
Beta, size and value effects on the JSE, 1994-2007

ABSTRACT

This paper builds on the observations of Van Rensburg and Robertson (2003a), who found persistent size and price-earnings effects in the cross-section of returns on the JSE, but surprisingly found that beta had, if anything, an inverse relationship with returns. Based on stock returns from January 1994 to October 2007, this portfolio-based study finds support for these earlier findings. However, when betas are estimated by the Dimson Aggregated Coefficients method with a lead and lag of at least three months, to compensate for the weaknesses of Ordinary Least Squares regression in the face of thin trading, the relationship between beta and return loses its statistical significance. We are left with the conclusion that beta has no predictive power for returns on the JSE, invalidating the CAPM, at least as it is commonly applied, based on a market proxy of the All-Share Index. We find further that the size premium is concentrated in the smallest stocks on the JSE, with no significant difference in returns between the four largest quintiles, and tentative evidence that it has been reducing over time.

1. INTRODUCTION

Two recurring threads in the international literature on capital asset pricing anomalies relate to firm size and investment value. With regard to the former, a substantial body of literature points to the outperformance of smaller firms (measured by market capitalization) compared to larger firms. Value effects are those linked to measures of the stock price being out of line with fundamental indicators of firm value, such as book value of assets or earnings. Key literature in both threads is discussed below.

It is important, when considering these empirical anomalies and the explanations advanced for them, to bear in mind what has come to be known as the joint hypothesis problem (as outlined in, for example, Roll, 1977). In effect, statistical tests in the anomaly literature are always testing the joint hypothesis of efficient markets and the specification of a particular pricing model, usually the Capital Asset Pricing Model (CAPM). Failure to support this joint hypothesis could therefore indicate either inefficiency or the misspecification of the model used (or indeed both). Crucially though, the degree of (in)efficiency of financial markets can never be concluded with certainty.

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The literature on the size premium, or small firm effect, begins with Banz (1981), who demonstrated the incompatibility of empirical data with Black's (1972) zero-beta CAPM. The next few years produced a wealth of investigation into the size effect (for example, Lustig and Leinbach, 1983; Brown and Barry, 1984) leading Reinganum (1984) to conclude that empirical evidence is incontrovertibly at odds with accepted theory, as embodied by the CAPM.

From this point onwards in the literature, the emphasis shifted from establishing the existence of the size effect to attempting to explain it. Proposed rationalisations included:

- (a) seasonality, noting the concentration of the size effect in the month of January in the United States (Keim, 1983; Tinic and West, 1984; Lakonishok and Shapiro, 1986) and in other countries (Gultekin and Gultekin, 1983; Berges, McConnell and Schlarbaum, 1984; Tinic, Barone-Adesi and West, 1987; Kato and Schallheim, 1985; Reinganum and Shapiro, 1987);
- (b) the influence of transaction costs (Stoll and Whaley, 1983; Schultz, 1983; Blume and Stambaugh, 1983);
- (c) thin trading of smaller stocks (Roll, 1981; Reinganum, 1982; James and Edmister, 1983);
- (d) use of an inefficient market proxy (Roll, 1977; Korkie, 1986; Berk, 1995);
- (e) differential information available to investors for small and large stocks (Barry and Brown, 1984, 1985; Zeghal, 1984; Brown and Kim, 1993; Freeman, 1987); and
- (f) the use of size as a proxy for true underlying risk

(Friend and Lang, 1988; Perez-Quiros and Timmermann, 2000; Chan, Chen and Hsieh, 1985; Huberman and Kandel, 1987; Chan and Chen, 1991).

In addition, some more recent studies have suggested signs of the disappearance or reversal of the size effect (Bhardwaj and Brooks, 1993; Fraser, 1995; Horowitz, Loughran and Savin, 2000; Griffiths, Turnbull and White, 1999).

Apart from the size effect, the most significant and persistent anomaly consistently reported in the literature of the past thirty years is what may be termed a value effect, where 'value' is typically defined as a ratio of price to a non-market (usually cash flow- or accounting-based) measure of firm value. The most common indicators are book-to-market equity or price-earnings ratios. Often the literature considers the size and value effects together, generally finding that they jointly comprise a significant additional explanatory variable beyond what the CAPM describes. Some prominent commentary on the existence of a persistent value effect in stock returns is provided by, for example, Reinganum (1981), Basu (1983), Cook and Rozeff (1984), Banz and Breen (1986) and Ball (1992).

The size and value effects discussed above are only anomalies in the sense that they are incompatible with models, such as the CAPM, which ascribe rewarded systematic risk to market covariance only. More flexible multifactor models, often following Ross' (1976) Arbitrage Pricing Theory, allow for the incorporation of the observed anomalies into an expanded equilibrium pricing model. The most comprehensive attempt so far to incorporate size and value effects into a single theoretical framework is the three-factor model developed by Fama and French (1992, 1993, 1995, 1996). Significantly, they find however that these two effects do not subsume the role of market covariance in explaining the cross-section of returns.

Early work examining the impact of firm size in explaining the cross-section of returns for South African equities was inconclusive, but tended, if anything, towards a rejection of market capitalization as a significant factor in the return-generating process. No evidence of a meaningful small firm effect was found by De Villiers, Lowlings, Pettit and Affleck-Graves (1986), Bradfield, Barr and Affleck-Graves (1988) or Page and Palmer (1993). However, Van Rensburg (2001), in his investigation into style-based factor returns, found that there was a significant risk-adjusted return to small firm stocks. Van Rensburg and Robertson (2003a) attributed the lack of significant findings in earlier studies to the small sample sizes

employed and the *de facto* exclusion of small stocks due to concerns about how thinly they are traded.

Evidence of the value effect in South Africa has been more uniformly positive. Page and Palmer (1993) were able to support the hypothesis of excess risk-adjusted returns to firms with high earnings yields (or low PE ratios). Following on this research, Page (1996) found that the value effect, measured by PE ratio, was robust not only to risk-adjustment in line with the CAPM, but also to an APT model with as many as five factors. This was further confirmed by Van Rensburg (2001), who found both earnings yield and dividend yield to be significant value indicators, and proposed earnings yield as a proxy for the value effect in a three-factor model which could describe JSE returns parsimoniously.

Van Rensburg and Robertson (2003b) applied the multiple regression methodology employed by Daniel and Titman (1997) to identify which style characteristics had a significant effect on returns. They used a sample of stocks covering the ten-year period from July 1990 to June 2000, which was filtered for thin trading by retaining only stocks with an average daily trade in a given month of more than 0.01% of market capitalization. Furthermore, the influence of outliers was limited by the winsorisation of the extreme 1% of observations. Testing was conducted on five measures of value, fourteen measures of future earnings growth and bankruptcy risk, and five measures of momentum and neglect.

Univariate tests identified a significant value effect on all five measures as well as a significant size effect; none of the other variables had a significant influence on return. Neither of these effects are anticipated within the CAPM framework, and must therefore be viewed as anomalies within that paradigm. It was further concluded that the two-factor APT model proposed by Van Rensburg (2002), where the relevant factors are the returns on the JSE Financial and Industrial Index and Resources Index respectively, is similarly unsuccessful in removing these anomalies.

Van Rensburg and Robertson (2003b) constructed a multifactor model to describe the cross-section of returns, testing the joint significance of combinations of individually significant factors. They found that returns were best described by a two-factor model based on the explanatory variables of market capitalization and price-to-earnings, which "capture the central intuition behind the international evidence of the style effects relating to value and size" (Van Rensburg and Robertson, 2003b:10). Given the high degree of correlation between the various measures of value, it is unsurprising that a parsimonious model can be

constructed with a single value factor in addition to the size factor, and such a model avoids the problem of multi-collinearity which would inevitably arise with multiple value factors.

Van Rensburg and Robertson (2003a) went on to take a portfolio-based approach to analyse the size and value effects, subdividing their sample of stocks at each month-end into quintiles by, respectively, firm size and PE ratio (the 'one-way sort'), and then subdividing each size quintile into further quintiles on PE ratio, thus ending with twenty-five portfolios (the 'two-way sort'). This latter approach allowed the measurement of the effect of the second attribute within a particular portfolio, controlling for the first. The problem of thin trading is handled in the same way as their previous paper.

The most striking feature of the 'one-way' results was the demonstration of a clear inverse relationship between firm size and return and between PE ratio and return, in line with the international literature, while simultaneously, and unexpectedly, showing an equally clear inverse relationship between market model beta and return. It was noted (Van Rensburg and Robertson, 2003a:9) that this may be the result of thinly-traded stocks remaining in the data set. The lowest beta quintile outperformed the highest beta quintile by an average of a statistically significant 0,90% for a one-month holding period, over the ten-year period of the study.

This surprising and somewhat counter-intuitive result raises a significant challenge to investment theory and practice in South Africa. It suggests that beta, if it plays any role at all, is a *negative* predictor of stock returns. This is a sufficiently unexpected and important observation to merit further analysis.

The 'two-way' results suggest that the value and size effects operate independently of each other, and that both are necessary to explain the generation of returns on the JSE. Neither does the size effect proxy for the value effect, nor vice versa (Van Rensburg and Robertson, 2003a).

An interesting extension of this work was conducted by Auret and Sinclair (2006), who found book-to-market equity ratio to be a better indicator of the value premium than PE ratio, contrary to the findings of Van Rensburg and Robertson (2003b). In fact, they found that book-to-market completely subsumed not only PE ratio, but also firm size, in multiple regressions on return. However, including book-to-market in a multifactor model did not result in greater explanatory power than a two-factor model using PE ratio and size, which they put down to the high correlation between

book-to-market and other candidate variables, while PE ratio and size have the virtue for modelling purposes of low correlation and hence a reduced multi-collinearity problem. This suggests that testing the replicability of the Van Rensburg and Robertson (2003a) results may reasonably use market capitalization and PE ratio as the size and value indicators respectively. Consideration of the role of book-to-market would however be a useful extension.

Likewise, Basiewicz and Auret (2009), using both portfolio-based and Fama and MacBeth (1973) regression approaches, found book-to-market to be a better indicator of value than PE ratio. Unlike Auret and Sinclair (2006), however, they found that the size effect was not subsumed by book-to-market, which they ascribed to the prominent size premium on the JSE over the period from 2003 to 2005, a timespan not included in the earlier work. They also found that the value effect was weakest among the smallest firms.

The significant contribution made by Basiewicz and Auret (2009) was the demonstration of the effects of applying restrictions on liquidity and price to mimic constraints that would be placed on portfolios in practice. These restrictions, which the authors argued acted as a proxy for transaction costs, significantly reduced the estimates of the size and value premia, suggesting that these are largely influenced by returns on stocks which would not be included in investors' portfolios in practice, at least not in significant volumes. They also formed value-weighted portfolios in addition to the equally-weighted portfolio approach which tends to be favoured in the literature, and found that size and value premium estimates tended to be lower based on value-weighted portfolio formation.

The aim of this paper is to establish whether the conclusions of Van Rensburg and Robertson (2003a) are robust and generally valid. In particular, we set out to answer the following questions:

1. Are the results sample-specific?
2. What is the impact of using 60 months' data to estimate beta, as is considered optimal in the literature, as summarised for example in Bradfield (2003), rather than a period of 12 to 30 months?
3. What is the impact of keeping a full data set and of not excluding thinly-traded stocks, but rather using beta estimation methods which are robust to thin trading?

4. Are results obtained by analysing three-, six and twelve-month holding period returns consistent with those from one-month returns?
5. Is further insight to be gained from the examination of intermediate quintiles, rather than basing conclusions only on differences between the extreme quintiles?
6. Are both the size and value effects jointly significant, and what is the nature of their interaction?
7. Are there any other possible explanations for the observed phenomena?

2. DATA AND METHODOLOGY

Closing prices, dividend yields, earnings yields, shares in issue and monthly volume traded were obtained from I-Net Bridge (I-Net), for each month-end from 31 December 1988 to 31 October 2007, for all stocks listed on the main board of the JSE at each of those dates within the period.

There were numerous concerns over the quality of the data. While the data for currently listed stocks generally appear accurate and complete, some delisted share data are concerning. A comprehensive analysis of the data yielded numerous anomalies and apparent inaccuracies, and a number of minor adjustments were made to the raw data.

One of the key remaining problems is the lack of information regarding the cause of delisting. Financial research is often silent on the approach taken to calculate returns in the period in which a stock is delisted, but a common approach in the absence of accurate return data, at least for portfolio-based studies such as this one, is to assume a complete loss on delisting, i.e. that a return of minus 100% is earned in this period. This is unsatisfactory insofar as delistings may be the result of merger activity (in which case it is possible, even likely, that shareholders earn a positive return in the month of delisting), while even where such delistings stem from companies being liquidated, it is possible that shareholders will realise some value from their holdings. If indeed small firms are subject to greater bankruptcy risk than larger firms, there should be greater numbers of delistings in the small firm portfolios and consequently this assumption should (subject to the comments below) provide a bias against detecting a size effect.

It has been suggested that the price immediately prior to delisting represents investors' expectations of the liquidation dividend, and hence it would be appropriate to assume a 0% return on delisting. This ignores, however, the possibility of an improvement in outlook for the firm, accompanied by the resumption of trading in the stock and a subsequent increase in its price. Four stocks had to be excluded from this analysis because of the undue influence the (extremely large) returns generated by such recoveries had on the equally-weighted portfolio returns. Assuming unbiased expectations, it must therefore be the case that the recovery by shareholders conditional on delisting is on average less than the share price prior to delisting.

In the absence of detailed delisting reason information, it is impossible to make an accurate allowance for these outcomes. The default minus 100% approach was therefore followed, with the analysis repeated assuming a minus 50% return for comparative purposes. This change in assumption does not make a material difference to the results, indicating that any differences in delisting probability between firms based on the criteria analysed here (notably size and PE ratio) are not economically significant. However, potential bias remains in the sample of delisted stocks if, given that a firm delists, the probability of bankruptcy being the cause is inversely related to firm size.

It should be noted that the data used by Van Rensburg and Robertson (2003a) were provided by the BARRA organisation and included actual monthly returns, thus removing the need to make such assumptions about the return on delisting. This data source was unfortunately not available for this study.

Month-end values for the FTSE-JSE All-Share Index (ALSI) were also obtained from I-Net. The FTSE-JSE Africa Index series replaced the JSE Actuaries Indices with effect from 24 June 2002, and the Total Return Index (TRI) was retrospectively restated back to 30 June 1995. As a slightly longer period was required for this study, returns in months prior to 30 June 1995 were calculated as the capital gain, measured by the movement of the restated ALSI as reported by I-Net, plus one-twelfth of the ALSI dividend yield for the month.

The portfolio formation, return calculations and statistical testing were coded in the free statistical software environment R (R Development Core Team, 2009).

Security returns were estimated by calculating the capital gain and adding to this one-twelfth of the dividend yield for the month. Ideally, actual dividend

data would have been used, but dividend payment dates were not available from the I-Net data. The approximation used is sufficiently accurate at portfolio level, given a spread of financial year-ends across the calendar year.

Betas were estimated for each stock by regressing security returns over the preceding 60 months against market returns, as measured by movement in the ALSI TRI. These are thus market model betas, rather than CAPM betas, consistent with the approach taken in Van Rensburg and Robertson (2003a); the two beta measures are in any event very similar provided there are no dramatic fluctuations in the risk-free rate over the estimation period. Thinly-traded stocks were not excluded from the database, but in addition to estimating beta by OLS, we considered two alternative techniques designed to correct for the thin trading problem.

The problem of errors in variables in econometric work has the potential to introduce bias into studies of the size effect, as smaller firms tend to have stocks which are traded less frequently, and a downward bias in their beta estimation will lead to the erroneous conclusion of higher risk-adjusted returns. Scholes and Williams (1977) proposed the following unbiased and consistent estimate of beta to overcome the problem of nonsynchronous trading:

$$\hat{\beta}_n = \frac{1 + b_n^- + b_n + b_n^+}{1 + 2\hat{\rho}_m}$$

where

b_n^- , b_n , b_n^+ are the OLS estimators of beta with the security return regressed against the market return from, respectively, the preceding, contemporaneous and succeeding periods, and $\hat{\rho}_m$ is the sampling estimator of the first-order autocorrelation coefficient of the market index.

The authors showed that this is a consistent and unbiased estimator of beta provided that the periods of no trading are independently and identically distributed. They further noted that nonsynchronous trading is a particular problem when estimating betas from daily data (Scholes and Williams, 1977:309). Although it is less of an issue when using monthly data, as is the case in this study, nevertheless it has the potential to cause distortions given the extremely thin trading on certain smaller JSE stocks. Importantly, it is only a consistent estimator of beta if trading takes place in every period. Even with data at monthly intervals, this is not the case for all JSE stocks.

An alternative approach was put forward by Dimson (1979), who proposed the aggregated coefficients (AC) method, based on regressing observed security returns on market returns in the synchronous period, as well as in several leading and lagging periods. He showed that beta can be estimated consistently for thinly-traded stocks by summing the slope coefficients from a multiple regression against all of these market returns:

$$\hat{\beta}_{i,t} = \sum_{k=-n}^n \hat{\beta}_{i,k}$$

where

$\hat{\beta}_{i,k}$ is the OLS estimator of beta obtained by regressing the security return against the market return with a lag of $-k$ months, obtained from the following multiple regression:

$$r_{i,t} = \hat{\alpha}_i + \sum_{k=-n}^n \hat{\beta}_{i,k} \cdot r_{m,t+k} + \epsilon_{i,t}$$

where

$r_{i,t}$ and $r_{m,t}$ represent the security and market returns in time period t respectively. The number of lead and lag terms n is variable and should be related to the degree of thin trading in the market.

Dimson showed his measure to be more efficient than the Scholes-Williams estimator based on UK data, and suggested that (again based on UK data) it was appropriate to use one leading and several lagged market returns to fully account for the effects of thin trading.

For this study, beta estimates were obtained by both the Scholes-Williams and Dimson methods (with lead and lag of two, three and five months in the case of the latter).

Given that 60 months' return data were required to estimate beta and that in the case of the Dimson method, security returns were regressed against market returns lagged by up to five months, it was only possible to form portfolios at month-ends from 31 December 1993 onwards. In view of the structural shift in the socio-political landscape of the country, and as a consequence its economy, from 1994, this is thought to be an appropriate period for analysis. In addition, I-Net's pre-1994 shares in issue data are incomplete, and hence the formation of portfolios on firm size would not have been possible prior to 1994 using this data set. As we wished to track portfolio returns over holding periods of up to twelve months, the last

portfolio was formed at 31 October 2006.

I-Net's practice of retrospectively changing Earnings Per Share (EPS) data (and related fields such as earnings yield and PE ratio) has the unfortunate consequence of giving rise to look-ahead bias in econometric work, as the PE ratios on which portfolios are formed in this study would not be those known to the market at the respective points in time. To compensate for this, a three-month lag was assumed between accounting and reporting dates, and PE ratio was consequently calculated as the price at a given month-end divided by the implied EPS from three months prior.

At each month-end in this portfolio formation period, quintile portfolios were formed on beta (all five methods), market capitalization and PE ratio. For purposes of forming portfolios on the latter, stocks with negative PE ratios were excluded from the database. In addition to these one-way sorts, two-way quintile-quintile portfolios were also formed: by splitting each size quintile into PE ratio quintiles, and by splitting each PE ratio quintile into size quintiles. Two sets of twenty-five portfolios were thus formed at each month-end for each of the two-way sorts.

The returns of each portfolio were then calculated for the ensuing holding periods considered (one, three, six and twelve months), and mean returns calculated for each quintile across the sample period. No statistical significance is reported for the longer holding periods, given the problem of non-independence between observations. For the one-month holding periods, the statistical significance of the difference in mean returns between quintiles was calculated using the form of Student's t-test proposed by Welch (1947), which does not require the assumption of equal population variances. This choice involves the sacrifice of some power, but this implies a bias against the detection of spuriously significant relationships.

A more significant loss of power is implied by the decision to test significance using a two-tailed test, rather than one-tailed; in other words, to test the null hypothesis against the alternative that a given difference in mean returns is not equal to zero, as opposed to greater than or less than zero. This decision was motivated by the desire for scientific neutrality. It would of course be possible to conduct a one-tailed test in the direction of the expected relationship based on theory and the literature, but this would yield no useful outcome for a relationship between beta and return which is the opposite of what theory suggests. Alternatively, the one-tailed test could be based on the relationships observed in the data, but this seems self-fulfilling and prone to

accusations of data mining. The practice employed by some researchers of using a one-tailed test if the direction of the relationship is as expected, and a two-tailed test if not, is equivalent to an increase of 50% in the level of significance (results shown to be significant at the 5% level, for example, can in fact only be said with confidence to be significant at the 7.5% level) (Nosanchuk, 1978; Abelson, 1995). The adoption of two-tailed tests throughout therefore seems appropriate. Once again, this choice provides an inbuilt bias against reporting statistical significance, with the result that greater confidence may be placed in the significance of the results reported.

This study follows the approach taken by Van Rensburg and Robertson (2003a) of quantifying size and value premia based on the gross unadjusted returns of the quintile portfolios, without accounting for risk. This is theoretically flawed in that it does not account for correlations between systematic risk, as measured by beta, and the size and value measures used. For example, if small firms tend to have higher betas, the size effect may simply be a beta effect. The results obtained by portfolio sorts on beta, both in Van Rensburg and Robertson (2003a) and in this study, however, indicate an inverse relationship between beta and return, suggesting that any beta risk-adjustment would only exacerbate the size and value effects reported. It was consequently deemed unnecessary to adjust the portfolio returns for beta risk.

3. RESULTS

Results are presented below as follows: for each of the one-way sort criteria (market capitalization, PE ratio and each of the five measures of beta) the average one-, three-, six- and twelve-month returns achieved over the sample period by each of the quintile portfolios are reported. The difference in mean returns between the first (smallest/lowest) and fifth (largest/highest) quintiles is then examined, as well as that between intermediate quintiles where such analysis yields further insights. A similar analysis is presented for the two-way sorts by market capitalization and PE ratio.

3.1 Firm size (market capitalization)

Mean returns, and differences in means between selected quintiles formed on market capitalization, are set out in Table 1. The mean returns are depicted graphically in Figure 1, which also shows the results reported for a one-month holding period by Van Rensburg and Robertson (2003a) for comparative purposes (labelled vR&R). It should however be borne in mind that these are based on a different sample, using different methodology.

As in Van Rensburg and Robertson (2003a), the difference in mean returns between the smallest and largest size quintiles is striking: 1,36% over a one-month holding period, 3,05% over 3 months, 4,67% over 6 months and 10,81% over 12 months. The one-month result is statistically significant at the 5% level. This is

clear evidence of a persistent size effect which is not merely an artefact of the sample used by Van Rensburg and Robertson (2003a).

Table 1: Results for portfolios formed on market capitalization

Portfolio	Holding period (months)			
	1	3	6	12
1	2,52%	6,59%	11,76%	24,68%
2	1,13%	3,30%	6,63%	16,34%
3	1,34%	3,97%	8,43%	16,84%
4	0,91%	3,22%	6,37%	14,13%
5	1,16%	3,53%	7,09%	13,87%
Differences between portfolios				
1-5	1,36%**	3,05%	4,67%	10,81%
1-2	1,39%**	3,29%	5,13%	8,34%
2-5	-0,03%	-0,23%	-0,46%	2,47%

In all tables, statistical significance of the difference in mean returns between pairs of quintile portfolios for a one-month holding period is indicated as follows:

* significant at 10% level

** significant at 5% level

*** significant at 1% level

Significance of the difference in means for longer holding periods is not reported due to the non-independence of observations, as discussed in Section 2 above.

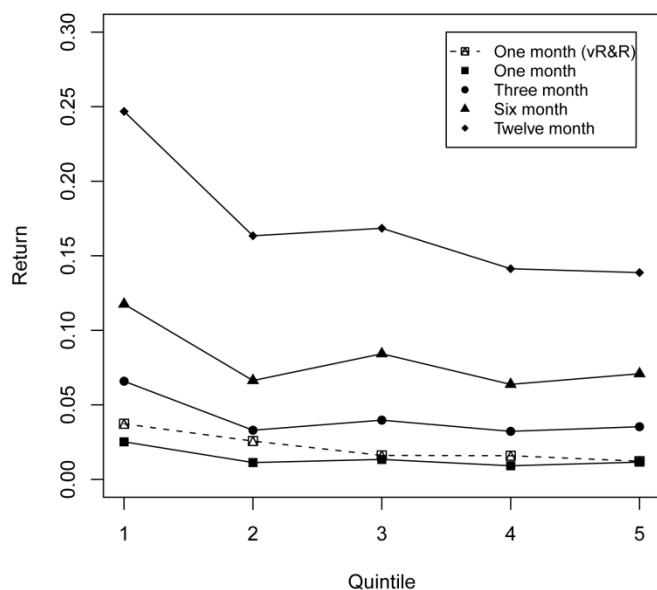


Figure 1: Mean returns by quintiles formed on market capitalization

Figure 1 shows a significant drop between quintiles 1 and 2, with a somewhat flatter graph between quintiles 2 and 5. This suggests that it would be instructive to consider differences between the mean returns of these intermediate quintiles, and the statistical significance thereof. Table 1 indicates that the difference between the mean returns of quintiles 1 and 2 is positive and significant at the 5% level for the one-month holding period. By contrast, the difference in returns between quintiles 2 and 5 is not significantly different from zero at the 10% level. This indicates that the size effect is primarily a function of excess return to the smallest quintile of stocks. Any attempt to build an equilibrium pricing model using market capitalization as one of the factors is consequently unlikely to yield reasonable results unless the factor has a binary quality: it does not appear to be a factor which is rewarded progressively across the entire stock universe.

This concentration in the smallest quintile is consistent with the observation of Basiewicz and Auret (2009) that estimates of the size premium are much reduced once price or liquidity filters are applied. Since it would be expected that many of the smallest stocks would be

those with the lowest share prices, as well as lowest trading volumes, the size premium observed in this study may likewise be removed or significantly reduced if the lowest-priced or least-traded stocks were screened out of the universe under consideration. This suggests that the size effect may reflect a premium for illiquidity, or a market inefficiency caused by constraints on institutional investment.

One of the possible explanations of the size effect is that it reflects the reward for bankruptcy risk. Such risk ought over time to be reflected in the statistical properties of the returns of small firm portfolios. Given that the CAPM is rooted in a mean-variance framework,

and that the size effect does not appear to be captured by the CAPM beta, we may perhaps expect this risk to be reflected in higher moments of the return distribution, notably skewness and kurtosis. However, analysis of these properties of the size quintile portfolios yielded no conclusive insights.

Figure 2 shows the size premium, measured as the difference in one-month holding period returns between extreme size quintiles for each month of the portfolio formation period of the sample, as well as the six-month moving average of the size premium and a linear trend-line fitted through its points.

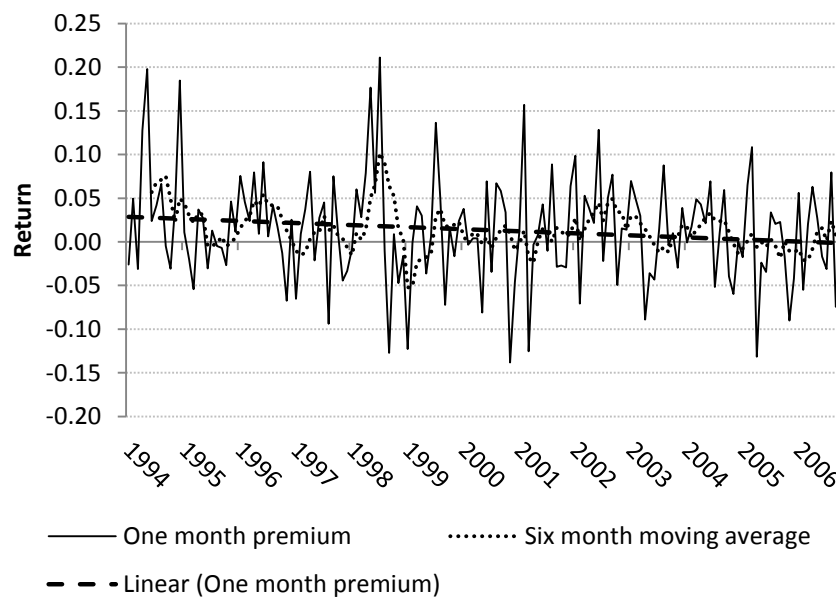


Figure 2: Trends in the size premium over time (one-month holding period)

While this figure indicates the volatility of the size premium, it also provides tentative evidence of its shrinking over time, with a clearly downward linear trend line. This is consistent with the hypothesis that the size premium historically has in part represented market inefficiency, presenting a statistical arbitrage opportunity which has been closed out over time by the participation of greater expertise in South African financial markets, increased asset management competition and access to quicker and more accurate information. However, the fitting of a simple linear model to such a noisy, volatile time series may not be appropriate, and is influenced by the starting and ending point. If we conclude that the size premium is disappearing, we must do so with caution.

There was no clear and discernible seasonal pattern in the quantum of the size premium by calendar month.

3.2 Value (price-earnings ratio)

Looking at the difference in returns between extreme quintiles in Table 2, it is clear that PE ratio is a statistically significant predictor of excess return at the 1% level for the one-month holding period. The evidence appears to weaken over longer holding periods, but this is difficult to quantify given the inability to perform standard statistical tests on this overlapping, and therefore dependent, data.

Figure 3 indicates a much smoother downward progression of returns as PE ratio increases than was the case for portfolios formed on market capitalization, suggesting that PE ratio is a factor which could be incorporated into a multifactor model in the usual way. None of the differences in mean returns between quintiles 1 and 3 or between quintiles 3 and 5 are statistically significant.

An interesting feature is that as the holding period increases, the excess return on the lowest PE ratio quintile seems to disappear, to the extent that quintile 1 in fact earns a lower return than quintile 2 over six- and twelve-month holding periods. Even assuming independence, however, these differentials would not be statistically significant.

3.3 Beta

Mean returns, and differences in means between selected quintiles formed on OLS beta, are set out in Table 3. The mean returns are depicted graphically in Figure 4, again showing the comparative Van Rensburg and Robertson (2003a) one-month holding period returns.

Table 2: Results for portfolios formed on PE ratio

Portfolio	Holding period (months)			
	1	3	6	12
1	2,43%	5,55%	8,91%	16,82%
2	1,40%	4,76%	10,09%	20,83%
3	1,48%	4,23%	8,15%	19,33%
4	1,04%	3,50%	7,08%	15,75%
5	0,72%	2,59%	6,08%	13,26%
Differences between portfolios				
1-5	1,71%***	2,96%	2,82%	3,55%
1-2	0,94%	1,32%	0,76%	-2,51%
2-5	0,77%	1,64%	2,07%	6,06%

Table 3: Results for portfolios formed on OLS beta

Portfolio	Holding period (months)			
	1	3	6	12
1	1,82%	5,07%	9,36%	19,40%
2	1,60%	4,88%	10,14%	22,37%
3	1,45%	4,42%	8,76%	18,87%
4	1,52%	4,32%	8,37%	17,40%
5	0,68%	1,95%	3,71%	7,94%
Differences between portfolios				
1-5	1,13%*	3,11%	5,64%	11,46%
1-2	0,30%	0,74%	0,99%	2,00%
2-5	0,83%	2,37%	4,66%	9,46%

Figure 4 indicates a general inverse relationship between beta estimated by OLS and portfolio return. This is consistent with the observations of Van Rensburg and Robertson (2003a), despite differences in the sample period and methodology. The difference in returns between extreme quintiles is significant only at the 10% level for a one-month holding period (5% using a one-tailed test). A noteworthy feature is that this excess return appears to be generated largely by

a low return in quintile 5, the portfolio of stocks with the highest beta; however, the difference between one-month holding period mean returns is not statistically significant either between quintiles 1 and 4 or between quintiles 4 and 5. The shortcomings of OLS regression in the face of thin trading suggest that alternative beta estimation techniques should be considered before attempting to draw conclusions from these results.

It might be expected that, if indeed thin trading and the consequent problems of robust beta estimation are contributors to the somewhat surprising results reported by Van Rensburg and Robertson (2003a), the inverse relationship between return and beta estimated by the Scholes-Williams method would be, at the very least, more muted than was the case with the OLS beta. The results however indicate that it is more pronounced. The difference in return between extreme quintiles is significant at the 5% level for a one-month holding period.

One possible explanation is that the earlier observations are not a function of thinly-traded stocks at all. However, it must be noted that the Scholes-

Williams method requires trading in every period for consistency. This is not the case on the JSE. The Scholes-Williams method is therefore unlikely to be appropriate for beta estimation on the JSE, and we must look to the Dimson Aggregated Coefficients method for further guidance on whether the use of methodology designed specifically to compensate for thin trading could explain, in part or in full, the observed inverse relationship between beta and return. Dimson betas were estimated using lead and lag of 2, 3 and 5 months respectively. Some improvement in results was achieved by extending the lag from 2 to 3 months, but further extensions made little difference; consequently, we report here only the 3 month results.

Table 4: Results for portfolios formed on Scholes-Williams beta

Portfolio	Holding period (months)			
	1	3	6	12
1	2,08%	5,98%	11,56%	25,57%
2	1,49%	4,19%	8,58%	19,10%
3	1,52%	4,28%	8,06%	17,31%
4	1,09%	3,55%	6,89%	12,16%
5	0,89%	2,63%	5,21%	11,77%
Differences between portfolios				
1-5	1,19%**	3,35%	6,35%	13,79%
1-2	0,59%	1,79%	2,98%	6,47%
2-5	0,60%	1,56%	3,36%	7,32%

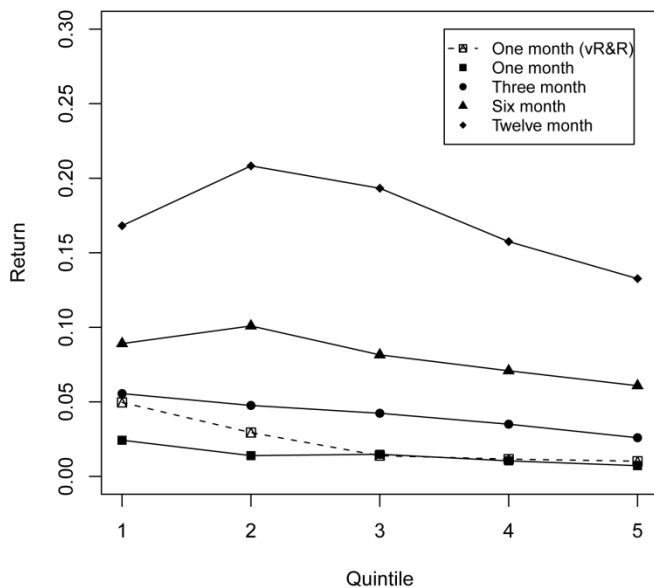


Figure 3: Mean returns by quintiles formed on PE ratio

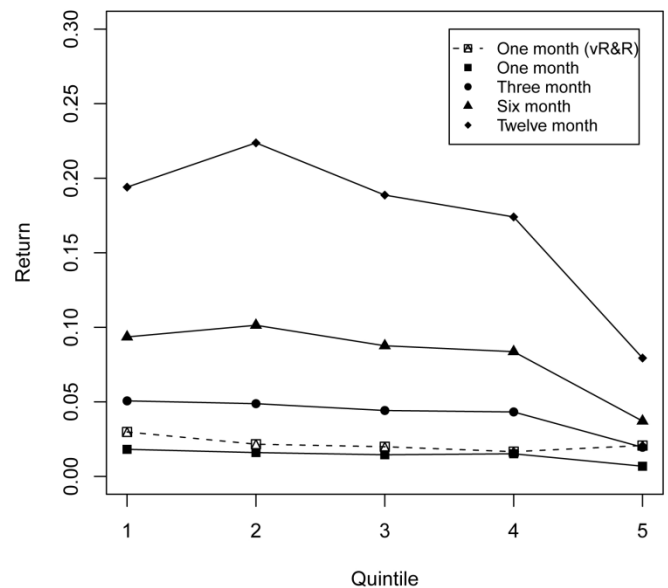


Figure 4: Mean returns by quintiles formed on OLS beta

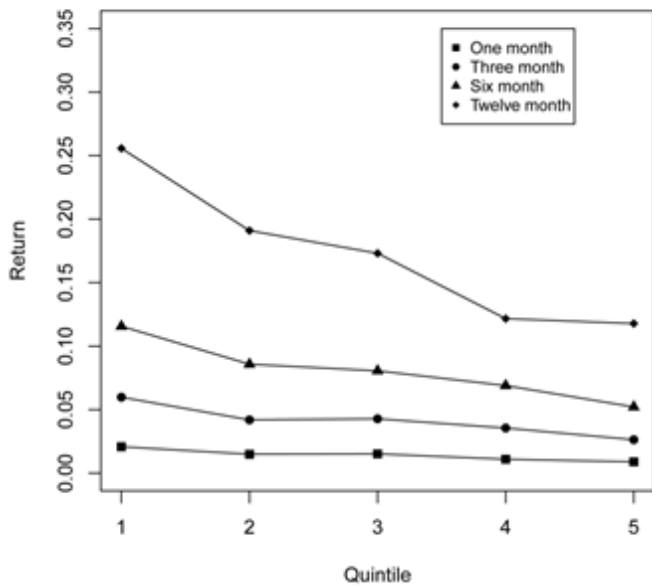


Figure 5: Mean returns by quintiles formed on Scholes-Williams beta

Using betas estimated according to the Dimson methodology, the statistical significance of the inverse relationship between beta and return disappears. This suggests that the result reported in Van Rensburg and Robertson (2003a) could owe its existence, at least in part, to the weaknesses of OLS estimation of beta for thinly-traded stocks (despite the application of the thin trading filter, many stocks would have remained in their universe which are sufficiently thinly-traded for OLS estimation to have been problematic, as conceded by the authors).

It is important to note, however, that application of the Dimson Aggregated Coefficients methodology removes only statistical significance from these results; it does not actually reverse the observed inverse relationship between beta and return. We may reasonably conclude from these results that beta has no role to play in the generation of returns on the JSE; there is certainly no evidence of the positive relationship which theory would suggest.

3.4 Two-way analysis: Size and value

Two-way analyses were conducted by splitting quintile portfolios formed on, respectively, market capitalization and PE ratio into further quintiles on PE ratio and market capitalization respectively, thus resulting in 25 portfolios for each two-way analysis. This allows us to analyse the effect of one factor while keeping the other constant.

Typically, the PE ratio and the logarithmic transformation of market capitalization would be the operational variables used in a multifactor model; the correlation coefficients between these was 0,3368, indicating some positive relationship but not sufficiently high to suggest that one may be a proxy for the other.

Figure 7 shows the return in each of the 25 portfolios where quintiles are formed first on market capitalization and then, within these, on PE ratio. Table 6 shows the difference in mean returns between extreme PE ratio quintiles within each size quintile (in other words, the return of the lowest PE ratio quintile minus the return of the highest, within each of the size quintiles; this isolates the value effect controlling for size).

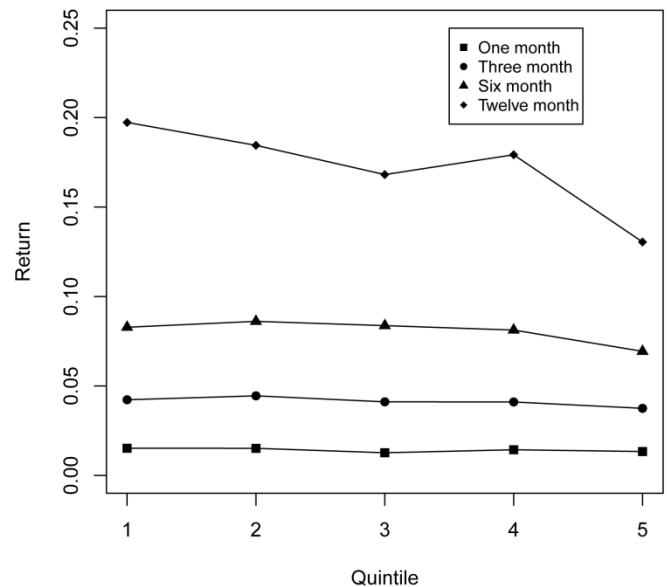


Figure 6: Mean returns by quintiles formed on 3 month Dimson beta

Within the intermediate size quintiles 3 and 4, PE ratio appears to have strong predictive power for returns. This is not however the case for quintiles 2 and 5, where the returns to low PE ratio are positive but not significant. The most interesting result is in the smallest size quintile, which the one-way analysis showed to be the source of almost all of the excess return to firm size. Here the excess return to low PE ratio is in fact negative for holding periods longer than one month.

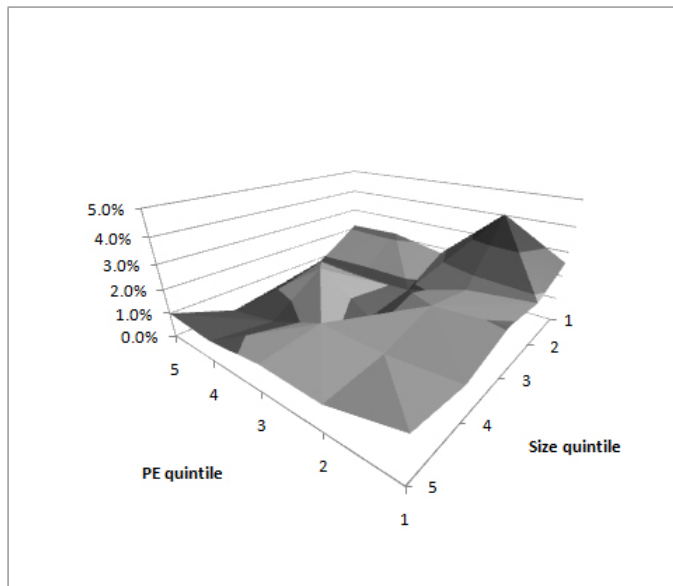


Figure 7: Two-way analysis: value within size quintiles

With the exception of quintile 5, these returns are consistent with those reported by Basiewicz and Auret (2009). They are suggestive of a size effect which dominates for the smallest quintile of firms, and a PE ratio effect which explains more of the variation in returns between larger firms.

Figure 8 shows the return in each of the 25 portfolios where quintiles are formed first on PE ratio and then, within these, on firm size. Table 7 shows the difference in mean returns between extreme size quintiles within each PE ratio quintile (similar to above, isolating the size effect controlling for PE ratio). We may conclude from the above that the size effect is generally more pronounced as the holding period increases, for all PE ratio quintiles apart from the smallest.

Table 5: Results for portfolios formed on Dimson beta

Size quintile	Holding period (months)			
	1	3	6	12
1	1,52%	4,23%	8,28%	19,73%
2	1,51%	4,45%	8,61%	18,45%
3	1,27%	4,11%	8,37%	16,81%
4	1,43%	4,10%	8,12%	17,92%
5	1,34%	3,75%	6,93%	13,05%
Differences between portfolios				
1-5	0,18%	0,48%	1,35%	6,68%
1-2	0,25%	0,12%	-0,09%	2,91%
2-5	-0,07%	0,36%	1,44%	3,77%

Table 6: Difference in returns between extreme PE ratio quintiles within size quintiles

Size quintile	Holding period (months)			
	1	3	6	12
1	0,41%	-2,62%	-7,22%	-17,39%
2	0,98%	1,79%	2,40%	1,00%
3	1,68%*	4,01%	4,82%	8,59%
4	1,38%*	3,99%	7,52%	11,55%
5	0,74%	1,78%	1,80%	2,40%

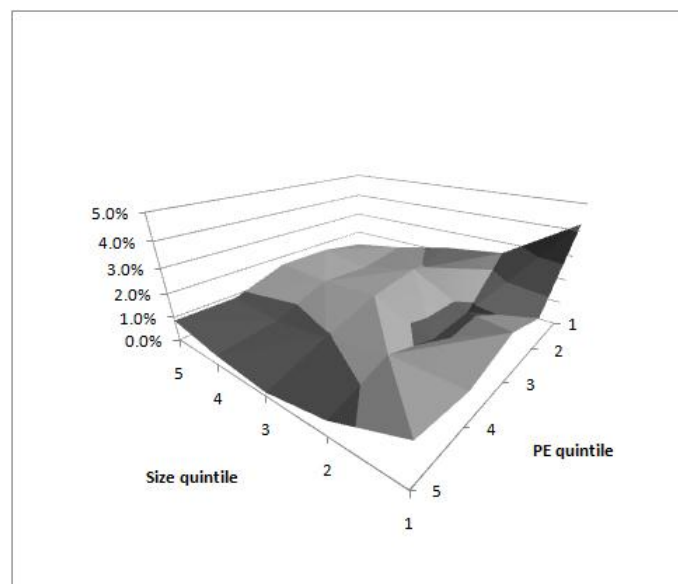


Figure 8: Two-way analysis: size within value quintiles

It therefore appears that both size and value effects are needed to explain returns on the JSE, and that one does not proxy for the other.

4. Possible explanations of results

One of the major criticisms of empirical work indicating anomalies within the CAPM framework is that they, of necessity, use market proxies which are narrower than the universe of all risky assets. This is an especially pronounced issue in a relatively small economy such as South Africa. The work of Van Rensburg (2002) suggested that the JSE All-Share Index is very probably mean-variance inefficient in a global context, and found that a two-factor APT model decomposing the JSE into Financial/Industrial and Resources components was more successful in describing the cross-section of returns than the single-factor CAPM.

Van Rensburg and Robertson (2003a) considered this possibility by repeating their study on a sub-sample of Financial and Industrial stocks, with betas estimated by regression against the Financial and Industrial Index returns (the sample of Resources stocks was too small to perform a comparable study). They found little material difference in results when compared to the sample of all stocks. This suggests that there is little to be gained from attempting to extend the analysis presented in this paper to betas from the two-factor APT model.

Table 7: Difference in returns between extreme size quintiles within PE ratio quintiles

PE quintile	Holding period (months)			
	1	3	6	12
1	2,95%***	4,03%	2,37%	2,00%
2	-0,28%	0,48%	2,42%	9,50%
3	0,35%	0,08%	0,34%	10,51%
4	0,49%	1,21%	3,99%	12,41%
5	0,80%	3,06%	6,03%	12,69%

Another plausible hypothesis is that the results are a function of mean reversion of stock returns, as reported by DeBondt and Thaler (1985) and, for the local market, Cubbin, Eidne, Firer and Gilbert (2006)¹. In a generally up-trending market, it is plausible that the stocks with the highest returns over the past five years will tend to be those with the highest estimated betas; if mean reversion is present in the market to the extent that these stocks tend to underperform going forward, then this would account to a large degree for the observed

inverse relationship between beta and return. This hypothesis was therefore investigated, albeit with little expectation of confirmation, given that the holding periods considered in this study (up to 12 months) are considerably shorter than those considered in De Bondt and Thaler (1985) and other prominent work on mean reversion. The formation of portfolios on past 60 months' return yielded no conclusive evidence of mean reversion, and we may thus conclude that mean reversion is not a significant cause of the observed inverse relationship between beta and return.

¹It should be noted for the latter though that their portfolios were formed on PE ratio rather than past return (the two measures are however expected to be positively correlated).

5. CONCLUSIONS

The size effect and value effect (based on the PE ratio) on the JSE are confirmed by the results of this study. These effects are significant and pervasive, and either indicative of some level of market inefficiency or, perhaps more likely, a misspecification of equilibrium pricing models such as the CAPM which assume that market covariance alone constitutes rewarded systematic risk. There is some tentative evidence of a reduction in the size premium over time, but this is not conclusive.

The most surprising observation of Van Rensburg and Robertson (2003a), namely that beta has, if anything, an inverse relationship with return, also finds support in this study. Their conclusion is clearly not sample-specific, as this analysis covers a later and longer period, nor is it a result of the short (12 to 30 month) period used for beta estimation, as betas estimated over a 60 month period lead to a similar result. The outcome is however robbed of its statistical significance when betas are estimated using the Dimson Aggregated Coefficients method, designed specifically to compensate for the effects of thin trading and the consequent shortcomings of OLS estimation, using a lead and lag period of at least three months. We may however at best conclude that beta is irrelevant as far as return generation on the JSE is concerned, at least based on the (possibly inefficient) market proxy of the FTSE-JSE All-Share Index; there is no support for the positive relationship espoused by finance theory.

Analysis of intermediate quintile portfolios yields further interesting insights. While the value effect appears to apply across the stock universe, with return a monotonically decreasing function of PE ratio, the size effect seems to be concentrated in the smallest quintile of stocks, with little difference in returns between the largest four quintiles. Any attempt to build these observations into a multifactor model would need to recognise this feature of the size premium.

The size and value effects appear furthermore to operate independently, given the variation within sub-quintiles in the two-way analyses. The value (size) effects are however not uniform over the size (value) quintiles, and again these features would need to be recognised in any sensible multifactor model.

These size and value effects may indicate market inefficiency, and the diminishing size premium over this study's sample period would be consistent with the rational actions of market participants closing out historic inefficiencies. It is however perhaps more likely that they reflect sources of risk which are not measured by market covariance, and hence point to a misspecification of the CAPM and the need for more sophisticated multifactor models to represent

the returns on South African equities.

It should be noted that this study takes no account of transaction costs. While many of the results presented in this dissertation are highly statistically significant, their economic significance may be nil if transaction costs prevent trading strategies from taking advantage of them. The incorporation of transaction costs would be a useful extension to this research.

The formation of value-weighted rather than equally-weighted portfolios may also provide some insight. This approach would obviate the need for the removal from the data set of penny-stock shares with an undue influence on portfolio returns. That being said, the reliability of the data used in this study, particularly for delisted stocks, is subject to question. It is likely that financial researchers across South Africa grapple with the same issues, and consideration ought to be given by finance academics to the (admittedly onerous) collaborative development of a reliable historic equity market data set which can then be maintained with comparatively little effort, along the lines of the database maintained by the University of Chicago's Center for Research in Security Prices.

Finally, an important extension to this research would be the construction of a multifactor equilibrium pricing model based on its insights, and those which may be provided by the research avenues suggested above. This study provides unambiguous confirmation of the evidence presented by Van Rensburg and Robertson (2003a) that the CAPM is unable to explain or describe the generation of returns on the JSE, and its contradiction creates a vacuum which begs to be filled.

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