L van Coller*

Investment Basics XLII. An introduction to swaps

1. INTRODUCTION – WHAT IS A SWAP?

Derivatives are a natural extension of cash markets because of the need to manage future cash flows efficiently. A swap is one of the most versatile interest rate derivatives. A standard swap involves two counter parties exchanging interest cash flows at predetermined dates in the future. For example, in South Africa a plain vanilla swap consists of one counter party paying the other quarterly interest cash flows based on a fixed interest rate which is decided upfront, while the second counter party pays the former quarterly cash flows based on the floating 3 month Jibar rate quoted daily by SAFEX. The floating rate is fixed at the beginning of each 3-month period and the interest cash flow is paid at the end. Only interest cash flows are exchanged – no notional amounts.

Swaps originated because different investors had comparative advantages in different markets. Thus an investor that could obtain a loan relatively cheaper in the fixed rate market than in the floating rate market would conclude the loan at a fixed rate and then swap this fixed rate into a floating rate exposure with another investor whose comparative advantage lay in the floating rate market. Today, swaps are used both to hedge existing interest rate exposures and to speculate on future interest rate movements. In South Africa they are traded mainly between banks, both local and foreign, but the institutions and larger corporates also use them for hedging purposes.

2. PRICING - VANILLA SWAP EXAMPLE

Consider a 1-year vanilla swap on R10 million where we are paying the fixed rate. In order to price this swap we need to determine the fixed rate that gives this swap a value of zero on day 0. This means that on day 0 the swap has no value to either counter party. Value will only be derived as interest rates change causing the yield curve to move. Since a swap is just a series of cash flows, we price the swap by present valuing these cash flows. In order to do this we need a yield curve that gives us the time value of money. Since a vanilla swap is an interbank instrument, we use an interbank yield curve to obtain this value.

Table 1 below shows the quarterly dates of the above swap, the discount factor for each day (assuming that we are currently on the 1 June 2001), the predicted forward rate corresponding to each quarterly time period (based on the current yield curve), and the relevant fixed and floating cash flows. At the end of each period, the net cash flow is exchanged between the two counter parties. Thus, at the end of periods 1 and 2 the fixed rate payer will pay the fixed rate receiver, and vice versa at the end of periods 3 and 4. The amount paid will be the value in column 6.

Let us consider Table 1 in more detail. We start with the quarterly dates for the swap beginning with the start date, 1 June 2001, and calculating the payment dates every 3 months until we reach the one-year point. We need discount factors for each of these dates. Since we are currently on 1 June 2001, the discount factor for this date is 1 as no discounting is necessary. Suppose the wholesale rates for depositing cash are 10,20%, 10,30%, 10,50% and 10,65% for 3, 6, 9 and 12 months respectively. We calculate the future value, FV, of an amount, PV, invested today as follows:

$$PV\,(1 + r_i\, d_i) = FV$$

where

r_i is the interest rate from today until the end of period i,
d_i is the number of days from today until the end of period i divided by 365.

Rearranging this equation in the form

$$PV = FV\,.\,dF_i$$

we obtain the expression for calculating discount factors from these cash rates:

$$dF_i = (1 + r_i\, d_i)^{-1}$$

where

dF_i is the discount factor at the end of period i,

Thus the discount factor at the end of period 2 is given by

$$dF_2 = (1 + 0{,}1030 \times 183/365)^{-1} = 0{,}95091.$$

We can then use these discount factors to obtain the forward rate (predicted floating rate) for each 3 month period. Forward rates are calculated as follows:

$$f_{ij} = (dF_i/dF_j - 1)\,/\,t_j$$

where

f_{ij} is the forward rate from the end of period i to the end of period j
t_j is the number of days in period j divided by 365.

*Investec Bank Ltd, PO Box 785700, Sandton 2146, Republic of South Africa

This expression comes from a rearrangement of the equation:

$dF_j = dF_i \,.\, (1+ f_{ij}\, t_j)^{-1}$

which is an alternate way of calculating discount factors when using forward rates instead of cash rates to obtain the yield curve.

For example, the forward rate f_{34} from the end of period 3 to the end of period 4 (i.e. for the fourth period) is given by

$f_{34} = (0{,}92717/0{,}90375 - 1) / (92/365) = 10{,}28\%$.

The floating cash flows are then calculated as follows

$c_i = N\, f_{i-1,i}\, t_i$

where

c_i is the floating cash flow at the end of period i
N is the notional amount of the swap.

The fixed cash flows are calculated in the same manner replacing $f_{i-1,i}$ by the fixed rate, f.

In order to value this swap today, we need to express all the predicted cash flows in common terms. We thus discount them in order to obtain their value in today's terms (column 6 multiplied by column 2 to give column 7). Summing these discounted cash flows gives the value of the swap. We start by guessing the fixed rate and then use trial and error (or Newton's Method) to modify it until the swap value is 0 – the fair value. When the swap value is zero, the fixed rate can be thought of as a weighted average of the expected forward rates.

Another way to price a swap is to split it into a fixed rate part and a floating rate part. If we add notional payments to the beginning and end of each part, then the swap looks like a combination of a fixed rate bond and a floating rate note. On day 0 the floating side will always discount to zero since it is just a floating rate note. We can thus obtain the value of the swap from the fixed rate bond. This is illustrated in Table 2.

Swap value = $N\,(1 - f\ \Sigma(t_i \,.\, dF_i) - dF_4)$

where

N is the notional amount
f is the fixed rate
dF_i is the discount factor for the end of period i
t_i is the number of days in period i divided by 365

Setting the above equation equal to 0 we can solve for the fair fixed rate, f. Again we can see that f is a weighted average of the discount factors, or alternatively, a weighted average of the forward rates.

In practice swaps hardly ever trade at fair value – there is always a bid/offer spread. In South Africa this is usually quoted around 10 points wide, but it is often possible to trade within this double. This spread can be seen as a transaction cost when trading swaps. It reflects the different cost of lending and borrowing money in the interbank market.

Table 1: valuing a swap by calculating the net present value of the net cash flows exchanged

1	2	3	4	5	6	7
Date	**Discount factor**	**Forward rate**	**Fixed cash flow**	**Floating cash flow**	**Net cash flow**	**PV of net cash flow**
01-Jun-01	1,00000					
01-Sep-01	0,97494	10,20%	-258 294,61	257 067,81	-1 226,80	-1 196,05
01-Dec-01	0,95091	10,14%	-255 487,06	252 696,14	-2 790,92	-2 653,91
01-Mar-02	0,92717	10,38%	-252 679,51	256 019,68	3 340,17	3 096,91
01-Jun-02	0,90375	10,28%	-258 294,61	259 127,87	833,25	753,05
					Swap value	**0,00**

Notional	10 000 000
Fixed rate	10,25%

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Table 2: Valuing a swap inclusive of the notional payments

Date	Fixed side	Floating side	Net cash flow	Fixed PV	Floating PV	Net PV
01-Jun-01	10 000 000,00	-10 000 000,00	-	10 000 000,00	-10 000 000,00	-
01-Sep-01	-258 294,61	257 067,81	-1 226,80	-251 821,10	250 625,05	-1 196,05
01-Dec-01	-255 487,06	252 696,14	-2 790,92	-242 944,80	240 290,89	-2 653,91
01-Mar-02	-252 679,51	256 019,68	3 340,17	-234 277,12	237 374,02	3 096,91
01-Jun-02	-10 258 294,61	10 259 127,87	833,25	-9 270 956,98	9 271 710,04	753,05
			Sum	**-**	**-**	**-0,00**

3. OTHER EXAMPLES

A number of variations on the above vanilla swap are possible.

- Other floating rates may be used. For example, prime may be used as the floating rate instead of Jibar. In this case the floating rate is an average of the daily prime rates and is usually paid monthly. This product is used to hedge prime-linked loans.
- RODS (Rand Overnight Deposit Swap) – here the floating rate is based on the overnight call rate, compounded monthly. Both fixed and floating cash flows only take place on maturity of the swap. This product is used to hedge South African call deposits.
- Cross-currency swap – for example between US dollar and Rand interest rates. Dollar and Rand notionals are also exchanged in this type of swap. This may happen only at the beginning and end of the swap, or at the end of each period. Both sides of the swap may have floating rates, or one side may be fixed and the other floating, or both may be fixed. These swaps are exposed to interest rate movements in two countries as well as the currency exchange rate between the two countries. Currency swaps are really just a combination of simultaneous borrowing of one currency and lending of another.
- Basis swap – both sides involve floating rates. It allows one to speculate on or hedge relative movements between two rates.
- Zero coupon swap – in this case the periodic cash flows are deferred to the final maturity date and are compounded at the end of each period.
- Roller coaster swap – in these swaps the notional amount on which the interest cash flows are based changes for each period. These swaps are often used to hedge the interest on loans that are being paid off over time.
- Forward starting swap – there is no reason for a swap to start today. We can price a swap now for a start date in the future and lock in the current interest rate levels.
- Total return swap – the non-floating rate side of the swap is the total return on an equity or fixed income instrument with a life longer than the swap.

The list of variations really depends on one's imagination. If the product can be broken down into a series of cash flows, then all we need is a yield curve in order to value these cash flows and price the swap.

4. RISK EXPOSURE

As mentioned above, if a swap is priced at fair value then on day 0 it has no value to either party. If interest rates were to follow the pattern predicted by the yield curve, and if we were able to reinvest the periodic cash flows at the rates given by the curve, then over time both parties would remain neutral – although any one cash flow may be non-zero.

But market interest rates and currency rates are never static. Supply and demand, political uncertainty, and various other market forces cause rates to increase or decrease. A swap is exposed to both absolute moves in rates and changes in the shape of the yield curve since the latter affects the forward rates (predicted floating rates) for each time period. If a bank is receiving fixed on a swap, then it will lose if rates increase as the floating payments will increase. This is comparable to owning a bond. If a bank is paying fixed on a swap, then it is similar to selling a bond – the bank is exposed to decreases in rates. Table 3 shows the sensitivity of the swap in Table 1 to a 1 basis point downward move in rates - its rand per point.

Table 3: The sensitivity of the value of a swap to a one basis point downward movement in rates

Date	Discount factor	Forward rate	Fixed cash flow	Floating cash flow	Net cash flow	PV of net cash flow
01-Jun-01	1,00000					
01-Sep-01	0,97496	10,19%	-258 294,61	256 809,28	-1 485,33	-1 448,14
01-Dec-01	0,95096	10,13%	-255 487,06	252 440,53	-3 046,53	-2 897,12
01-Mar-02	0,92724	10,37%	-252 679,51	255 766,79	3 087,28	2 862,65
01-Jun-02	0,90384	10,27%	-258 294,61	258 869,28	574,67	519,41
					Swap value	**-963,19**

The similarity between swap exposures and bond exposures is not a coincidence as we saw above. In fact, swaps and government bonds of similar maturity are frequently used to hedge each other as they have similar rand per point exposures. This kind of trading does generate other risks called basis risks. There is the risk that government bond rates and interbank swap rates move in relation to each other, and there is the risk that the floating rate on the swap (3 month Jibar) moves relative to the carry rate on the bond.

5. SWAPS IN PRACTICE

The need to exchange one type of interest payment for another is fundamental to the running of many businesses, hence the popularity of swaps. The swap markets in many first world countries are even more liquid than the government bond market. In fact, it is the swap markets that link the capital markets of major currencies due to the relative ease of trading a swap rather than an asset.

Swaps are used in a number of ways:

- To hedge interest rate exposures (such as in asset/liability management)
- To speculate on future interest rate movements
- In financial packaging (structured financing)

For example, suppose a company is paying floating rate interest at Jibar plus a margin on a loan. It can fix its cost of financing via an interest rate swap to protect against adverse exposure should interest rates rise. In some ways it is like buying insurance against an increase in rates – the cost in this case is the loss of reduced interest payments should rates decrease instead.

Issuers of fixed rate Eurobonds swap the liability of these bonds to floating rate liabilities using swaps. The floating rate they are able to get in this way is often more attractive than the rate they could have obtained domestically using the usual bank credit lines.

Swaps are also used to construct more complicated financial packages to meet the individual needs of a borrower or investor. They are often used in conjunction with options to create the desired profile.

Banks and trading boutiques also trade swaps speculatively depending on their view of where interest rates will move in the future. Yield curves change shape as well as move up and down. Portfolios of swaps are used to construct a variety of yield curve plays, which attempt to benefit from these movements. For example, if we think that the 2 year swap rate is going to increase relative to the 1 year rate (yield curve steepening), then we would buy a 2 year swap (pay fixed), and sell a 1 year swap (receive fixed). We can trade the volumes of these two swaps in such a way that their rand per point exposure to parallel shifts in the yield curves is the same. We would then have very little exposure to small parallel shifts in the curve, but would make money if the yield curve steepened in this area, and lose money if it were to flatten instead. Using a variety of swap maturities and volumes it is possible to construct fairly complicated exposures to yield curve movements. Other products that can be added to these portfolios are FRA's (forward rate agreements which are just forward starting, single period swaps), caps and floors (options on FRA's), and swaptions (options on swaps).

Swaps are one of the most liquid derivatives because of their versatility and because of their low capital requirements. Because they are traded in large volumes (usually notionals greater than R50million) pricing them accurately is important. Yield curve generation is thus fundamental to financial engineering.

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