1 - s)}$$

$$= \frac{f}{1 - d(\frac{1}{2} - s)} + \frac{e}{1 - e(1 - s)}.$$

Ignoring third and higher powers of d, e and f, we find that:

$$y_1^{LM} \approx f\{1 + d(\tfrac{1}{2} - s)\} + e\{1 + e(1 - s)\}. \qquad \text{(A1)}$$

Now y may be found as the solution to the equation:

$$V_0(1 + y) = V_1 + \tfrac{1}{2} c \left\{ (1 + y)^{1-s} + (1 + y)^{\frac{1}{2} - s} \right\};$$

i.e.:

$$y = f + \tfrac{1}{2} d \left\{ (1 + y)^{1-s} + (1 + y)^{\frac{1}{2} - s} \right\}.$$

Again ignoring third and higher powers of d and f, we find that:

$$y \approx (f + d)\{1 + d(\tfrac{3}{4} - s)\}. \qquad \text{(A2)}$$

From (A1) and (A2), ignoring differences between the second powers of d and e, we find that:

$$y_1^{LM} - y = -\tfrac{1}{4} df + e(1 + e) - d(1 + \tfrac{3}{4} d).$$