D Taylor*

Mathematical finance in South Africa

1. INTRODUCTION

1.1 What is mathematical finance?

It is tempting to say that Mathematical Finance consists merely of a study of the Black-Scholes equation (and its variants) and, for the purposes of many academic programmes in "Financial Engineering", this may well be true. In fact, this also holds true for many practitioners in Mathematical Finance. However, the study of derivative securities and their pricing models, which have been developed over the past 30 years, is now much more complicated than merely finding solutions of a linear, second-order partial differential equation with a variety of boundary conditions. Any complete degree course on martingale or risk-neutral pricing methodology cannot ignore the contributions from stochastic calculus and analysis, numerical approximation theory and, most recently, optimal stopping theory, to the study. Consequently, the mathematical tools that are needed by a Quantitative Analyst in an investment bank have become wide-ranging and complicated. With the increasing sophistication of our understanding of market behaviour comes a concurrent increase in sophistication of our mathematical description of it. Mathematical Finance offers a classically applied mathematical opportunity to invent new mathematics to deal with new problems.

Mathematical Finance is not Actuarial Science. This is not to say that Actuarial Science does not contribute to the research and debate in Mathematical Finance. However, it is clear from our experience, and from analyzing many degree programmes outside of South Africa, that there is an academic distinction between the disciplines.

Mathematical Finance is also not finance in the traditional sense. Although some of the models do draw on the mathematical foundations of 20th century finance, it is the insights of Merton, Black & Scholes that drive and motivate Mathematical Finance. In particular the ideas of *market completeness* and derivative *replication through portfolio design* lead to the theory of no-arbitrage and the Black-Scholes analysis. As a consequence, much of the advancement in the field has been driven by work done by pure and applied mathematicians. In this sense then, Mathematical Finance should be regarded as a truly modern branch of applied science.

*Programme in Advanced Mathematics of Finance, School of Computational and Applied Mathematics, University of the Witwatersrand-Johannesburg, Republic of South Africa.
As with all review articles, numerous people have assisted with this work. The author would like to thank Mr Glenn Brickhill, Dr Jaco van der Walt, Prof Dawie de Jongh, Prof Riaan de Jongh, Prof Mike Murray, Prof Willie Conradie and Dr Peter Ouwehand for their contributions.

Email: taylor@cam.wits.ac.za

2. WHY IS IT TAUGHT?

The trivial answer to this question is that there is a demand in the job market for graduates of Mathematical Finance programmes. In reality the demand is driven by a number of factors which are interlinked and interdependent. Globalisation has led to country specific markets being incapable of insulating themselves from international effects. Although South Africa does retain some version of exchange control, recent years have seen a gradual relaxation of these controls and a(n externally and internally driven) move towards a freely traded currency. Under circumstances of an open economy and sporadic, but aggressive, interest in emerging markets as a source of investment, it is essential that the participants in these markets are as sophisticated in training as their international counterparts.

Most financial markets seem eventually to lead to derivative markets. In most cases the initial impulse for an attendant derivative market seems to be the need for an "alternative" form of insurance. In some instances, the depth of reserves in financial markets makes them cheaper than traditional insurance options. However, it is usually the presence of hedgers, speculators and arbitrageurs that enhances and deepens the derivative markets. In contrast to ordinary financial markets, the pricing and/or hedging of derivatives is invariably dependent on an arbitrage-type argument. It is clear, then, that money can be made or lost through ignorance and mis-pricing. As a consequence it becomes increasingly important that financial institutions in South Africa, and elsewhere in emerging economies, employ financial engineers with highly developed quantitative skills.

In the short time since the birth of derivative pricing theory, the number of derivative products, as well as their complexity, has increased dramatically. The size of the market, in both value and notional amounts, is staggering. (OTC market: Notional = US$220 trillion (thousand billion), Value = US$6 to 7 trillion. Exchange Traded Options: Notional = US$312 trillion.[1]) In many cases, the notional amounts far exceed the value of the underlying market. Pricing and hedging the risk of these (sometimes highly non-linear) financial products is rapidly becoming an extremely non-trivial

[1]Source: BIS (Bank of International Settlements) Quarterly Review, March 2005.

mathematical exercise. Consequently, it will be those institutions that are best prepared for this that will prosper.

3. THE SOUTH AFRICAN DERIVATIVES MARKETS

The South African derivatives market is extremely liquid in some areas and under-developed in others. The investment community tends to be conservative in their approach to derivative use. The derivative disasters of the past ten years or so, and the community's reaction to them, have done little to assist their growth. There is also an extensive lack of knowledge and skill with regard to pricing and hedging derivative positions, even within investment banks.

The local **interest rate** (IR) market is supported by a mature base of short-term money-market instruments (deposits and NCDs) as well as longer term government-issued fixed-rate bonds. The derivatives that reference this market extend from the vanilla sector Forward Rate Agreements (FRAs) and swaps to overnight indexed swaps (Rand Overnight Deposit Swaps (RODS)), caps, floors and swaptions.

Overnight indexed swaps exist out to one year. The South African FRA market currently lies in the short term (zero to two years). Swaps are extremely liquid from one to ten years, with the liquidity beyond this dropping off very rapidly to around twenty years. Caps and floors are quoted out to five years but are only deeply liquid out to three years. Bond options tend to be very short dated and trade out to one year. This is because demand for bond options is driven by speculators who do not have very long investment horizons.

The interest rate models which are applied to the valuation and risk management of the above derivatives tend to be standard market models.

The **equity derivatives** market is significantly more complex in its product offering than the interest rate market. Some of the more popular products that are sold include cliquets (with additional collar or fence characteristics), look back options, basket options, as well as put and call options with exotic underlying assets, e.g. Unit trusts or funds of funds. The equity derivative market is a rich source of academic and commercial research but the markets tend to be over-the-counter (OTC) and reliable data/volatilities are hard to find.

Although in international markets, **currency derivatives** tend to produce the most exotic product offerings, this is not the case in South Africa. An important reason for this is the existence of Reserve Bank imposed exchange controls which restrict the outflow of currency from South Africa. There are also regulatory restrictions prohibiting the sale of derivatives that might have speculative currency exposures in lieu of hedging an existing currency exposure. As a result, the suite of products available tends to be plain vanilla: liquid FX forwards out to one year; FX options out to two years; and cross-currency basis swaps out to fifteen years.

It is generally anticipated that when exchange controls are completely lifted, there will be a substantial increase in the volume of the existing products that are traded, as well as an increase in the number of other, more speculative, investor-type products e.g. FX range accruals.

4. AT WHAT LEVEL SHOULD IT BE TAUGHT?

In almost all programmes outside of South Africa, it seems that the core of Mathematical Finance is taught at the post-graduate level. In the USA, Europe and Britain, the emphasis is generally on a professional Masters level degree (akin to an MBA.) Sometimes this may stifle progression to the PhD, so the Masters degree is often offered through, or in conjunction with, the Mathematics department (or some version of it.)

In South Africa there are a number of examples of the Mathematics of Finance being taught from first-year undergraduate level. At the University of Johannesburg (formerly Rand Afrikaans University), the North-West University (formerly Potchefstroom University), the University of KwaZulu-Natal (formerly University of Natal – Durban) and at Pretoria University these programmes are often coupled with actuarial training. In many instances, though, the students have to make a decision about their future path before reaching the fourth-year of study.

At two universities, the study of Mathematics of Finance takes place at the post-graduate level only. These are the University of the Witwatersrand and the University of Cape Town. The reason for this is both historical and market related. At the University of the Witwatersrand the lecture courses that formed the core of Mathematical Finance were originally taught within the Honours degree of the School of Computational & Applied Mathematics as an "area of interest". It subsequently became a separate, professional degree. Discussion with the employers of graduates, at the time, suggested that an undergraduate curriculum was undesirable and even unsuitable. At the University of Cape Town the degree is at the Masters level. This was partly motivated by a desire to avoid competition with the established Honours degree at the University of the Witwatersrand and partly because the creation of professional Masters degrees was in vogue at the time.

5. WHERE SHOULD IT BE TAUGHT?

From an analysis of European and American programmes, it would appear that geography plays little role in determining the existence or success of Financial Engineering programmes. The reasons for this may be manifold but obviously lie partly in the size of the graduate student community in developed economies. The minimum requirement for entry into the quantitative financial community in London or New York is now a PhD. This is not the case in South Africa.

It is, however, no coincidence that the more successful Financial Mathematics programmes throughout the world are those at universities near their country's financial centre(s). The success of Columbia and NYU in New York, Chicago University in Chicago, ETH in Zurich, and Imperial and Kings' Colleges in London are testament to this. The situation in South Africa is comparable.

The degree programmes offered at the University of the Witwatersrand, Pretoria University, the University of Johannesburg and the University of Cape Town produce some of the more successful graduates. Graduates of the Financial Mathematics programme at Stellenbosch University are, in general, absorbed by the Cape Town financial community. North-West University's Centre for Business Mathematics and Informatics benefits from being relatively close to Johannesburg, and from a demand for its graduates from ABSA (Amalgamated Banks of South Africa).

It is clear, however, that there are two considerations which influence the success of a programme. One is access to the industry itself; the other, the resources that are available to the university. Access to the industry is often a product of geography. The resources available to the universities may be augmented by the financial community itself. The Centre for Business Mathematics & Informatics at the North-West University and the Programme in Advanced Mathematics of Finance at the University of the Witwatersrand have both received support from the banking industry which runs into millions of South African rands. It is unsurprising then that these programmes are amongst the more successful.

6. SOUTH AFRICAN ACADEMIC PROGRAMMES

The University of Cape Town first offered their MSc degree in Mathematics of Finance in 2000. The curriculum was developed by Mr. Hardy Hulley, who was a PhD student in the Department of Mathematics & Applied Mathematics at the time. Mr. Hulley subsequently taught and researched at the University of the Witwatersrand.

The MSc is a collaboration between the Departments of Mathematics & Applied Mathematics and Statistical Sciences, although it was initially situated in the Department of Mathematics & Applied Mathematics. The degree runs over two years. An intensive and extensive series of lectured courses in the first year is followed by a research report component in the second year. Many students elect to complete the mini-dissertation component while employed in the finance industry. Coursework and dissertation are equally weighted, and a pass on both components must be obtained for the degree to be awarded.

The annual intake of students varies, ranging from 12 to 25, but the drop-out rate is high, and approximately 50% of students fail to complete the coursework component. Approximately 15 students have obtained the degree during the first 5 years of the programme's existence. The programme struggles to produce research because of the twin restrictions of limited resources and over-stretched staff.

The programme has benefited from limited external funding from Investec Bank in particular, but also Rand Merchant Bank.

Stellenbosch University's Department of Statistics and Actuarial Science is the home of their Bachelor of Commerce (Mathematical Sciences) programme with a focus area in Financial Risk Management. The department started an undergraduate Institutional Investments programme in the middle nineties. In 2000 the first Honours programme in Financial Risk Management was introduced. During 2002 and 2003 the curriculum of the under- and post-graduate programmes were redesigned, leading to the current programmes.

Teaching resources in the programme are augmented by part-time professionals who bring market experience and enthusiasm to the curriculum. As mentioned, the graduates tend to be absorbed by the asset management industry in Cape Town.

The University of KwaZulu-Natal currently offers an Honours degree in Mathematics of Finance. This degree grew out of the efforts of Professor Michael Murray to create a centre of excellence in actuarial studies.

The KwaZulu-Natal area produces many excellent school matriculants who would previously have had to leave the province in order to pursue studies in Actuarial Science or Financial Mathematics. The university now offers an undergraduate curriculum which prepares the graduates for further study in either Actuarial Science or Financial Mathematics. This curriculum has been offered to Bachelor of Science and Bachelor Business Science students since 1999.

In order to ensure that graduating students are able to enter a postgraduate programme in either Actuarial Science or Financial Mathematics a fully fledged Honours programme was developed. The programme now has three permanent members of staff that are responsible for the teaching of the eight Honours courses and all three years of the undergraduate curriculum. The demands that this makes on them has curtailed any research programme.

The North-West University's Mathematical Finance programmes are concentrated around their Centre for Business Mathematics and Informatics (BMI).

In 1997, ABSA and Potchefstroom University established the Centre for BMI as a joint initiative. The Centre for BMI is built on a similar concept to that between RABO bank and the Vrije Universiteit in Amsterdam, and aims to be a centre of excellence in Financial Risk/Reward Management and Analysis. It trains graduates for professional careers in this area and engages in research that will benefit the financial services and the banking industries.

The BMI offers degrees in Mathematical Finance from undergraduate level to PhD and has an extensive research programme in mathematics, statistics and risk management.

The University of Johannesburg has an established undergraduate programme in Financial Mathematics. This programme was first introduced in order to provide preparation for post-graduate studies in Actuarial Science but has subsequently changed focus to mathematical finance.

The Honours degree in Mathematics of Finance is being offered for the first time in 2005 in collaboration with the University of the Witwatersrand. Students register at the University of Johannesburg, but attend some classes in financial mathematics from the programme at the University of the Witwatersrand. The rest of the curriculum is taught through the Department of Statistics & Mathematics.

The university has limited resources and no external funding which makes research and development difficult.

The University of Pretoria has a well established programme in Financial Engineering which was developed in the late 1990s. The programme provides training for both Actuarial Science and quantitative finance from undergraduate to PhD level, and involves academics in the Department of Mathematics & Applied Mathematics. The loss of key staff in recent years has had an effect on the post-graduate teaching and research. The university has made resources and funding available for the development of the area.

The University of the Witwatersrand - Johannesburg has the oldest programme in Financial Mathematics in South Africa. It owes this to the vision of Dr Dawie de Jongh, formerly of ABSA Investment Bank and now at the North-West University. Dr de Jongh approached the university in the late 1980's with the suggestion that they form an area of study in financial engineering. Dr de Jongh is certainly the "father" of Financial Mathematics in South Africa.

The programme commenced in 1990 - which makes it one of the oldest in the world! In 1997 the programme registered with the Department of Education as a separate Honours degree in Advanced Mathematics of Finance and the first intake was in 1998. The increase in interest in the degree was exponential. The number of enrolling students in the full-time Honours degree has remained fairly uniform at about 25. The reasons for controlling the class size are the staffing resources available and the level of quality that the programme has tried to maintain.

The staff complement has increased with the development of the area, and the programme now employs four teaching staff and one administrative assistant. Once the teaching programme at Honours level was established, the emphasis moved to a research Masters degree.

The programme has been extensively funded by the banking community throughout its existence.

8. ACADEMIC AND COMMERCIAL RESEARCH

Success in a research initiative is much more difficult to establish and maintain, but is probably more vital to the reputation and survival of a programme than the teaching component. The key factors here are funding and stability of supervising staff. The latter has been the major inhibiting issue for most programmes in South Africa.

Staff and students of Mathematical Finance programmes are employment targets for the commercial industry. Coupled to this is an alarming level of emigration of skilled labour from post-apartheid South Africa. This is an issue that affects all emerging markets and seems to have no immediate solution.

Commercial research at most financial institutions tends to centre around a small number of obvious, common problems: implementation and tailoring of existing models; coordination of front- and back-office modelling systems; coordination of trading and risk-measurement; data scarcity and irregularity; and adjusting systems to incorporate local idiosyncrasies.

In addition to the above, all local banks face the following problems.

1. Implementation of the Basel accord

Consequences of this already include the improvement of the credit modelling process and the recognition and measurement of operational risk. Several hurdles still remain, which include, *inter alia*, the measurement of interest rate risk in the banking book where embedded options and irrational exercise are commonly found.

The next frontier in banking research probably lies in the integration of all these risk types into a common model.

2. IAS139 (called AC133 in South Africa)

South Africa was one of the first countries to implement the new accounting rules for derivatives. This not only creates jobs for auditors, but also requires the input and insights of financial mathematicians. The additional regulatory focus on risk and marking-to-market of risk positions has made corporates quite wary of any form of derivative activity. On the selling side of the derivative business it will be an immediate challenge to develop derivative solutions that provide effective risk management without alarming the accountants. The interaction between Mathematical Finance and the rest of the commercial environment is set to increase dramatically because of these regulations.

3. Structurally lower interest rates

Following the structural improvements in the South African economy since its re-integration into the world economy in 1994, it has followed the example of Australia in moving towards *structurally* lower interest rates. Internationally such a decline has been met by an unprecedented demand for yield enhancement. Inevitably there have been a variety of new investment derivative products created to cater to these demands. Within the South African market a similar yield demand is establishing a fertile foundation for the introduction of more advanced derivative investor products. This environment brings a unique set of challenges and opportunities in financial modelling and requires significant mathematical finance skills within the banking and investor community.

Academic research in Financial Mathematics in South Africa is relatively minor. Most consequential research is being pursued at universities in the developed world. There is a niche for research into the consequences of thin-trading environments and developing markets but, in general, these are not core mathematical problems.

Most research into financial modelling uses continuous-time finance as a starting point. This is possibly accurate when modelling very liquid markets and quite inaccurate when modelling illiquid markets. Unfortunately, the latter describes most of the interesting markets in South Africa. This makes many models which have been developed simply unusable.

Many academic programmes either do no mathematical finance research at all (preferring to remain purely mathematical or statistical in their research) or couple their research to clearly defined commercial goals. One such programme is that of the Centre for BMI at the North-West University. Here the Risk Analysis Research Group follows a directed research programme where academic researchers in mathematics, statistics, computer science and economics collaborate to solve industry proposed problems. The programme concentrates on the following: modelling of financial time-series, dynamic portfolio theory, derivative pricing models, and enterprise-wide risk management.

In contrast, the majority of the research done at the University of the Witwatersrand is concentrated in the masters degree programme. MSc students either pursue co-research with one of the academics involved, or follow an independent line of enquiry. To alleviate the shortage of experienced supervisors, the programme either co-opts an additional supervisor from industry, or uses collaborative supervision with programmes in the UK, Europe and Australia. Some additional research has been produced and published internationally and locally by the academic faculty.

REFERENCES

North-West University:
http://www.puk.ac.za/bmi/index.html

Stellenbosch University
http://academic.sun.ac.za/economy/statsact/index.html

University of Cape Town:
http://www.mth.uct.ac.za/graduatestudies/financialmaths/index.html

University of KwaZulu-Natal:http://www.ukzn.ac.za/department/default.asp?dept=finmathsund

University of Johannesburg:
http://general.uj.ac.za/maths/EngHome.html

University of Pretoria:
http://www.up.ac.za/academic/maths/

University of the Witwatersrand:
http://www.cam.wits.ac.za/mfinance/

Bank of International Settlements:
http://www.bis.org/index.htm

Reserve Bank of South Africa:
http://www.reservebank.co.za/