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Constraints on investment weights: What mandate authors in concentrated equity markets such as South Africa need to know

ABSTRACT

This article demonstrates the importance of setting risk budgets and constraints mindful of the nature of the chosen benchmark and the investment environment. The asymmetrical inefficiency of the long-only constraint when applied to a concentrated investment environment such as the South African equities market is illustrated. We estimate the optimal distribution of investment weights in each security in the context of standard portfolio construction techniques and typical South African equity benchmarks and market conditions. These distributions provide guidance to mandate authors who are considering allowing limited shorting in their net long portfolios as to the amount of gearing that is likely to be required. These estimates also show authors of long-only mandates the circumstances and assets for which their restrictions are materially binding.

1. INTRODUCTION AND LITERATURE REVIEW

Modern portfolio theory, developed by Markowitz, Sharpe, Tobin and others, was revised by the use of benchmarks and the introduction of the notion of active or benchmark-relative performance and risk. Grinold and Kahn (1994) amongst others contributed to Modern Portfolio Theory by examining the source of excess risk-adjusted return (Information Ratio) for an investment portfolio. Their Fundamental Law (Grinold (1994)) relates the accuracy and breadth of views on individual assets to the overall success of a portfolio's performance.

Since then, Clarke, De Silva and Thorley (2002) generalised the law to consider the impact of implementation on the performance of a portfolio of assets. The authors proposed that, in addition to the accuracy of investment forecasts and the breadth of these same forecasts, the efficiency with which these views are implemented in a portfolio contribute to or detract from its performance. Clarke *et al.* (2002) introduced the notion of the Transfer Coefficient (TC) to the Fundamental Law. The TC quantifies the extent to which the active weights in a portfolio reflect the investment view(s) of the portfolio manager.

Several empirical studies, such as Clarke, De Silva, and Sapra (2004) and Martielli (2005), have demonstrated the impact of investment constraints on the performance of the portfolio, using the TC to quantify the extent of the value lost in implementation. These empirical studies show that, of all the typical mandated fund constraints, the prohibition on short positions in a portfolio accounts for the greatest loss of value between view creation and portfolio implementation.

Short extension products have proved to be one solution to this problem in international markets. The typical, fully-invested, equity portfolio in a pension fund's overall investment portfolio is constrained to be "long-only" (i.e. hold no short positions in any assets) and fully invested (i.e. 100% of the fund is invested without gearing). Short extension products offer this segment of the market, hardest hit by the inefficiency of size and investment constraints, an opportunity to be 100% invested with only a partial relaxation of the short constraint. For example, a so-called 130/30 portfolio allows for a maximum of R30 of every R100 invested to be held in short positions and R130 to be invested long.

This so-called "short extension" can be used to increase the risk and gearing of a previously long-only fund. However, in keeping with the original intention of this investment product and in the context of this article, short-extension has as its purpose to increase the transfer of investment view from the fund manager to the fund for the same level of active risk as its long-only counterpart.

Analytic Investors[i1] (chaired by Roger Clarke) are considered to be the pioneers in short extension products and have been managing short-extension products on the S&P 500 and Russel 1000 since 2002. Many others have followed suit, resulting in an increasing demand for these partially short products amongst international pension investors.

The empirical studies of Clarke, De Silva, and Sapra (2004) and Martielli (2005) amongst others, have shown the improvement to TC offered by these short-extension products, confirming the relationship between ultimate performance and both the accuracy of the fund manager's views and their ability to appropriately transfer these views to the portfolio. These studies also show the extent to which high market concentration in large capitalization stocks and increased target active risk exacerbate the detrimental effect of the long-only constraint on the TC and ultimately the performance of an otherwise well-managed fund.

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[1] http://www.aninvestor.com/index.asp

In a small and highly concentrated equity market such as South Africa where roughly half of our equity index is comprised of only five securities, the effect of constraints on investment weights (such as the long only constraint) can be even more detrimental than has been shown in international literature, which is typically based on indices of 500 or more securities. Short extension products have only recently (2009) been available to South African pension funds and as yet no published track record exists for funds invested in this way. In considering these funds as a viable alternative to long-only investment, South African investors and pension fund sponsors have been challenged to reconsider the restrictions they typically mandate on securities in their investment universe.

Clarke *et al.* (2008) concerned itself with the quantitative boundaries and guidelines required by mandate authors to accommodate short-extension funds but the analysis in this article applies to all security investment weight restrictions and, as such, provides useful models for mandate authors of actively managed equity funds in general. Clarke (2008) provides a generalised mathematical model of the distribution of optimal unconstrained investment weights in every asset across various investment views. This derivation clarifies the relationship between the extent of the optimally required short positions in a portfolio and a) the number of assets in the benchmark, b) the concentration of the benchmark, c) the overall risk of each asset, d) the overall level of correlation among assets and e) the target active risk.

The premise of Clarke *et al.* (2008) is that investment views at any point in time on any particular stock can be described as a random normal variable. Consequently, the optimal unconstrained investment weights are also random normal with an expected value which is a function of the benchmark, the fund's risk budget and market risk conditions.

This article begins with an examination of the asymmetry in investment opportunities with which long-only fund managers are presented, particularly when their benchmarks are highly concentrated with many securities of low weighting. A very simple metric is introduced to compare the opportunity sets implicit in several commonly used South African benchmarks.

In the following section, the derivations published in Clarke (2008) are used to determine the distribution of the optimal investment weights under various market conditions. While fund managers are concerned with the efficient implementation of their investment view into an investment portfolio, this analysis is particularly intended for the use of mandate authors who are concerned with finding the reasonable boundaries to fund manager activities. The methodology and empirical analysis in this section provide insights as to the likely distribution of optimal investment weights across various investment views and subject to benchmark choice and risk budget. The article highlights the importance of the choice of benchmark and risk budget in determining the distribution of optimal investment weights in the portfolio and, by implication, the extent of the sub-optimality of certain security-specific restrictions in the light of these two choices.

The analysis extends to determine the likely optimal fund gearing or short-extension under various market conditions and risk budgets for those mandate authors who are considering relaxing the long-only constraint. The article concludes thereafter.

2. THE INVESTMENT OPPORTUNITY SET

2.1 The asymmetry of investment opportunities for long-only funds

This section presents a very simple measure of the effect of restrictions on asset weights in a portfolio on the fund manager's opportunity set. Specifically, the conventional long-only fund's prohibition against selling individual assets short has an asymmetrical effect on the active fund manager's opportunity set. Active fund managers' expresses their investment view on the assets in their investment universe by holding an over- , neutral or under-weight position ($w_{a,i} > 0$, $w_{a,i} = 0$ or $w_{a,i} < 0$) in these assets relative to the manager's assigned benchmark.

Equation 1: Definition of active weight in asset i

$$w_{a,i} = w_{f,i} - w_{b,i}$$

where

$w_{a,i}$ is the active weight of the fund in asset i,

$w_{f,i}$ is the weight of the fund in asset i and

$w_{b,i}$ is the weight of the benchmark in asset i.

The extent to which the fund manager can expand a positive active weight is limited only by particular mandate restrictions[2] and the ability to finance the total

[2] Regulation 28, for example, which regulates the investment of South African pension funds, requires that no more than 15% of a pension fund may be invested in a large capitalisation listed stock, and10% in any single other stock. The proportions apply to the whole fund to which the same regulation restricts the total equity investment to 75%. Considering a maximum equity portion of a pension fund in isolation then, regulation 28 allows a generous 20% (15% of 75%) and 13% (10% of 75%) maximum investment in individual large capitalisation and other listed stocks respectively. The regulatory constraints are not usually binding on a typical, mandate-constrained equity portfolio and the individual mandates which describe the limits of the fund manager's investments are typically more restrictive when it comes to the maximum allowable active or investment weight of the portfolio in any individual stock.

positive active positions in the portfolio with sufficient negative positions in other assets. The hedge fund manager, or a similarly unconstrained investor, enjoys a symmetrical investment opportunity set with respect to implementing negative active investment weights. By contrast, the conventional long-only fund manager can expand a negative active position only to the point of excluding the asset from the fund (i.e. $w_{f,i} \geq 0$). Therefore the most negative active weight possible in any particular asset in a long-only fund is the negative of the asset's benchmark weight ($w_{a,i} \geq -w_{b,i}$). In this way, the long-only fund manager has greater scope for expressing positive investment views in each asset than negative views. Hence the asymmetry in the long-only active manager's opportunity set.

2.2 The transfer coefficient

The transfer coefficient (TC) is defined as the cross-sectional correlation of the risk-adjusted forecasts across assets and the risk-adjusted active portfolio weights in the same assets.

Equation 2: Definition of transfer coefficient (TC)

$$\mathrm{TC} = \mathrm{Correlation}\left(w_a\sigma_e, \frac{\alpha}{\sigma_e}\right)$$

where

$w_a\sigma_e$ is a vector of risk adjusted active weights ($w_{a,i}\sigma_{e,i}$),

$\sigma_{e,i}$ is the residual risk for each asset i.e. the risk of each asset that is not explained by the benchmark portfolio,

α is a vector of forecast active returns i.e. forecast returns in excess of benchmark-related return.

In this way the TC essentially measures the manager's ability to invest in a way that is consistent with their relative views on the assets in their investment universe (refer Equation 2). A perfectly consistent investment portfolio would have a TC of one. Any inconsistency in implementation, including the compliance to mandated limits on investment weights, will reduce the TC below one.

Calculating the TC can assist the portfolio manager to assess the efficiency of their implementation. But, more importantly, TC enables an assessment of the inefficiencies introduced by the mandated restrictions of the portfolio. It is this latter application of TC that can assist fund sponsors and mandate authors in their understanding of the impact of constraints on the portfolio's performance.

In order to accurately measure the TC of any fund at any particular point in time, one would need to know the forecasts and model estimates of residual risk for every asset at the time. Since only the portfolio managers themselves usually have access to this kind of information, the fund sponsor requires a method of measurement with less onerous information requirements. Furthermore, the effect of mandated constraints on portfolio weights will vary to some extent with what the forecast returns are. For example, in a long-only fund, a negative view of larger assets is more easily implemented than a positive view on large assets because the negative active weight in larger assets frees up financing for a variety of positive positions. By contrast, a positive active weight in large assets requires many small negative active weights to finance such a position, irrespective of whether those positions are justified by the forecasts, thus reducing the TC of the resulting portfolio. The larger the active weights in the portfolio (which are usually as a consequence of a larger risk budget), the greater the asymmetry of the investment opportunity set and the smaller the TC of the portfolio.

2.2.1 The implied transfer coefficient

This article suggests a simple, forecast-independent metric for the inefficiency in implementation implied by portfolio weight constraints. This metric requires only the weights of the benchmark assets and the investment weight constraints. We refer to this as Implied Transfer Coefficient (ITC) because it is a measure of the active fund manager's ability to implement an investment view as implied by the benchmark composition and the security-level constraints. Although the ITC is no substitute for an accurate measure of TC, it provides fund sponsors with a quick, easy and intuitive calculation which can assist in the mandate setting process without requiring information specific to the investment view of the manager.

The ITC, as described by Equation 3, compares the sum of the range of active positions on each asset in a long-only fund (the numerator) to that of an unconstrained fund (the denominator).

Equation 3: Definition of implied transfer coefficient (ITC)

$$\mathrm{ITC} = \frac{\mathrm{N}w_a^{max} + \mathbf{1^T}.\min[\mathbf{w_b}, w_a^{max}]}{2\mathrm{N}w_a^{max}}$$

where

w_a^{max} is the maximum permitted active weight in absolute terms as prescribed by the mandate limits,

$\min[\mathbf{w_b}, w_a^{max}]$ is the N x1 matrix of the smaller of each asset's weight in the benchmark or the maximum investment weight permitted,

1^T is a 1x N vector of ones and

N is the number of assets in the investment universe.

The denominator of Equation 3 ($2\mathrm{N}w_a^{max}$) represents the full permissible active weight range of the unconstrained fund manager in each of N assets in the investment universe: from w_a^{max} to $-w_a^{max}$. The long-only manager has a similar range in terms of the positive active weights in each of the assets ($\mathrm{N}w_a^{max}$). However, in terms of the possible negative active positions, the long-only manager can only hold the maximum negative active positions ($-w_a^{max}$) without having to sell the asset short if the asset's benchmark weight is larger than the active weight limit (i.e. for assets such that $w_{b,i} \geq w_a^{max}$). For smaller assets (i.e. assets such that $w_{b,i} < w_a^{max}$) the maximum possible negative active positions possible without incurring short positions is $-w_{b_i}$.

Figure 1 illustrates the ITC using a typical equity benchmark for South African pension funds, the SWIX (Shareholders-weighted Index)[3]. The SWIX, much like the JSE All Share Index, reflects the highly concentrated nature of the South African equity capital markets, with the largest five stocks comprising almost 30% of the total weighting in this index. Only 24 out of the 165 members of this index have weightings larger than 1%. The asymmetry and loss of opportunity as measured by the ITC is therefore evident in long-only funds at very conservative active weight limits and worsens with greater permissible active weights. At a maximum of 1% active weight, for example, approximately one third of the opportunity set is unavailable to the long-only active manager who has an ITC of 0,67.

Using this simple ITC statistic, which is independent of investment view, market conditions and all risk and portfolio self-financing considerations, we attempt to quantify the benefits of a very small short-extension to the long-only fund manager. To this end, we generalise the ITC in Equation 4 to allow for greater range in the maximum negative active weights as would be the case in a short-extension fund.

Equation 4: Generalised ITC allowing for short extension (X)

$$\mathrm{ITC} = \frac{\mathrm{N}w_a^{max} + \mathbf{1}^{\boldsymbol{T}}.\min\left[w_{b,i} + X.\mathbf{1}, w_a^{max}.\mathbf{1}\right]}{2\mathrm{N}w_a^{max}}$$

where

X is the maximum permissible sum of short positions in the fund e.g. a so-called "130/30" fund has an X of 30% and a long-only fund has an X of zero.

The denominator is unchanged as the comparison to the unconstrained fund remains. The number of assets for which the maximum active position is permissible, increases because of the short extension (X): all the assets for which $w_{b,i} + \mathrm{X} \geq w_a^{max}$ can be held at the maximum negative active position ($-w_a^{max}$). Likewise, there are fewer assets for which the full range of active weights cannot be achieved: assets for which $w_{b,i} + \mathrm{X} < w_a^{max}$ can only have maximum negative active positions equal to the benchmark weight plus the short extension i.e.$-\left(w_{b,i} + \mathrm{X}\right)$.

2.2.2 A comparison of South African equity indices

Returning to Figure 1 then, the ITC's for four very modest short-extension products on the SWIX are depicted alongside the long-only fund to illustrate the improvement in the opportunity set for net long active managers for whom the short-selling restriction is very slightly relaxed. For example, at a 3% active weight limit, the long-only manager is at a 0,57 ITC, a substantial disadvantage compared to their unconstrained peers. However, a mere 2,5% short extension will improve this manager's ITC to 0,65 and a 5% short extension will essentially put the active manager on equal footing with their unconstrained counterparts in terms of their opportunity set.

The consequences of limits on active weights are less for fund managers with a more specialized universe and a larger weight per asset. Figure 2 depicts the same ITC and weight limits as Figure 1 but based on a benchmark of the largest forty stocks only. The loss of efficiency and asymmetry in investment opportunities is less for a fund with a SWIX 40 benchmark (median weight 1,42%) compared to a more diverse SWIX benchmark (median weight 0,19%). Consequently, the need for short extensions to improve the efficiency of these portfolios is less.

Table 1 is a summary of well-known FTSE/JSE indices and comprises the ITC for each of these indices under long-only constraints. On the whole, the smaller indices with fewer members and larger overall weightings per index member present the long only active manager with a well distributed opportunity set for active allocation. The RESI20 resource index is a notable exception: this index has only 20 members but more than half of these have less than 1% weight in the index thus providing a RESI20 benchmarked active manager with very limited scope for taking negative active investment views.

[3]The share-weighted index attempts to represent the collective free-float investments of all South African investors i.e. excluding foreign holdings.

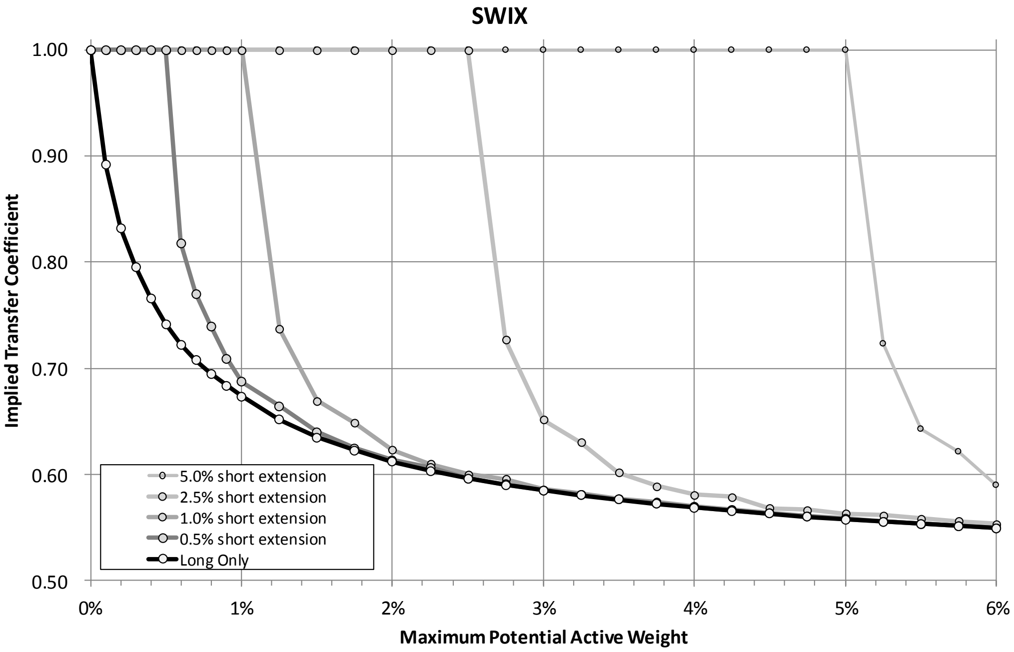

Figure 1: Implied transfer coefficient for a SWIX[4] equity fund

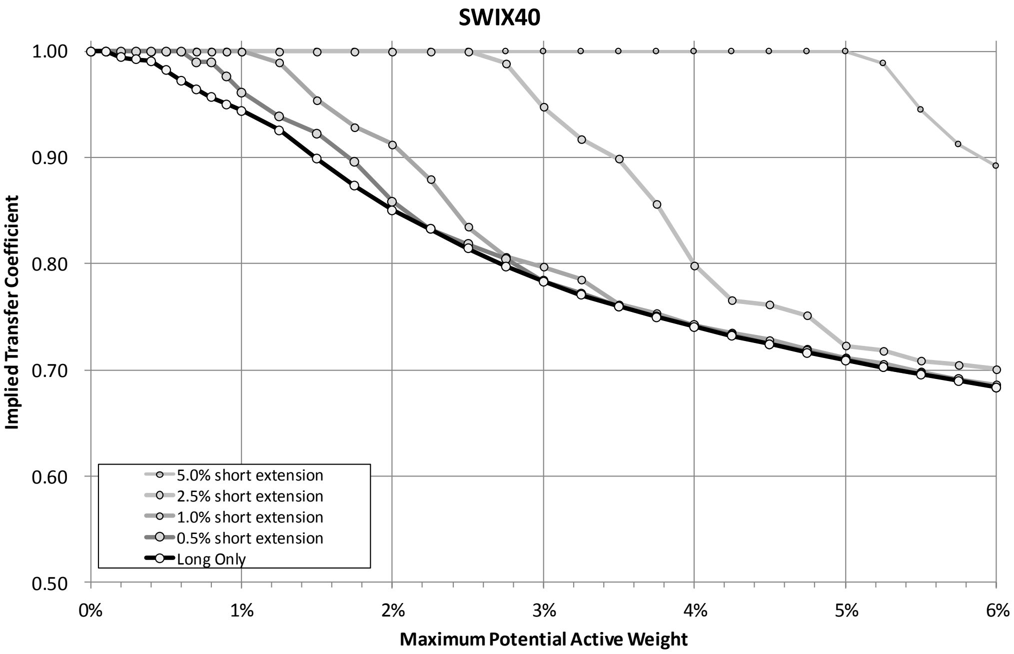

Figure 2: Implied transfer coefficient for a SWIX 40 equity fund

[4]As of August 2010

Table 1: ITC of long-only funds with various FTSE/JSE benchmarks

w_a^* (%)	SWIX J403	ALSI J203	CAPI J303	Value J330	Growth J331	SWIX40 J400	ALSI40 J200	CAPI40 J300	RAFI40 J260	MID CAP J201	SMALL CAP J202	RESI20 J210	INDI25 J211	FINI15 J212	FINI30 J213
-	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
0,10	0,89	0,86	0,86	0,88	0,88	1,00	1,00	1,00	1,00	1,00	1,00	0,96	1,00	1,00	1,00
0,20	0,83	0,80	0,80	0,81	0,81	0,99	1,00	1,00	1,00	1,00	1,00	0,90	1,00	1,00	1,00
0,30	0,80	0,76	0,76	0,77	0,77	0,99	0,99	0,99	0,99	1,00	1,00	0,87	1,00	1,00	1,00
0,40	0,77	0,73	0,73	0,73	0,73	0,99	0,98	0,98	0,99	0,99	0,99	0,85	1,00	1,00	1,00
0,50	0,74	0,71	0,71	0,71	0,71	0,98	0,97	0,98	0,98	0,98	0,98	0,83	1,00	1,00	1,00
0,60	0,72	0,69	0,70	0,69	0,69	0,97	0,97	0,97	0,98	0,97	0,97	0,81	1,00	1,00	1,00
0,70	0,71	0,68	0,68	0,68	0,68	0,96	0,96	0,96	0,97	0,97	0,97	0,80	1,00	1,00	0,99
0,80	0,69	0,67	0,67	0,67	0,67	0,96	0,95	0,95	0,96	0,96	0,96	0,79	0,99	1,00	0,99
0,90	0,68	0,65	0,66	0,66	0,66	0,95	0,93	0,94	0,95	0,95	0,95	0,78	0,99	1,00	0,98
1.00	0,67	0,64	0,65	0,65	0,65	0,94	0,91	0,93	0,94	0,94	0,93	0,78	0,98	1,00	0,98
1,25	0,65	0,63	0,63	0,63	0,63	0,93	0,88	0,89	0,92	0,91	0,91	0,76	0,97	1,00	0,96
1,50	0,63	0,61	0,62	0,61	0,61	0,90	0,85	0,86	0,89	0,88	0,88	0,75	0,96	1,00	0,95
1,75	0,62	0,60	0,60	0,60	0,60	0,87	0,82	0,84	0,87	0,86	0,86	0,74	0,94	0,99	0,92
2,00	0,61	0,59	0,60	0,60	0,60	0,85	0,80	0,82	0,84	0,83	0,84	0,74	0,92	0,99	0,89
2,50	0,60	0,58	0,58	0,58	0,58	0,81	0,77	0,79	0,81	0,79	0,79	0,73	0,87	0,97	0,85
3,00	0,58	0,57	0,57	0,57	0,57	0,78	0,75	0,76	0,78	0,75	0,76	0,72	0,83	0,96	0,81
4,00	0,57	0,56	0,56	0,56	0,56	0,74	0,71	0,72	0,73	0,70	0,70	0,71	0,78	0,93	0,76

In this section, we have provided some empirical indication of the material restriction under which long-only, benchmarked fund managers are placed, particularly when the benchmarks have high concentration in a few members and a large number of smaller weightings in others. In this context and using this very simple metric, we've presented the benefits of small short-extensions to the investment range of these same active fund managers. The evidence suggests that small short extensions (less than 5% overall) may substantially improve the opportunity set for these managers.

3. ACTIVE WEIGHTS AND THE RISK BUDGET

When setting mandates, the fund sponsors must consider the necessary long term objectives and constraints independently of a particular investment view which varies in the short term. In the previous section, we examined the boundaries of the active opportunity set given particular benchmarks and constraints. In this section, we attempt to quantify likely active positions in each security in the context of standard portfolio construction techniques and particular benchmarks. Using this analysis, we can estimate the extent of the short-positions that will likely be required by an optimal active portfolio. These estimates provide some guidance to mandate authors who are considering allowing limited shorting in their net long portfolios. They are also an indication to authors of long-only mandates of the circumstances under which their restrictions are materially binding.

3.1 Optimal distributions of security weights

Our starting point is the unconstrained[5] portfolio optimization problem in a benchmark relative framework, namely to maximize the forecasted active return of the portfolio, while achieving a particular active risk target and ensuring that the portfolio is self-funding. This problem has a well-known, unique solution for any given active risk target (refer Equation 5).

[5] Although there is clearly a self-financing constraint imposed on this optimization problem, it is generally referred to as an "unconstrained" optimization on account of there being no constraints placed on individual weights in the portfolio.

Equation 5: Unconstrained optimal active weights for a given target active risk

$$w_a = \sigma_A \frac{\sigma^{-1}\alpha}{\sqrt{\alpha^T \sigma^{-1} \alpha}}$$

where

w_a is an n x 1 vector of active bets i.e. portfolio weights in excess of the benchmark's weights which must add to zero in order for the portfolio to be self-financing,

$\boldsymbol{\alpha}$ is an n x 1 vector of forecasted active returns (i.e. forecasted returns in excess of benchmark) for each of n securities,

$\boldsymbol{\sigma}$ is an n x n estimated covariance matrix of the returns of these securities, and

σ_A is the target active risk of the portfolio.

Equation 5 shows that the size and sign of the active weight of any particular stock is directly related to the size and sign of the forecasted active return when the portfolio is unconstrained. The target active risk magnifies the extent of this bet or active weight while the risk in the denominator has the opposite effect. Thus a positive excess-of-benchmark return expectation (relative to the other forecasts) for a particular security would lead a rational, unconstrained and optimal fund manager to a positive active position in that same stock. The greater the target active risk (i.e. the lower the risk-aversion) of the investor, the greater the active positions in their portfolio will be. Conversely, the extent of this active position is reduced by the uncertainty of the security's prospects.

Sorensen *et al.* (2007) demonstrated how Equation 5 can be rewritten to determine the condition under which any particular stock will be held short (i.e. have a weight of less than zero in the portfolio) by an optimal unconstrained fund manager.

Equation 6: Condition for a short position in an unconstrained optimal fund

$$\alpha < -\frac{w_b}{\sigma_A} \frac{\sqrt{\alpha^T \sigma^{-1} \alpha}}{\sigma^{-1}}$$

The Sorensen (2007) derivation in Equation 6 yields several insights. Firstly, the optimal unconstrained portfolio is more likely to have a short position in a security when the security itself has a low weighting in the benchmark (small w_b). The implication being that, irrespective of the fund manager's investment view at any given time, short-extension fund administrators should be more concerned with the ability to short smaller securities when preparing prime broking arrangements and scrip lending agreements than the larger, more liquid securities. This is particularly true of small stocks with higher residual risks.

The second important insight yielded by Equation 6 is that the likelihood of an optimal short position in any security is increased across the board by higher risk budgets i.e. higher target active risk. The implication for fund sponsors being that, with greater active risk expectations[6] must come either a greater loosening of the typical constraints on fund managers or an increasingly sub-optimal, asymmetric portfolio construction setting.

The active weight (and the potentially short holding) in any security is therefore not purely a function of the manager's view at any particular time and the risk characteristics of the assets under consideration, but is also dependent on the risk budget, the benchmark composition and each security's weight in the benchmark. Fund sponsors, by way of their choice of benchmark and their setting of the fund's risk objectives, therefore directly affect the distribution of likely weights assigned to assets during the portfolio construction process.

3.2 The distribution of active weights

Considering then the sponsor's role in the portfolio construction process, the mandate authors need to be mindful of the distribution of active weights in the portfolio implied by the choice of benchmark and risk budget. To this end, an understanding of the distribution of likely security weightings in the portfolio is necessary.

Grinold (1994) proposed the "alpha generation" formula which Clarke *et al.* (2006) generalised in the following form into their underlying components:

Equation 7: Alpha generation a la Grinold (1994)

$$\alpha = IC\sigma^{\frac{1}{2}}S$$

where

IC is the information coefficient, a measure of manager skill: the correlation between their forecasted returns and the subsequent (realized) returns and,

S is an Nx1 vector of randomized standard normal scores.

Equation 7 therefore presents forecasted excess returns as generated by a random normal process, scaled by skill and risk. Clarke *et al.* (2008) and Sorensen *et al.* (2007) argue that, if forecast excess returns follow a random process, the distribution of

[6]Presuming that the investment universe is not expanded in order to achieve greater active risk.

optimal active weights which result from these forecasts can be derived accordingly. Using multiple simulations of the score ($\boldsymbol{S}$) in combination with their current risk model ($\boldsymbol{\sigma}$) and risk budget (σ_A) and solving for the unconstrained active portfolio, it is possible then for mandate authors to derive distributions of active weights (and by extension, also the investment weights) for each asset in the investment universe. These simulated distributions of asset weights, consistent with a range of likely forecasts and the risk model, can then provide sound justification for various weight constraints which are appropriate for each asset and across changing investment views. This process can also inform the likelihood and extent of short selling optimally required in each stock thereby informing mandated constraints on short-selling and preparing prime broking requirements in anticipation of future shorting requirements.

3.3 Distribution of active weights under simplified conditions

Simulation can be a very useful tool for approximating complex analytical situations such as the active weight distribution under any possible set of forecasts. In this section we use a simplification provided by Clarke *et al.* (2008) to illustrate the relationships between the various parameters of fund management and the probability distribution of active weights. Clarke *et al.* (2008) uses a simplified two-parameter variance-covariance matrix by setting all individual asset variances equal to a single value, □, and all pairwise correlations to the same value, □. Under these assumptions, the authors show that the unconstrained optimal active weights are normally distributed with a mean of zero and a variance which is proportional to the active risk.

Equation 8: Optimal unconstrained active weight distribution under the two parameter covariance assumption

$$\boldsymbol{w}_a \sim N\left(0, \frac{\sigma_A}{\sqrt{N}} \frac{1}{\sigma\sqrt{1-\rho}}\right)$$

where

σ is the standard deviation of returns of each security,

ρ is the correlation of the returns of each pair of securities.

This derivation relates the required scope for active positions to both market and mandate conditions. Increasing volatility and decreasing correlation (i.e. increasing cross-sectional variation) result in a narrower distribution of active weights and a lower probability of needing short positions to optimally achieve a particular active risk target.

Furthermore, Equation 8 demonstrates the role that the mandate author plays in the distribution of active weights by way of their choice of risk budget and benchmark. All things being equal, a wider distribution of active weights in each security is required with greater active risk targets. This increase in active weight spread is exponentially increased by a reduced investment universe (smaller N). These two insights alone confirm that, given the same active risk targets, funds managed on a smaller asset universe or benchmark will likely be more aggressive in their individual active weights per asset than funds managed in a more diverse universe. Furthermore, the wider the distribution of active weights, the more likely short positions in the smaller stocks will be required in the optimal construction of a portfolio and, by implication, the more materially binding the long-only constraint will be on the portfolio's construction.

The insights gained from Equation 8 demonstrate the importance of setting risk budgets and constraints mindful of the nature of the chosen benchmark and the changing cross-sectional variation in the investment environment.

3.4 Empirical demonstration of the distribution of active weights using South African indices

By way of empirical example, we've used the SWIX as a benchmark and its 165 component stocks as the investment universe (only the largest forty and smallest five stocks in the SWIX are displayed) and we have adopted the two-parameter simplification for illustration purposes. Figures 3 and 4 depict the 95% confidence interval for the optimal unconstrained holdings in a selection of the SWIX constituents under a particular set of market conditions and a target active risk of 4% p.a. The circles represent the neutral position or benchmark holding which, following from Equation 8, is also the most likely position. Figures 3 and 4 represent market conditions where the cross-sectional dispersion is low (17% p.a.) and high (52% p.a.) respectively, representing two historical extremes in the realized cross-sectional dispersion of the JSE[7]. Figures 5 and 6 represent the same scenarios but use the SWIX40 as a benchmark instead.

[7] Refer Raubenheimer (2011).

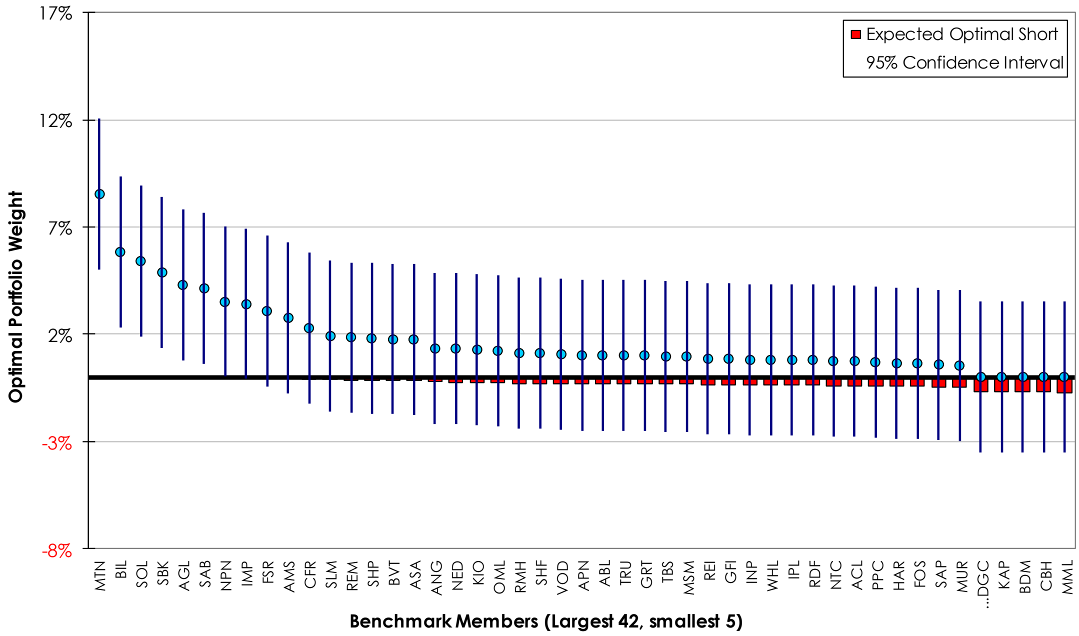

Figure 3: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ($\sigma\sqrt{1-\rho}$=0,17, active risk=4% p.a., Benchmark: SWIX)

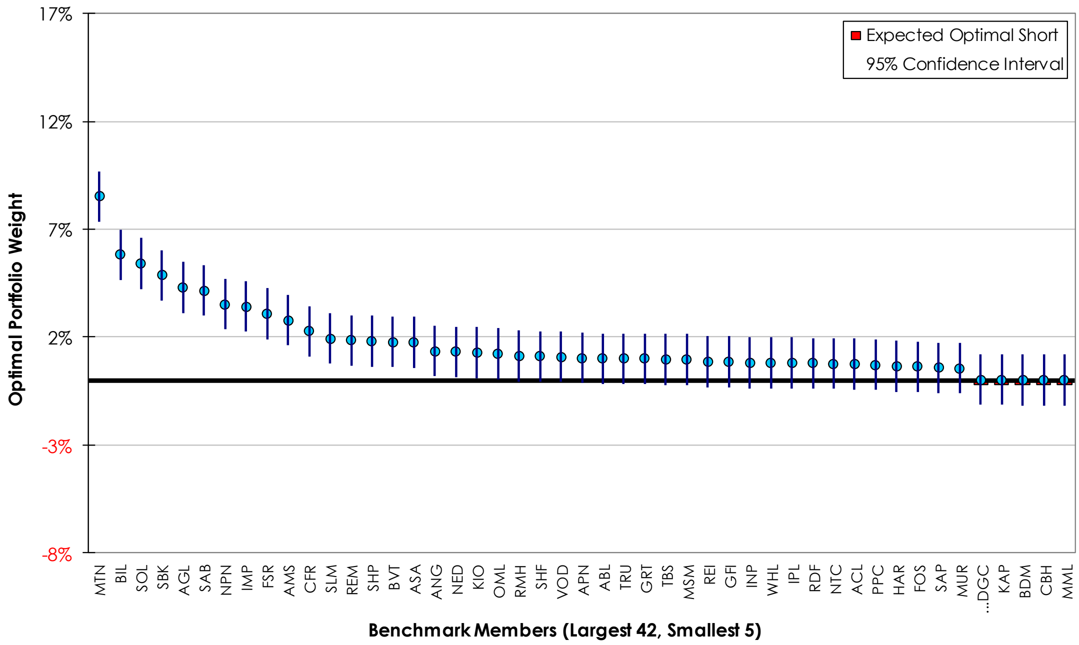

Figure 4: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ($\sigma\sqrt{1-\rho}$=0,52, active risk=4% p.a., Benchmark: SWIX)

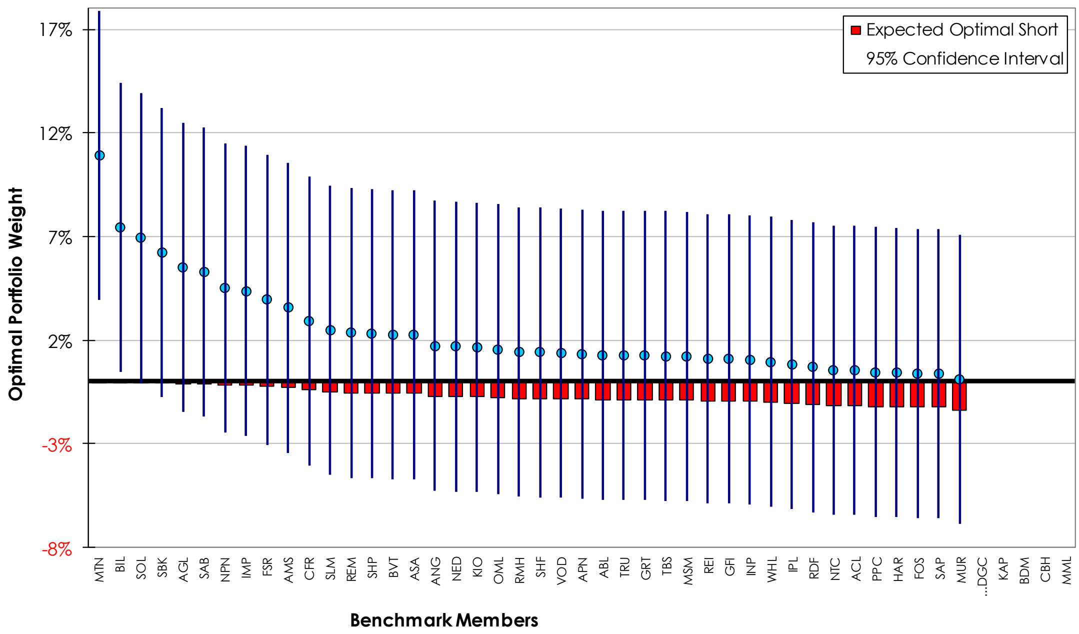

Figure 5: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ($\sigma\sqrt{1-\rho}$=0,17, active risk=4% p.a., Benchmark: SWIX40)

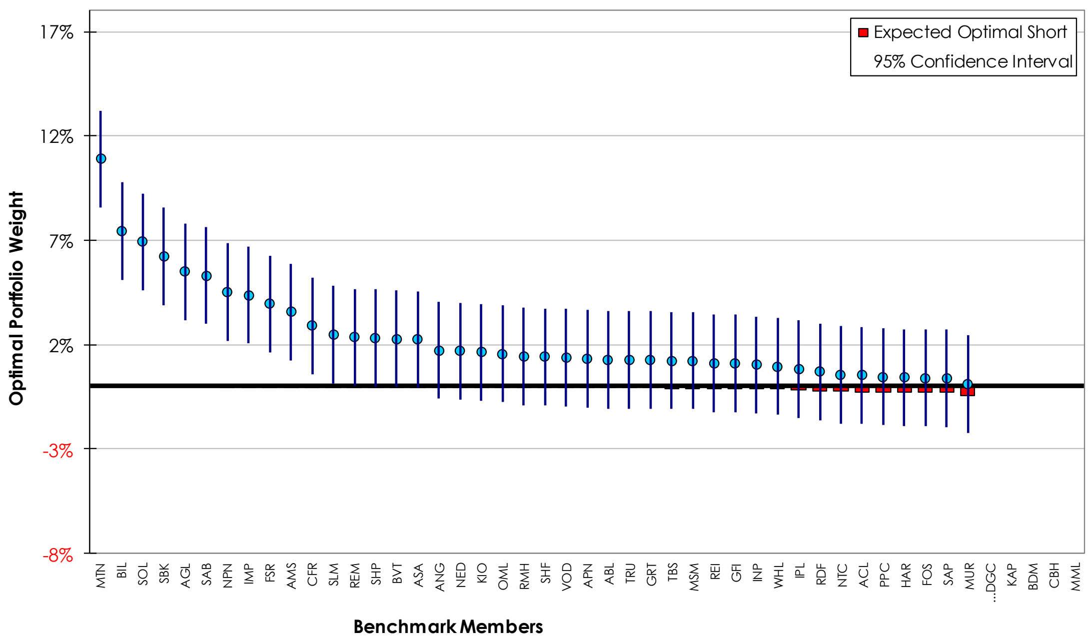

Figure 6: 95% Confidence Intervals for Optimal Unconstrained Holdings in each Benchmark Security ($\sigma\sqrt{1-\rho}$=0,52, active risk=4% p.a., Benchmark: SWIX40)

One of the most striking features of these figures is the greater short positions covered by the stocks with a smaller weight in the benchmark. The implication for fund managers is that, when prime broking is sought for short extension products, it is the smaller stocks which will most often be required to be held short. Conversely, trustees with concerns regarding the wholesale lifting of short sale constraints should note that not all the stocks in the investment universe require the long-only constraint to be removed. Figures 3 and 4 show that the top ten stocks, comprising almost half the weight of the investment universe, may be constrained to be held long only in a SWIX fund with a fairly aggressive risk budget, even in a low dispersion market without material detriment to the optimality of the investment. In fact, to allow short selling in these larger securities could be considered reckless.

Another feature of these figures is the difference in the spread of the distribution of optimal weights across assets under different market conditions. When the cross-sectional dispersion is high, the risk budget is more easily attained without substantial shorting. Under these conditions, a long-only manager is not as handicapped as they would be in a low dispersion market where optimal conditions will likely require short positions across a wide variety of assets. Thus a rigid long-only constraint across all assets will vary in its detrimental impact on the TC as the dispersion in the market's opportunities varies.

Although the analysis in this section makes use of a simplification that is not cognisant of the manager's risk model, it provides a good understanding of the inappropriateness of one-size-fits-all mandated constraints on asset weights that is relatively easy to implement. In this way mandate authors are able to quantify the distribution of optimal weights implied by their choice of benchmark and risk budget. Using various market scenarios, they can therefore observe the extent to which their imposed constraints on asset weights are likely to be binding.

3.5 Fund level constraints on gearing and the risk budget

The simulations envisioned in section 3.2 and the resulting distribution of asset weights can be aggregated to estimate optimal average fund gearing. Using the distribution of asset weights, we can find the most likely short position of each asset, conditional on the asset being held short at all. These likely short positions can be aggregated to provide an estimate of the total likely short positions on a fund level.

To illustrate empirically, we once again make use of the two-parameter covariance matrix simplification introduced in Clarke *et al.* (2008) and the resulting derivation of the expected short weighting on an asset level (refer Equation 9).

Equation 9: Expected short-selling weight conditional on short sales being required (Clarke *et al.*, 2008)

$$E(w_i|w_i < 0) = cf\left(-\frac{w_{b,i}}{c}\right) - w_{b,i}P\left(z < -\frac{w_{b,i}}{c}\right)$$

where

$$c = \frac{\sigma_A}{\sqrt{N}}\frac{1}{\sigma\sqrt{1-\rho}}$$

and

$f(.)$ is the normal density function.

The thick negative bars in Figures 3 through 6 represent the conditional expected short position in each of these securities. In other words, if each security is to be held short, this is the extent of the short position we expect on average given these market and investment conditions. Notice once again how the expected short position on larger assets is zero but that the expected short positions increase with decreasing asset weight. In an unconstrained SWIX fund (as represented by Figures 3 and 4) there are a large number of small expected short positions spread among 150 to 125 smaller stocks whereas an optimal unconstrained SWIX40 fund (as represented by Figures 5 and 6) would likely required larger short positions over a smaller number of assets. The expected short positions increase across the board with decreasing cross-sectional dispersion.

Adding up the expected short positions per security provides us with the expected level of shorting in the portfolio as a whole. This in turn allows mandate authors to form expectations as to the level of gearing that will likely be required over various investment views, consistent with the choice of benchmark and risk budget. By implication, again, mandate authors for long-only funds can use this as a metric for the sub-optimality of their choice of risk budget and benchmark: the more gearing is required for an optimal unconstrained fund under the same circumstances, the more inappropriate the choice of benchmark and risk budget is for a long-only fund.

Tables 2 and 3 provide some examples of these estimates under a variety of market conditions and target active risk levels for both a SWIX and a SWIX40 active fund respectively. Under cross-sectional volatility conditions (i.e. $\sigma\sqrt{1-\rho}$) that vary from 0,2 to 0,7, and risk budgets that vary from enhanced equity funds (σ_A=0,5%) to aggressive active funds (σ_A=4%), these tables describe the optimal, unconstrained likely level of gearing in the fund as a consequence of the likely short sales. For example, in a market where the cross-sectional volatility is 0,4 and the target active risk is 4%, the optimal unconstrained SWIX fund would expect 31,8% short sales (i.e. R31,80 of every R100 invested would be in short positions and R131,80 would be invested long). By contrast, the same requirements of a SWIX40 fund would require a very modest average of only 1,5% gearing confirming the more efficient long-only implementation of these requirements in a SWIX40 than a SWIX environment.

This analysis confirms the increased gearing allowance required for funds with decreasing cross-sectional dispersion in their market, increasing risk budgets and highly concentrated benchmarks with a large number of small securities. By implication, the detrimental effect of the long-only constraint on the active fund manager's ability to implement their investment views worsens under the same conditions.

Table 2: Sum of expected short positions in an unconstrained optimal active SWIX fund

	Target Active Risk (%)	0,5	1,0	1,5	2,0	3,0	4,0	5,0
Cross-sectional Volatility	0,20	4,5	12,4	21,7	31,8	53,3	75,9	99,2
	0,30	2,4	6,9	12,4	18,5	31,8	46,0	60,7
	0,40	1,5	4,5	8,2	12,4	21,7	31,8	42,3
	0,50	1,0	3,2	5,9	9,1	16,0	23,7	31,8
	0,60	0,7	2,4	4,5	6,9	12,4	18,5	25,0

Table 3: Sum of expected short positions in an unconstrained optimal active SWIX40 fund

	Target Active Risk (%)	0,5	1,0	1,5	2,0	3,0	4,0	5,0
Cross-sectional Volatility	0,20	0,0	0,3	0,7	1,5	3,8	7,0	10,7
	0,30	0,0	0,1	0,3	0,6	1,5	2,9	4,8
	0,40	0,0	0,0	0,1	0,3	0,7	1,5	2,5
	0,50	0,0	0,0	0,1	0,1	0,4	0,9	1,5
	0,60	0,0	0,0	0,0	0,1	0,3	0,6	1,0

4. CONCLUSIONS

The article began with a simple empirical measurement of the boundaries and asymmetries of the active equity fund manager's opportunity set given some typical South African equity benchmarks. We have shown that the long-only constraint, when applied to benchmarks with a large number of small members, implies a substantial asymmetry in the opportunity set of active weights, particularly when active weights are larger. The results also imply a substantial improvement to this opportunity set when small collective short positions are permitted.

Using the derivations published in Clarke *et al.* (2008) we describe how mandate authors could simulate the distribution of optimal investment weights to assist them in finding reasonable asset-level constraints that are consistent with their choice of benchmark and risk budget. By implication, mandate authors can use these techniques to determine the extent of the sub-optimality of certain security-specific restrictions, such as the long-only constraint.

Using a simplification of the risk matrix and two common South African equity benchmarks (SWIX and SWIX40), we illustrate the importance of the choice of benchmark and risk budget in determining the distribution of optimal investment weights in the portfolio. The more concentrated the benchmark and the higher the active risk target, the wider the distribution of asset weights will be and the more binding the typical restrictions an asset weightings will be.

Furthermore, constraints on short positions are more binding on assets with low-weightings in the benchmarks illustrating the asymmetric and sub-optimal effect of constraints that are applied equally to all assets in the investment universe.

Using the same techniques, mandate authors who are considering relaxing the long-only constraint can determine the likely optimal fund gearing or short-extension under various market conditions and risk budgets. The evidence presented here suggests that small short extensions (less than 5% overall) may substantially improve the capacity for large cap managers (SWIX40 benchmark) to achieve aggressive risk targets (4% active risk) in a more optimal way but that broader funds with benchmarks such as the SWIX would require substantial short-extension (to the order of 30%) to achieve the same risk budget efficiently.

The analysis presented in this article confirms the increased gearing allowance required for funds with decreasing cross-sectional dispersion in their market, increasing risk budgets and highly concentrated benchmarks with a large number of small securities. By implication, the detrimental effect of the long-only constraint on the active fund manager’s ability to implement their investment views worsens under the same conditions.

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