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Credit risk models in the South African context

1. INTRODUCTION

Credit risk is the risk that one party to a financial transaction who is under obligation to make a payment, fails to do so. We specifically consider the risk that a company issuing a fixed-income instrument (bond), defaults. Such risk needs to be quantified so that contracts can be hedged and priced.

The first class of credit risk models, based on Merton, imposes assumptions on the evolution of the firm's value and its liability structure. These structural models are based on the Black-Scholes pricing framework. The idea is to obtain a partial differential equation which can be solved (analytically or numerically) to find a price for default-risky bonds. From this we can obtain the credit spread – the difference between the yield on the risky bond and the yield on a same maturity risk-free bond. The defaultable security should be sold at a lower price (higher yield) than the risk-free one.

In short, the idea behind structural models is as follows:

Suppose a firm has a single issue of debt in its capital structure, namely a zero-coupon bond with face value B and maturity T. We assume that the value of the firm at time t, V(t), equals equity value F(t) plus debt value D(t):

$$V(t) = F(t) + D(t)$$

with

$$D(T) = B.$$

If V(T) > B the debt will be paid, otherwise the company defaults and bondholders take over the company. Equity holders thus hold a call option on the assets with pay-off max [V(T) – B,0].

F(t) can then be valued by the Black-Scholes formula and D(t) = V(t) – F(t) gives the price of the risky bond.

The basic Merton model (Merton, 1974) with constant interest rate has been extended to incorporate stochastic interest rates, coupon bonds and bond indenture provisions (Brennan and Schwartz, 1980; Cox, Ingersoll and Ross, 1985; Duffie and Singleton, 1997; Hull and White, 1993; Kim, Ramaswamy and Sundaresan, 1993; Longstaff and Schwartz, 1995; Shimko, Tejima and van Deventer, 1993).

An alternative approach to this "structural" or "contingent claims analysis" approach, is the "reduced form" approach. Such models (Jarrow and Turnbull, 1995; Jarrow, Lando and Turnbull, 1997) impose assumptions directly on the bankruptcy and recovery rate processes instead of on the value of the firm.

2. THE MODEL OF MERTON

With the dynamics of the value V of the firm given by

$$dV = \alpha_{(V)} V dt + \sigma_{(V)} V dz \qquad \text{... (1)}$$

and that of the equity value F given by

$$dF = \alpha_{(F)} F dt + \sigma_{(F)} F dz \qquad \text{... (2)}$$

we find, applying Itô's lemma to F = F (V,t), the following relations:

$$\alpha_{(F)} F = \frac{1}{2}\sigma^2_{(V)} V^2 F_{VV} + \alpha_{(V)} V F_V + F_t$$
$$\sigma_{(F)} F = \sigma_{(V)} V F_V$$

The expected rate of return and variance of return per unit time (for firm and equity values) are denoted by $\alpha_{(\cdot)}$ and $\sigma^2_{(\cdot)}$ respectively. Processes (1) and (2) are geometric Brownian motion processes with dz denoting a standard Gauss-Wiener process. Subscripts to $F(F_V, F_{VV}, F_t)$ in equation (3) denote partial derivatives of F. The usual perfect market assumptions (Merton, 1974) are made and we assume there are no coupons, cash dividends, new issue or repurchase of equities during the relevant time period.

Following the standard riskless portfolio argument we deduce the parabolic partial differential equation for *F*:

$$-F_\tau + \frac{1}{2}\sigma^2_{(V)} V^2 F_{VV} + rVF_V - rF = 0 \qquad \text{... (4)}$$

where r is the constant riskless rate of interest and $\tau = T - t$ is the time to maturity.

With the initial condition

$$F(V,0) = \max[V - B, 0] \qquad \text{... (5)}$$

one clearly has the correspondence of *F* with a European call option. Using the well-known Black-Scholes pricing formula for call options and the fact

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that the debt $D(V, \tau) = V(\tau) - F(V, \tau)$, we have the price of the risky bond:

$$D(V, \tau) = Be^{-r\tau}\,[N(h_2) + \frac{Ve^{r\tau}}{B} N(h_1)] \quad \dots (6)$$

where

$$h_2 = -\left[\frac{1}{2}\sigma^2_{(V)}\tau + \ln\left(\frac{Be^{-r\tau}}{V}\right)\right] / \sigma_{(V)}\sqrt{\tau}$$

$$h_1 = -\sigma_{(V)}\sqrt{\tau} - h_2.$$

$N(\cdot)$ is the cumulative normal distribution function.

Now writing $D(V,\tau) = Be^{-R(\tau)\cdot\tau}$ where $R(\tau)$ denotes yield-to-maturity on the bond, we can formulate equation (6) in terms of the credit spread or risk premium:

$$R(\tau) - r = -\frac{1}{\tau}\ln\left[N(h_2) + \frac{V(\tau)e^{r\tau}}{B}N(h_1)\right] \quad \dots (7)$$

For a given maturity we see that the credit spread depends on $\sigma_{(V)}$; $V(\tau)$ and B.

While the volatility can be estimated from historical data or be obtained as an implied volatility, the value $V(\tau)$ is not easily obtainable because the present market value of debt is not usually observable (Hull, 2000). We can however observe $F(\tau)$ and estimate $\sigma_{(F)}$. Now, equations (3) and (6) give relations between $F, \sigma_{(F)}$ and $V, \sigma_{(V)}$. This enables us to solve for $V(\tau)$ and $\sigma_{(V)}$ in terms of $F, \sigma_{(F)}$ and B.

Although we are mainly interested in the credit spread, from expression (6), we can also calculate the probability of default

$$P(V(T) < B) = N(-h_2)$$

as well as the recovery rate

$$\delta = \frac{V(\tau)N(h_1)}{Be^{-r\tau}N(-h_2)}$$

One may anticipate that the model will not perform well in empirical tests because of the unrealistic assumptions of constant interest rates and the way bankruptcy is settled. Coupon bonds can also be a problem. Other observations (eg. Shimko, Tejima and van Deventer, 1993) show that credit spreads from Merton's model are significantly lower than implied by data observed in the markets.

3. APPLICATION OF THE MERTON MODEL TO SOUTH AFRICAN BONDS

We investigate a range of 20 South African companies, rated *AAA* to *BBB*. We use the actual debt of the company and analyze the debt financing amount at different maturity dates. One year, three year and five year debt terms were used as shown in Table 1. It is clear that the credit spread increases with volatility of asset value and with the face value of debt (expressed in terms of leverage ratio d = debt/firm value). For a debt term of 5 years the calculated spread varies from 6 bp to 297 bp in the banking sector and from 3 bp to 85 bp in other sectors. More specifically, using a debt term of 3 years, the credit spreads of *AA*-rated companies vary between 2 and 12 bps whereas observed spreads in the South African market at the time of writing and in general, vary between 30 and 60 bps.

Our findings confirm the remarks of researchers in other parts of the world regarding the too low spreads predicted by the Merton model. (Jarrow and Turnbull 1995, Kim, Ramaswamy and Sundaresan 1993, Shimko, 1999).

4. THE SHIMKO, TEJIMA AND VAN DEVENTER (STvD) MODEL

This model (Shimko *et al.*, 1993) generalizes the Merton model to allow for stochastic interest rates, specifically the Vasicek process for the short-term riskless rate of interest *r*. The Vasicek process for *r* means that *r* is mean-reverting in the long run to mean γ with speed *k*, and has constant instantaneous volatility $\sigma_{(r)}$:

$$dr = k(\gamma - r)dt + \sigma_{(r)}d\tilde{z} \quad \dots (8)$$

This implies that the price $P(\tau)$ of a risk-free zero coupon bond with time-to-maturity τ is given by (Shimko et al, 1993).

$$P(\tau) = \exp\left[\frac{1-e^{-k\tau}}{k}\left(\gamma - \frac{1}{2}\frac{\sigma^2_{(r)}}{k^2} - r\right) - \tau\left(\gamma - \frac{1}{2}\frac{\sigma^2_{(r)}}{k^2}\right) - \frac{\sigma^2_{(r)}}{4k^3}(1-e^{-k\tau})^2\right] \quad \dots (9)$$

We still assume that $dV = \alpha_{(V)}Vdt + \sigma_{(V)}V\,dz$ and, very importantly, that $dzd\tilde{z} = \rho dt$. This means there is a correlation between the factors driving the return on assets and those driving the risk-free rate. It can then be shown that the partial differential equation satisfied by the price of the risky bond, is:

Table 1: Credit spread for Merton's and Shimko's model using various debt terms

Company	Rating	Volatility of assets	Leverage ratio	Credit spread (basis points)							
				1 year		3 years			5 years		
				Merton	Shimko	Merton	Shimko	Difference	Merton	Shimko	Difference
A	AAA	32%	25%	0,0	0,0	7	11	4	28	43	15
B*	AA+	19%	37%	0,0	0,0	1	3	2	6	15	9
C	AA	27%	24%	0,0	0,0	2	3	2	11	19	9
D*	AA	5%	84%	0,2	1,1	7	37	31	18	73	55
E*	AA	4%	87%	0,2	1,9	7	44	37	18	80	61
F*	AA	7%	82%	0,4	1,7	12	43	31	29	84	54
G*	AA	3%	91%	0,2	3,6	6	59	53	15	91	76
H	AA-	17%	41%	0,0	0,0	0	2	1	3	11	8
I	AA-	20%	46%	0,0	0,0	5	11	6	19	38	19
J	A+	21%	37%	0,0	0,0	2	4	2	10	21	11
K	A+	29%	38%	0,6	0,9	33	46	13	85	115	30
L*	A+	3%	93%	0,4	6,8	7	69	62	17	100	83
M*	A+	7%	84%	0,7	2,5	15	51	36	36	95	58
N*	A-	4%	89%	0,4	2,8	8	54	46	21	90	69
O	A-	24%	36%	0,0	0,0	6	11	5	24	41	17
P	A-	41%	13%	0,0	0,0	3	4	2	18	27	9
Q*	BBB+	30%	49%	16,0	19,1	160	191	31	297	346	49
R*	BBB	15%	69%	1,6	2,7	34	59	26	77	123	46
S*	BBB	18%	67%	7,4	10,3	84	116	33	169	220	51
T*	BBB	22%	60%	9,8	12,7	108	139	31	211	260	49

* Companies marked with an asterisk are from the banking sector

$$-D_\tau + \frac{1}{2}\sigma^2_{(V)}V^2D_{VV} + \frac{1}{2}\sigma^2_{(V)}D_{rr} + \rho\sigma_{(r)}\sigma_{(V)}VD_{rV} + k(\gamma - r)D_r + rVD_V - rD = 0 \qquad \ldots (10)$$

with initial condition $D\ (\tau = 0) = \min(B, V)$.

The solution to this is

$$D(\tau) = V - N(h_1) + BP(\tau)N(h_2) \qquad \ldots (11)$$

where

$$h_1 = \frac{1}{\sqrt{\Sigma}}\left[\ell n\frac{V}{P(\tau)B} + \frac{1}{2}\Sigma\right]$$

$$h_2 = h_1 - \sqrt{\Sigma}$$

$$\Sigma = \tau\left(\sigma^2_{(V)} + \frac{\sigma^2_{(r)}}{k^2} + \frac{2\rho\sigma_{(V)}\sigma_{(r)}}{k}\right) + \frac{e^{-k\tau} - 1}{k^3}\left(2\sigma^2_{(r)} + 2\rho\sigma_{(r)}\sigma_{(V)}k\right) - \frac{\sigma^2_{(r)}}{2k^3}\left(e^{-2k\tau} - 1\right).$$

Σ can be considered as the integrated instantaneous variance of the risk debt function D over the life of the risky bond.

Now we write, as previously,

$D(\tau) = Be^{-R(\tau).\tau}$

and

$P(\tau) = e^{-r(\tau).\tau}$

so that the credit spread is

$$R(\tau) - r(\tau) = -\frac{1}{\tau}\ell n\frac{D}{B} + \frac{1}{\tau}\ell nP = \frac{1}{\tau}\ell n\frac{PB}{D}. \qquad \ldots (12)$$

For a given maturity we see that the credit spread depends on $\sigma^2_{(V)}, \sigma^2_{(r)}, \rho, k, \gamma$ and the leverage ratio.

Shimko et al (1993), show that the credit spread increases as leverage increases and also increases with increasing $\sigma_{(V)}, \sigma_{(r)}$ and ρ. They do not apply the model to real-world data however, but choose a hypothetical situation. Shimko (1999) does report better credit spreads than can be obtained with Merton.

5. APPLICATION OF THE STVD-MODEL TO SOUTH AFRICAN BONDS

Using the same 20 bonds as for the Merton model, with $\sigma_{(r)} = 5\,\%$ and $\rho = 30\,\%$, we find (Table 1) that the credit spreads increase significantly compared to the output from Merton's model. For a debt term of 5 years, the calculated credit spreads now vary from 15 bp to 346 bp in the banking sector (companies marked with an asterisk in Table 1) and from 11 bp to 115 bp for the other sectors.

Comparing *AA*-rated companies with a debt term of 3 years, the credit spreads vary (with one exception) between 37 and 59 bp. This compares well with observed credit spreads in the market. The STvD-model is much more responsive to low volatilities and high leverage ratios.

6. COMPARISON BETWEEN THE TWO MODELS

Figures 1 and 2 show the difference in spreads and their dependence on asset volatility and leverage ratio. It appears that for leverage ratios of 50% and above, the STvD-model is the more sensitive and gives much higher spreads. Also, for low asset volatilities (under 10%), the STvD-model seems to perform better than the Merton model. This can be seen from Table 1 and Figure 2.

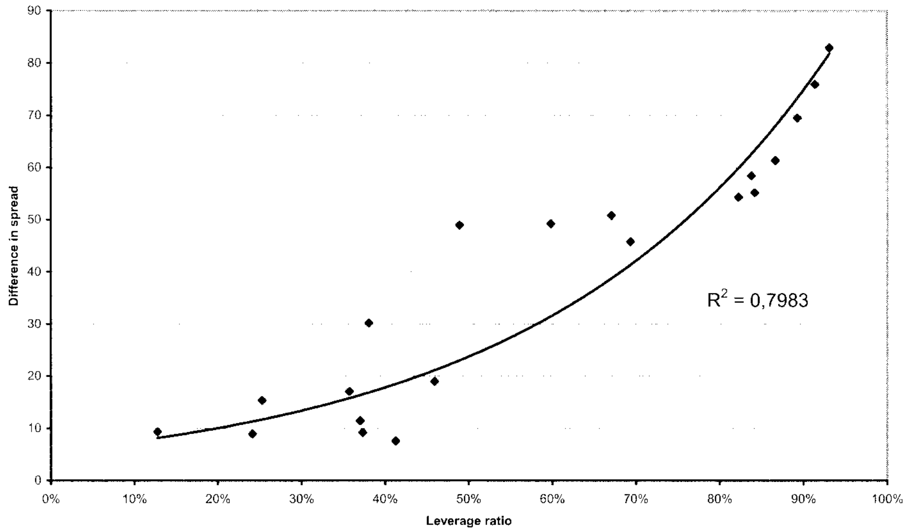

Figure 1 : Shimko vs Merton Models: Difference in credit risk spread relative to leverage ratio

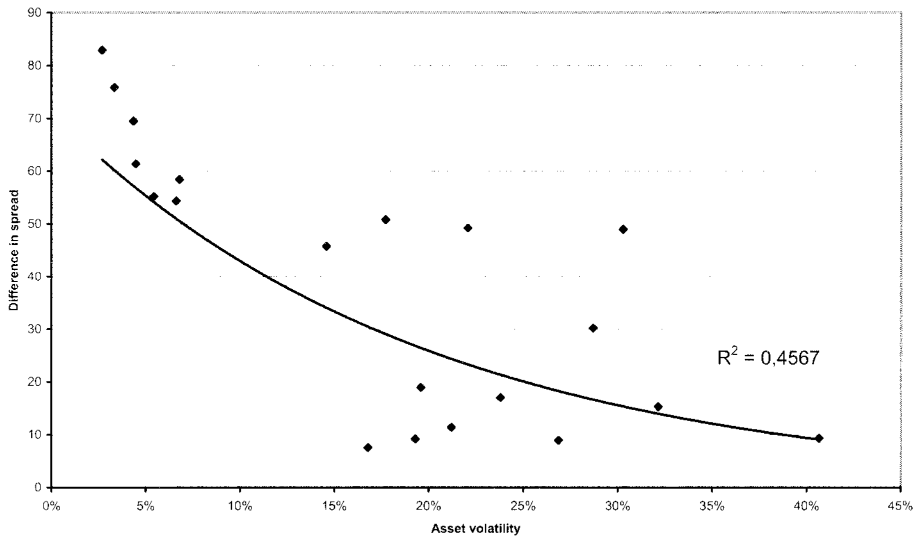

Figure 2 : Shimko vs Merton Models: Difference in credit risk spread relative to volatility of assets

7. CONCLUDING REMARKS

All in all the STvD-model, which extends Merton's model by including stochastic interest rates, yields credit spreads that fit the observed spreads better. Our study, which tests two credit risk models in an emerging market environment, further validates the use of such models to value risky debt.

The results of this study are important, not only for South African role players, but for all users of credit risk models. The reason is that such models should be subjected to wide-ranging critical analysis involving countries with different market conditions to those in the US or in Europe.

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