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Optimising the global/local equity split

1. INTRODUCTION

International portfolio diversification is often advocated as a means of enhancing portfolio performance through the reduction of portfolio risk. The expanded investment universe provides an enlarged set of assets whose returns often show a low degree of correlation with domestic assets. The configuration of an efficient portfolio is determined by the inter-country correlations of returns as well as the expected returns and variance of return for the various countries with all return data converted into local currency. The literature suggests that expected returns are unlikely to be strongly correlated to past returns but that past variances and correlations may be useful in predicting the future correlation structure for an efficient portfolio (Jorion, 1985; Eun and Resnick, 1984; Elton and Gruber, 1995).

An examination of efficient *ex-post* international portfolios based on the Markowitz mean-variance approach, shows that substantial benefits could potentially have been derived by South African investors who had (hypothetically) diversified internationally in the past. However, even if world capital is represented by just the four largest country equity markets, for combination with the South African market, the inter-country correlation structure is so variable over time, that the configuration of a future efficient portfolio is unlikely to be predictable (Swart, 1999).

In this study the allocation problem is further simplified by representing the world market by the MSCI World Index and the local market by JSE-All Share Index. The approach is more practical than in previous studies and seeks investment strategies that can be easily implemented and maintained. It is felt that only assessing portfolio performance over a single total study period is not adequate in today's volatile markets. Risk-adjusted performance is therefore considered not only over the entire 30 year study period, but also over shorter review periods of three and five years. The re-balancing of *ex-ante* portfolios only takes place at the beginning of each review period, thus avoiding the high cost associated with monthly re-balancing and making the policies suitable for passive investors. In addition to volatility, non-symmetric and truly downside risk measures are also employed (see section 4.2). Simple *ex-ante* strategies are identified in section 4.4 that showed significant outperformance of the South African market (see section 5).

2. LITERATURE REVIEW

Studies of the benefits of international diversification, from a South African perspective, have been carried out by various authors including van den Honert and Affleck-Graves (1985), Barr (1986), Barr and Affleck-Graves (1987), Bhana (1986, 1987, 1990), Patrick and Ward (1996) and Swart (1999). The earlier investigations followed the release of the De Kock Commission report in 1985 and apart from Bhana (1987), considered only the *ex-post* benefits of international diversification with international equity markets restricted to the US and UK.

Patrick and Ward (1996) enlarged the opportunity set to include 27 countries that had at least ten years of index data available. The South African market was represented by the disjoint segments consisting of the mining sector (All Gold Index) and the financial and industrial sector (Financial and Industrial Index). Empirical evidence was provided that South African investors could have benefited from international diversification during the period of study. Three *ex-ante* strategies were investigated over a 19 year period ending in 1994. The first, 'buy-the-market' approach, invested equal amounts in each of 27 countries (two sectors for South Africa), the second, 'naïve diversification', combined the two South African indices with the MSCI World Index and the last, 'world one-third rule', combined a portfolio consisting of equal investments in the three regions North America, Europe and Pacific with the South African indices mentioned above. The only strategy that showed benefits in terms of both higher return and lower risk was the 'buy-the market' approach. The other two strategies provided better risk-adjusted performance, due to a lowering of the volatility, provided the South African portion was at least 50%. It should be noted that each of the *ex-ante* strategies based on monthly data would have required the investor to re-balance the portfolio once a month, a daunting task especially when 27 sectors are involved!

Swart (1999) considered an international universe consisting of the four biggest capital markets, representing 75% of world capital, to combine with the local market, using as proxy the JSE-All Share Index. An analysis of historical data confirmed significant potential benefit from diversification for South African investors. However, the lack of intertemporal stability of the inter-country correlation structure results in substantial changes in efficient portfolio structures over adjacent time intervals, thus making it difficult to

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predict the configuration of future efficient portfolios and hence to formulate near efficient *ex-ante* investment strategies. Further investigation by the author has shown that in the above case, it is unlikely that simple passive investment strategies could be identified that would yield consistent risk-adjusted outperformance of the local market over the entire study period, as well as over shorter review intervals. For this reason the problem is further reduced in this study to a simple two-asset problem consisting of the South African equity market and the world market.

3. OBJECTIVES OF THE STUDY

The main objective is to determine simple *ex-ante* investment strategies that do not require frequent rebalancing and that are likely to benefit passive South African investors. Portfolios based on these strategies should perform well in risk-adjusted terms, both in the long term and fairly consistently over shorter review periods. The outperformance should occur whether risk is measured by volatility or retracement.

4. RESEARCH METHODOLOGY

4.1 Data

The benchmark MSCI World Index, computed by Morgan Stanley Capital International is used as a proxy for the world market and the JSE All Share Index is the proxy for the South African market. These are both total return indices. The period of the study extends from 1 January 1970 to 31 May 2002 and thus includes the entire period during which the MSCI World Index and the market-capital-weighted JSE All Share Index co-existed. The MSCI World Index was launched in 1970 and the JSE All Share Index was replaced by a new free-float-weighted index on 24 June 2002. Monthly total return data converted to rands at the appropriate exchange rate were used in all the calculations. All returns shown are annual compound rates and standard deviations are expressed in annualised terms. In this study the ratio of the compound annual return over the annualised standard deviation is referred to as the *Sharpe ratio*.

4.2 Risk measures

There is no unique definition of portfolio risk accepted by all market players. Adams (1989:9) distinguishes between the definitions used by financial economists ('uncertainty of future returns on investments') and actuaries ('ability to meet liabilities when they fall due'). In this study it is assumed that investors have considered their liabilities and risk profile and have decided to commit a portion of their surplus funds to the equity markets. It is believed that in this case the financial economist's definition of risk, in terms of uncertainty of investment returns, is adequate.

Some investment practitioners criticise the use of volatility as the measure of risk in Modern Portfolio Theory, as developed by Markowitz (1952, 1959). Amongst other arguments put forward is that risk is more properly associated with adverse outcomes and that an asymmetric measure of risk such as the semi-variance may be more appropriate. Another appealing measure of downside risk is the Average Maximum Retracement (AMR) (Schwager, 1996:712), which is discussed in the appendix. The associated performance measure is the Return Retracement Ratio (RRR)—it is defined as the annualised compound return per unit of retracement (see appendix).

In this study the above three risk measures will be considered, namely the standard deviation of returns, the semi-variance and the AMR.

4.3 Historical market performance

Over the period of the study (January 1970 to May 2002) the total return on the JSE All Share index was 18,1% compounded annually. The corresponding MSCI return was 19,9% in rand terms. The relatively good performance of the JSE is perhaps somewhat surprising as the rand depreciated by a compounded 8,4% annually over this period. This result clearly indicates that improved risk-adjusted performance can only be achieved by risk reduction. Figures 1 and 2 highlight the high-risk nature of the JSE, both in terms of volatility and retracement. A combination strategy where 30% of funds are placed in the JSE at the beginning of each three-year period, without any further adjustment within that period, is included for comparison. The reason for illustrating the 0,3:0,7 local to foreign equity split will become clear in the sequel where it is shown that this ratio is near optimal.

The average correlation coefficient of the rand returns of the two markets over all 354 three-year periods is 0,24, with a range between –0,4 and 0,6, confirming that considerable volatility reduction should be possible by diversifying across the two markets.

4.4 Investment strategies

Two basic strategies that require infrequent rebalancing are investigated: the first places a constant fraction of the available funds in each market, whereas the second uses the portfolio structure of a historical minimum variance portfolio for the next investment period.

4.4.1 Constant fraction strategies

Two types of 'constant fraction' strategies are discussed. The first is the classical *constant fraction* investment strategy, where it is assumed that the fraction of funds in each market is kept constant throughout the investment period. In practice this strategy is not feasible for long term passive investors, as monthly or even continuous rebalancing would be required to maintain the fractional investments in each of the two markets at a given value. This is of course the assumption made in the classical theory, so that if one calculated the configuration of a historical minimum variance portfolio, it would be based on that assumption. The corresponding simple investment strategy consists of making the same initial constant fractional investment in each market, but re-adjustment to that value takes place only at the beginning of the chosen shorter review periods. In this study the review periods considered are three or five years in duration and no adjustment takes place to the investment portions during these periods. Such strategies will be referred to as *variable fraction* strategies.

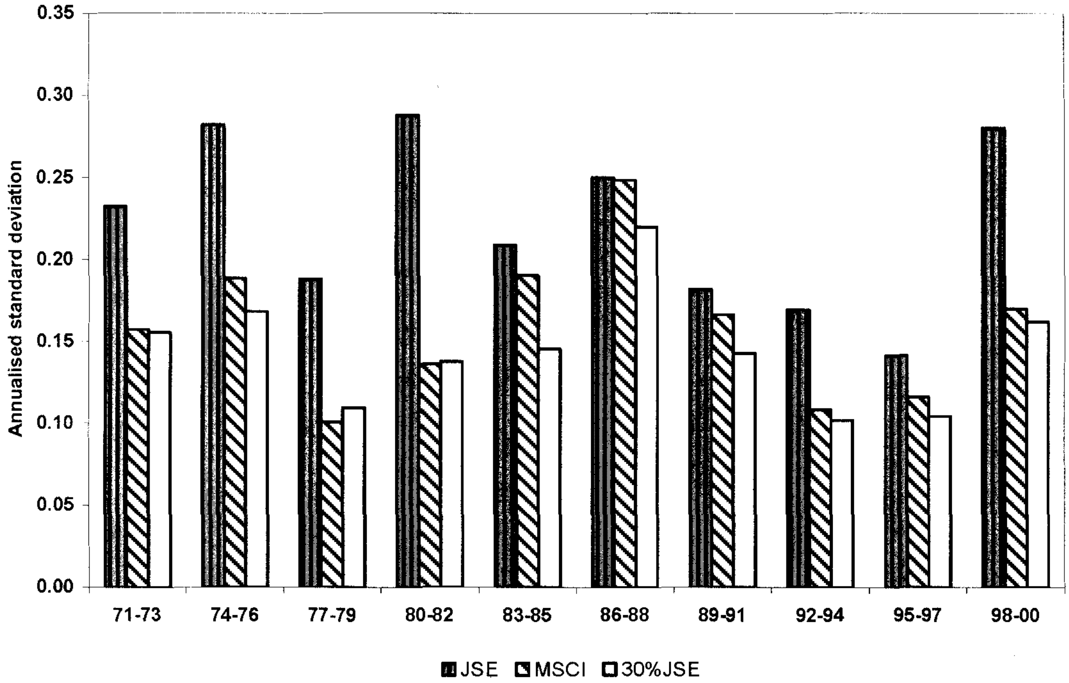

Figure 1: Annualised standard deviations over 36 month review periods

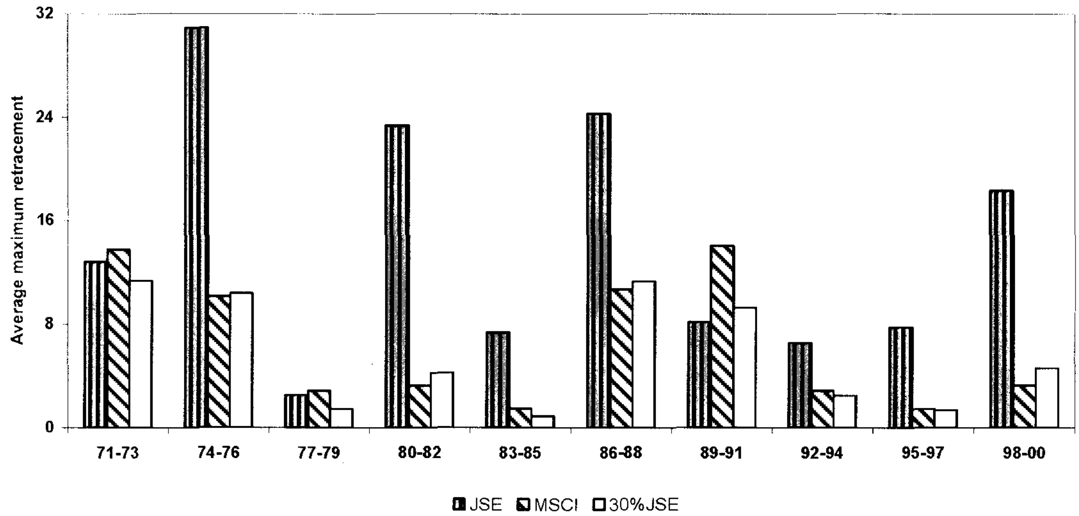

Figure 2: Average maximum retracement over 36 month review periods

The time variance of the 0,3 variable fraction investment strategy for the case of five-year review periods is illustrated in Figure 3, Initially 30% of funds is invested in the local market and this value is adjusted only at five-year intervals during the thirty-year investment period. The value of the fraction invested in the local market one month after adjustment to 0,3 is highlighted. It is clear that a considerable amount of rebalancing would have been required if the fraction had been kept even approximately constant at all time.

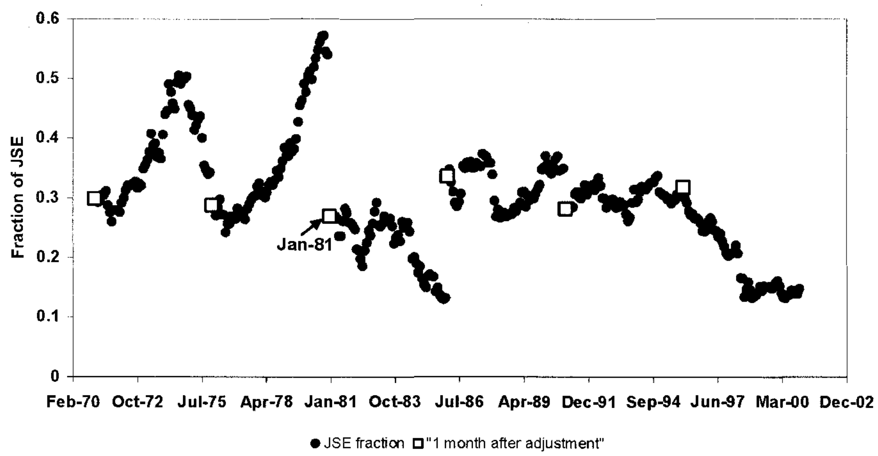

Figure 3: The time variance of the 30% JSE variable fraction, adjusted five-yearly highlighting the value one month after adjustment

In section 5.2, optimisation techniques are applied to determine the optimal *ex-ante* constant fraction strategies over the study period.

4.4.2 Strategies based on the MVP

Parameter sensitivity analyses indicated that the configuration of the Mean Variance Portfolio (MVP) is reasonably insensitive to small parameter changes, but that the same is not true of the historical tangent portfolio. This suggested the investigation of an investment strategy where the configuration of the historically efficient MVP is used as an investment strategy for the next period. The policy named *Previous Mean Variance Portfolio* (*PMVP*) requires yearly rebalancing. Once a year, the configuration of the historical MVP is calculated based on the previous 36 months return data. This configuration is then used for the following year when the process is repeated.

A variation of the above policy is the *Adjusted Previous Mean Variance Portfolio* (*APMVP*), where the fraction JSE in the historically efficient MVP, identified by the three-year analysis, is modified based on the assumption that this parameter is likely to regress to its long-term average. A record is kept of the historical average value of the fraction JSE in the MVP, based on all previous 36-month periods. The target fraction is then set to the average of the fraction identified by the latest 36-month analysis and the long-term average fraction JSE in the historical MVP. This calculation of the target fraction and corresponding rebalancing of the portfolio, is carried out annually. The fraction of the APMVP invested in the JSE is shown in Figure 4 and illustrates the time variance of the variable fraction about the target value.

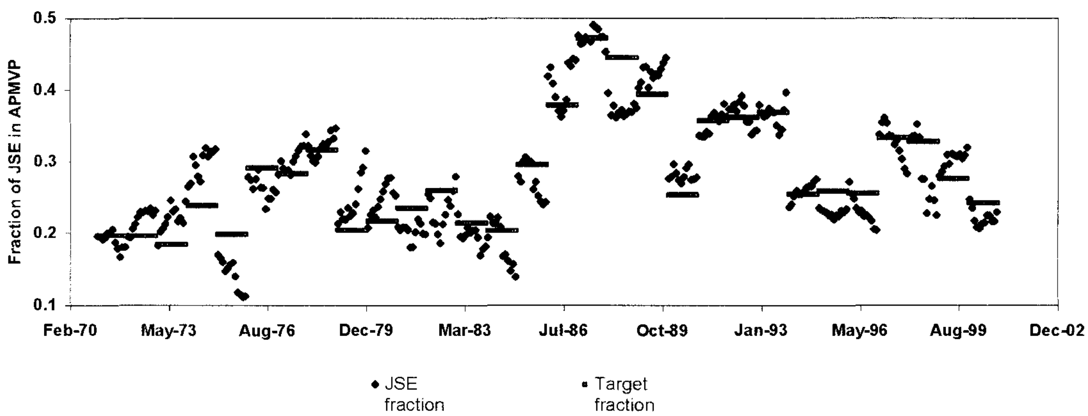

Figure 4: The time variance of the JSE variable fraction in the APMVP, adjusted annually

5. RESULTS

Table 1 illustrates the results obtained by applying the investment strategies described above during the 30-year period January 1971 to December 2000, subdivided into ten disjoint three-year periods. The fractional strategies reported are the easily implementable variable fraction ones, with a review period of three years. The two strategies based on the MVP require yearly rebalancing within the review period and clearly seek to optimise risk-adjusted return by minimising the standard deviation of return. There is no guarantee that such strategies will yield portfolios of low retracement, but the results in Table 1 indicate good performance in this regard. The two superior strategies (30% JSE fraction and APMVP) show an increase in the Sharpe ratio of more than 66% over the JSE. In terms of return per unit of retracement, RRR, the improvement is over 300%. The outperformance of the world market (MSCI) by these strategies is less impressive but also substantial. Results obtained by extending the review interval to five years for the variable fraction strategy and considering corresponding disjoint five year periods are similar, e.g., the 60 month Sharpe ratio in the last line of Table 1 now reads 0,92; 1,44; 1,61; 1,62; 1,29; 1,54 and 1,60. Classical constant fraction assumptions lead to results that are quite similar to the above.

Table 1: Risk-adjusted performance over the period January 1971 to December 2000, divided into 10 disjoint 36 month review periods.

Jan 1971 to Dec 2000	JSE	MSCI	Hist MVP	30% JSE	70% JSE	PMVP	APMVP
Compound Return	19,6	21,5	20,3	22,2	21,4	21,4	21,7
Ave 36m std deviation	22,2	15,8	13,9	14,5	17,4	14,8	14,4
Ave 36m semi-variance	24,0	16,8	15,1	15,7	18,6	16,1	15,8
Average 36m AMR	14,2	6,4	5,8	5,7	8,8	6,4	5,9
Average 36m RRR	3,42	9,32	9,07	11,89	5,85	10,71	12,13
Average 36m Sharpe ratio	0,98	1,44	1,59	1,66	1,36	1,56	1,63

5.1 Consistency of the outperformance

The results in Table 1 reflect the long term performance of the various investment strategies. It is likely that even long term investors would wish to review the performance of their portfolios at regular intervals of say three or five years. In Table 2, the risk-adjusted performances of various variable fraction policies are indicated, relative to the performance of the JSE, over these review periods. For the first run, the thirty year period from January 1970 to December 1999 was divided into ten non-overlapping three year review intervals. Both the Sharpe and the RRR measures were used as performance measures.

The first row of the table indicates that a strategy where 30% of the funds were invested in the JSE and the remainder in the MSCI, outperformed the JSE in 9 of the 10 review periods in terms of the Sharpe measure and in 8 of the 10 periods, using the RRR measure. Notice again that portfolio adjustment occurs only at the beginning of each review period.

The starting date was then moved forward by one month and the process repeated for the next thirty year period, split into ten non-overlapping three year periods. The results are recorded in the second row. The process was repeated for thirty different starting dates as indicated in the table. The last row expresses, in percentage terms, the average number of times that the various policies outperformed the JSE over such non-overlapping review periods. Thus, for example, the 30% JSE fraction outperformed the JSE on average, in almost 8 out the 10 review periods.

The results for five year review periods are computed in a similar manner. Thirty year periods are now divided into six non-overlapping review periods of five years each and rebalancing occurs only at five year intervals. The variable fraction policies which had between 30% and 70 % invested in the JSE, outperformed the JSE in 90% of the review periods. The results are summarised in Table 3.

5.2 Optimising the constant fraction

A natural question arises as to what the optimal constant or variable fraction investment in the JSE would have been over the study period in order to have maximised the Sharpe ratio. This problem was first solved as a programming problem in the case of a constant fraction, using MS Excel's built-in solver routines.

Suppose a fraction a is invested in the local market and denote the monthly returns on the JSE and MSCI by r_i and s_i respectively. The problem is to maximise

$$\prod_{i=1}^{360}\{a(1+r_i)+(1-a)(1+s_i)\}$$

subject to $0 \leq a \leq 1$.

Table 2: Consistency of outperforming the JSE over three year review periods.

	Sharpe analysis				RRR analysis			
Starting date of 30 year period #	%JSE 0	30	50	70	0	30	50	70
Jan-70	8	9	9	9	6	8	8	9
Feb-70	6	9	10	10	7	8	9	9
Mar-70	7	9	9	10	8	9	9	9
Apr-70	8	9	9	10	8	9	9	9
May-70	8	9	9	9	8	9	9	9
Jun-70	8	9	9	9	8	9	9	9
Jul-70	7	9	9	9	7	9	9	9
Aug-70	7	9	9	9	7	9	9	9
Sep-70	7	8	8	8	7	8	9	9
Oct-70	7	8	8	8	7	8	9	9
Nov-70	7	8	8	8	7	8	9	9
Dec-70	7	7	7	7	7	7	8	8
Jan-71	7	7	7	7	7	7	8	8
Feb-71	7	7	7	7	7	8	8	8
Mar-71	7	7	7	8	7	7	8	8
Apr-71	7	7	7	7	7	7	7	8
May-71	7	7	7	7	7	7	7	8
Jun-71	7	7	7	7	7	7	7	8
Jul-71	7	7	7	7	7	7	7	8
Aug-71	7	7	7	7	7	7	7	8
Sep-71	6	7	7	7	6	7	7	8
Oct-71	7	7	7	7	6	7	7	8
Nov-71	7	7	7	7	7	7	8	9
Dec-71	7	7	7	8	6	8	8	9
Jan-72	6	7	8	8	6	8	8	9
Feb-72	7	7	8	8	7	8	8	9
Mar-72	7	7	8	8	7	8	8	8
Apr-72	7	8	8	8	7	8	8	8
May-72	7	7	7	8	7	7	8	8
Jun-72	7	7	7	7	7	7	7	8
Average percentage of three year non-overlapping review intervals in which policy outperformed the JSE	70	77	78	80	70	78	81	85

Each thirty year period was divided into ten disjoint 3 year intervals. The figures in the table reflect the number of times in the ten periods the JSE/MSCI portfolio outperformed the JSE.

Table 3: Consistency of outperforming the JSE over five year review periods.

	Sharpe analysis				RRR analysis			
	%JSE 0	30	50	70	0	30	50	70
Average percentage of non-overlapping five year review intervals in which policy outperformed the JSE	71	89	96	96	70	91	96	96

The optimal constant fraction over the entire study period was found to be 0,301 and over the thirty-year interval from January 1971 to December 2000, it was 0,299. In practice rebalancing would have to occur at monthly intervals.

The more difficult optimisation problem for the corresponding variable fraction was solved next and it yielded an optimal a value very close to 0,3 as in the case of the constant fraction. Once again the modelling was done within MS Excel. Using the same notation as above, the initial capital of 1 has value c_1 after 5 years, where

$$c_1 = a\prod_{i=1}^{60}(1+r_i)+(1-a)\prod_{i=1}^{60}(1+s_i)$$

At this stage the portfolio is re-balanced so that the capital invested in the JSE is ac_1 and that invested in the MSCI is $(1-a)c_1$. After a further 5 years, the total capital has value

$$ac_1\prod_{i=61}^{120}(1+r_i)+(1-a)c_1\prod_{i=61}^{120}(1+s_i)$$
$$=\left\{a\prod_{i=1}^{60}(1+r_i)+(1-a)\prod_{i=1}^{60}(1+s_i)\right\}\left\{a\prod_{i=61}^{120}(1+r_i)+(1-a)\prod_{i=61}^{120}(1+s_i)\right\}$$

Continuing in this manner, it is easily seen that the optimisation problem is that of maximising

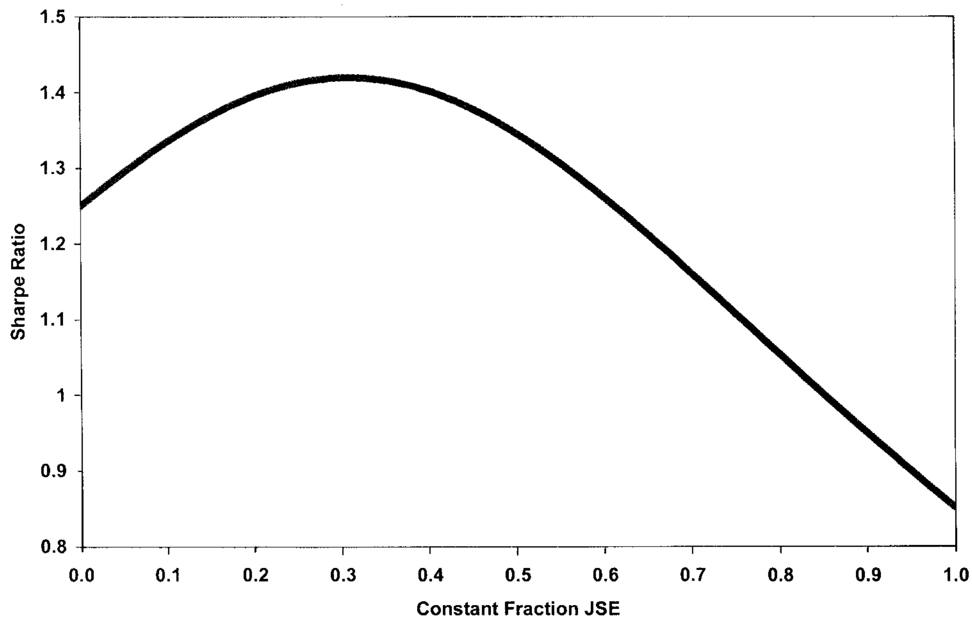

Figure 5: The Sharpe ratio over the study period corresponding to various constant fraction investments in the JSE

$$\prod_{j=1}^{6}\left\{a\prod_{i=1+60(j-1)}^{60j}(1+r_i)+(1-a)\prod_{i=1+60(j-1)}^{60j}(1+s_i)\right\}$$

subject to $0 \le a \le 1$.

In order to illustrate the effect of a constant fraction investment on the Sharpe ratio, Figure 5 was generated. This explains to some extent the low risk performance of the 30% JSE fraction over time as illustrated in Figure 1. .

5.3 A hypothetical question

The question then arises as to theoretically how much the result could be improved on if the variable fraction is not necessarily reset to the same constant value at the beginning of each five-year review period. Denoting the variable fractions by a_i $(i = 1,...,6)$, the optimisation problem becomes: maximise

$$\prod_{j=1}^{6}\left\{a_j\prod_{i=1+60(j-1)}^{60j}(1+r_i)+(1-a_j)\prod_{i=1+60(j-1)}^{60j}(1+s_i)\right\}$$

subject to the constraints $0 \le a_j \le 1$ for $j = 1,...,6$.

The results are shown in Table 4. It shows that even with perfect hindsight, this more complicated strategy, which could not easily be implemented, only improves on the simple one by a relatively insignificant amount.

Table 4: More complicated optimal fractions over five-year review periods.

Type of fraction	Sharpe Ratio	Optimal Fraction
30% JSE variable fraction policy	1,62	
Optimal variable fraction (constant)	1,62	0,299
Optimal variable fraction (non-constant)	1,68	values below

5 year period starting in January	**71**	**76**	**81**	**86**	**91**	**96**
Fraction JSE	0,23	0,62	0,13	0,57	0,26	0,00

5.4 Relaxation of exchange control

The study period extends over a long period of strict exchange control and a shorter period of more relaxed control measures. Since July 1997, private investors have been able to invest a certain amount offshore. An analysis of the 24 three-year periods between this date and May 2002, shows that the 30% JSE variable fraction delivered a higher Sharpe ratio than the JSE, 83% of the time and a higher RRR, 92% of the time. These values are higher than the corresponding values over all intervals tested but due to the small sample size no meaningful conclusion can be drawn.

6. CONCLUSION

The objective of this study was to determine simple beneficial investment strategies for South African investors in the equity markets. These strategies had to be easily implementable, have a good track record over the long term and provide consistent outperformance, in risk-adjusted terms, of the local equity market over shorter review periods.

The variable fraction strategies satisfy the above criteria and could be implemented by investing in low cost unit trust funds or index-shares that track the appropriate indices. The investor would be well diversified. Re-balancing need to occur only at three- or even five-year intervals.

Somewhat more complicated strategies based on the historical configuration of the MVP, promise performance similar to optimal constant fraction policies. Access to monthly return data would be required and certainly more effort would be required to implement the yearly rebalancing. Associated with more frequent rebalancing are higher trading costs.

The correlation structure is certainly not time-invariant, yet the policies identified in the study outperform the efficient historical MVP over the thirty-year period in terms of both the Sharpe ratio and other non-symmetric risk-adjusted performance measures.

The simple rule of '30% local and 70% foreign' could be used as a starting point for more active long-term investment strategies but the details are outside the scope of this simple investigation.

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Appendix: The Average Maximum Retracement (AMR)

The AMR is the average of the maximum retracements, MR_i, experienced at the individual data points i over a given investment period consisting of n months. The maximum retracement, MR_i, at an arbitrary point is the larger of the following two measures:

(i) the maximum percentage retracement from that point to a subsequent equity low, RL_i

(ii) the maximum percentage retracement from a previous equity high, RH_i.

The AMR measures an average worst case scenario by considering the worst case at each point in time. The approach is more meaningful than methods that employ only the single worst case or maximum draw down. More precisely,

$$MR_i = \max\{RL_i, RH_i\}, i = 1, 2 \ldots, n$$

where

$$RH_i = \frac{100(\max E_i - E_i)}{\max E_i}$$

$$RL_i = \frac{100(E_i - \min E_i)}{E_i}$$

and where E_i denotes the equity at the end of month i, $\max E_i$ denotes the maximum equity at time prior to i and $\min E_i$ is the minimum equity subsequent to time i.

The return retracement ratio, RRR, is the annualised compound return divided by the AMR.