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Evaluation of GARCH-based models in value-at-risk estimation: Evidence from emerging equity markets

ABSTRACT

This paper evaluates the forecasting performance of a range of volatility models in Value-at-Risk estimation in the context of the Basle regulatory framework using stock index return data from a selection of emerging markets. It extends the current research in these economies by including a range of GARCH models and their long memory extension, in addition to some standard statistical methods often used by financial institutions. The results suggest that models with long memory or asymmetric effects or both are important considerations in providing improved VaR estimates that minimise occasions when the minimum capital requirement identified by the VaR process would have fallen short of actual trading losses. In addition, the results highlight the relevance Basel regulatory framework, and of using out-of-sample forecast evaluation methods for the identification of forecasting models that provide accurate VaR estimates.

1. INTRODUCTION

The diversification and return benefits provided by emerging markets have attracted significant investors' attention which, in turn, have led to significant portfolio equity inflows into these economies. In addition, these developments have also motivated the study of various aspects of stock return behaviour in these markets. An important and topical strand of recent empirical research has focused on the calculation of value-at-risk (VaR) in these markets.

VaR models were developed to estimate the exposure of a portfolio to market risk. This methodology focuses on the maximal potential losses of a portfolio (or institution) and derives from modern finance techniques which were developed in order to evaluate the risks of financial failure. More formally, the VaR of a portfolio can be defined as the maximum loss expected to occur over a target horizon with a given probability (Jorion, 2007).[1]

In addition, VaR has also emerged as standard quantitative measure of market risk within most financial institutions; moreover, this method also forms the basis for a host of risk controls (e.g., position limits and margin requirements) (IMF, 2007). From a regulatory perspective, the VaR framework has been advocated by the Bank of International Settlements (BIS) for financial institutions to use in the evaluation of their market risk, and, as a consequence, for setting market risk capital requirements (BIS, 1996).[2] This framework requires financial institutions to 'backtest' their in-house VaR models by comparing actual daily trading outcomes (i.e., profits or losses) with the estimated VaR and recording the number of 'exceptions' (i.e., days when the VaR estimate was insufficient to cover realised losses). The frequency of 'exceptions' then provides a basis for the regulatory concern and action (often in the form of penalties) to the relevant financial institution.

An important parameter in the calculation of VaR relates to the derivation of accurate asset-return volatility estimates (in particular, forecasts). The ubiquitous RiskMetrics model expresses the volatility as an exponentially weighted moving average of historical squared returns. However, this formulation goes against the substantial evidence of time-variation in the conditional volatility of financial asset returns (e.g., Bollerslev, Chou and Kroner, 1992). Furthermore, recent empirical work has found that many (high-frequency) financial time series exhibit long memory (or long-range dependence) in their volatility. Indeed, long memory in the volatility of emerging equity markets has been widely documented (e.g., Assaf and Cavalcante, 2005; DiSario, McCarthy and Saraoglu, 2008 and McMillan and Thupayagale, 2009).

Despite the extensive research into the empirical aspects of VaR estimation in the major international financial markets, little is known about these dynamics in the context of emerging securities markets. Recent research (Brooks and Persand, 2003; So and Yu, 2006; McMillan and Speight, 2007) have investigated the applicability and accuracy of a variety of volatility forecasting models in VaR calculation for Asian stock markets. However, there appears to be a gap in the extant literature with respect to VaR measurement within other emerging markets.

The aims of this paper are threefold. First, to evaluate the performance of a wider range of volatility forecasting models – from simple statistical models to a variety of conditional variance models and their long memory extensions - in estimating VaR of stock

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[1] Related to the concept of VaR is the notion of the minimum capital risk requirement (MCRR) which is, in turn, defined as the minimum amount of capital required to accommodate all but a specified (small) proportion of expected future losses.

[2] VaR is the measurement basis for the Basel Internal Models approach for setting market risk capital requirements (BIS, 1996).

market indices compared to many previous studies. Operational evaluation takes the form of backtesting volatility forecasts and exception reporting as enunciated by BIS guidelines (BIS, 1996). Second, to extend the empirical literature by considering data from a more diverse array of emerging markets. Focus is on markets for which evidence on VaR estimation appears to be limited. Attention is therefore on some African equity markets - Egypt, Kenya, Nigeria and South Africa – in addition to stock markets in Brazil, China, India, Russia and Turkey, along with the US as a benchmark comparator.[3] Third, to implement diagnostic tests to assess the robustness of the VaR estimates. The study employs the Kupiec LM test (1995) and the dynamic quantile (DQ) test of Engle and Manganelli (2004), which are tests of unconditional and conditional VaR accuracy. The former examines whether there is an excessive number of exceptions and the latter tests for autocorrelation in the sequence of exceptions.

The rest of this paper is organised as follows. Section 2 presents an introduction to VaR and describes its calculation (and evaluation) in the context of the Basle regulatory framework. Section 3 presents the methodological approaches used in the study for the estimation of VaR across various confidence levels. Special focus is given to the various empirical models used for forecasting volatility in the calculation of VaR, which range from standard statistical models to various conditional variance models and their long memory extensions. Furthermore, operational evaluation takes the form of calculation of the VaR failure rate such that the preferred model is identified in terms of minimising the number of exceptions (i.e., days when the VaR is insufficient to cover actual trading losses). Section 4 describes the data set and discusses the results from the VaR estimation. Section 5 presents some diagnostic tests to verifying the adequacy of the obtained results. Section 6 concludes this analysis.

2. CALCULATING AND EVALUATING MEASURES OF VAR

VaR models estimate the exposure of a portfolio to market risk. This quantitative technique has been developed for financial institutions primarily for the purposes of assigning capital to operational risk exposures in compliance with regulatory capital requirements (BIS, 1996). The introduction of JP Morgan's RiskMetrics model in 1994 brought the use of VaR into mainstream business practice. Furthermore, the BIS has sanctioned the use of VaR models for financial risk management purposes. Formally, VaR measures the worst expected loss of a portfolio over a target horizon at a given confidence level, due to an adverse movement in the relevant security price (Jorion, 2007). For instance, if a stock's (or a stock portfolio's) one-day estimated VaR is 10 million dollars, at a 95 percent confidence level, then, this means that a loss of 10 million dollars or more is expected on only five trading days out of 100.

There are various methods, or approaches, to measure VaR. Differences among these approaches arise from the model applied to the estimation of the expected changes in prices. Typically, these methods are divided into two categories: local valuation and full valuation, where the former method relies on the assumption of normality and the latter does not.[4]

While the assumption of normality has important disadvantages, Bams, Lehnert and Wolff, (2005) evaluate alternative distributional assumptions against more standard procedures. Their results suggest that more sophisticated tail modelling procedures entail VaR measures being estimated with a higher degree of uncertainty given the paucity of tail observations (ECB, 2007).[5] In addition, Jorion (1995a,b) argues that the parametric normal approach may be preferable even when normality does not hold.[6] Indeed, the RiskMetrics approach is conditional on the assumption of normality and has emerged as the primary quantitative measure of market risk within most financial institutions.

Therefore, in line with previous research and current professional practice, this study assumes asymptotic normality to characterise the underlying risk factors. The Basle Accord mandates that VaR be computed as the higher of the preceding daily calculation of VaR, or the average of estimated daily VaR over one quarter and that regulatory capital is between three and four times the VaR measure (the precise value being determined by the regulator's assessment of the accuracy of the financial institution's approach to VaR estimation). In addition, the Basel market risk framework prescribes that for purposes of establishing bank regulatory market risk capital, a 99 percent (one-tailed) confidence interval be used (using daily data) over a minimum sample period of one year (equivalent to 250 trading days), and that VaR estimates be updated at least every quarter (60 trading days).

[3] Egypt, Kenya, Nigeria and South Africa account for more than 90 percent of Africa's stock market capitalisation.

[4] The normality assumption simplifies VaR calculation since only the mean and the variance-covariance matrix of price changes are required to calculate the maximum loss within a given confidence interval.

[5] In contrast to the delta-normal method, Monte Carlo simulation requires less strong assumptions but is more computationally engaging.

[6] While it is widely recognised that VaR does not adequately capture episodes of extreme volatility and market illiquidity, the alternatives are typically data intensive, difficult to verify through backtesting, and hard to explain to senior management. Accordingly, financial institutions tend to complement VaR measures with stress tests to evaluate the impact of tail events (IMF, 2008).

Against this background, this paper follows previous studies for measuring VaR by employing the delta-normal specification which is expressed as

$$VaR = N_{\alpha}\sigma 3V \quad \dots (1)$$

where N_{α} is appropriate standard normal deviate (e.g., 2.326 for the 99 percent confidence level), σ is the volatility estimate or forecast, the number three represents the minimum regulatory Basle multiplicative factor and V is the initial portfolio value. While the Basel Accord prescribes a 99 percent probability, this study also examines performance at the 97,5 and 95 percent confidence levels for greater comparability and consistency with previous studies (N_{α} is 1.960 and 1.654, respectively).

3. VOLATILITY MODELLING AND FORECASTING

Accurate volatility estimates are essential for producing robust VaR estimates. This is especially true for the generated out-of-sample forecasts that are used to produce accurate VaR estimates. Accordingly, this study compares the forecast performance of a variety of volatility models with a view to determine which models deliver the most accurate volatility forecasts and, therefore, lead to superior VaR estimation.

The models investigated range from simple statistical models to a variety of standard GARCH models and their long memory extensions. In analysing the performance of the different models for producing accurate and reliable VaR estimates, this study examines how many times the actual loss exceeds the estimated VaR (referred to as an exception). Different estimation methods are tested in order to measure their impact on the accuracy of the estimated VaR. The RiskMetrics model forms the base model for this analysis since it is a standard tool to measure market risk within most financial institutions. Furthermore, the estimation modalities presented here are consistent with the Basle Committee rules for the calculation of VaR calculation techniques.

3.1 Statistical methods

Equally weighted moving average model

The most familiar measure of volatility is given by the variance or standard deviation of a time series. In order to estimate (or forecast) volatility using this method, all that is required is an estimate the unconditional (or long run) variance of the data. This is equivalent to estimating an equally weighted moving average model and is given by:

$$\sigma_t = T^{-1}\sum_{i=1}^{T}\sigma_{t-i} \quad \dots (2)$$

where

T is the moving average period (or rolling window).

In line with previous empirical studies, this paper starts by using the 5 most recent years of daily data to estimate the standard deviation over the evaluation sample period (which is set at one trading quarter), in line with the precepts of the Basle Committee. The sample is updated every 60 observations (i.e., this recalculation, in turn, provides the forecast for the next evaluation period and this process continues until the entire sample is exhausted). This procedure is applied to all the volatility models subsequently presented.

Semi-variance model

The semi-variance is similar to variance; however, it only considers observations below the mean (of a data set); as such, semi-variance corrects for problems of asymmetry and downside risk. By focussing on the negative fluctuations of an asset return series semi-variance estimates the average loss that a portfolio can incur. This in turn, implies that minimising semi-variance would reduce the probability of a large loss portfolio loss. In addition, the semi-variance approach is based on the notion of the lower partial moment, which has been analysed extensively in finance (e.g., Grootveld and Hallerbach, 1999); and it can be calculated as

$$\hat{\sigma}^2_{Lt} = \left[T_L / \left(T_L - 1\right)\right]\sum_{x_t<0} x_t^2 \quad \dots (3)$$

which provides an asymptotically unbiased and strongly consistent estimator for a sample of size T, where T_L denotes the number of observations for which the (mean-adjusted) squared return, $x_t^2 = \left(r_t - \tilde{\mu}\right)^2$.

Risk metrics model

JPMorgan's VaR methodology (i.e., RiskMetrics) was introduced in 1994. This method helped to popularise VaR estimation and, therefore, bring it into mainstream risk management practice. Under this method the volatility process is expressed as an exponentially weighted moving average (EWMA) which is given by:

$$\hat{\sigma}^2_t = \phi\hat{\sigma}^2_{t-1} + \left(1-\phi\right)\sigma^2_{t-1} \quad \dots (4)$$

where $\hat{\sigma}^2_t$ is the estimated (or forecast) variance, the smoothing parameter, ϕ, is such that $0 \le \phi \le 1$ and σ^2_{t-1} denotes the past observed volatility.

Previous research using this model suggests that a value of $\phi = 0{,}94$ be used since this value has been shown to deliver the most accurate volatility forecasts (e.g., Morgan, 1996; Fleming, Kirby and Ostdiek, 2001).

3.2 Models of changing variance

Following the seminal contributions of Engel (1982) and Bollerslev (1986), modelling of financial asset returns has been cast in the generalised autoregressive conditional heteroskedasticity (GARCH) framework.[7] For asset returns, the GARCH class of models involves the estimation of an equation for asset returns and a conditional variance (h_t) specification. The dynamics of h_t for a wide range of financial asset returns has been found to be adequately modelled as a GARCH(1,1) process.

These models allow the conditional variance to change over time as a function of past errors and volatility, leaving the unconditional (or long-run) variance constant. Under these models the returns process is generated as $r_t = \mu + \varepsilon_t$, where r_t is the returns process, μ the conditional mean, which may include autoregressive and moving average terms, and ε_t is the error term, which can be decomposed as $\varepsilon_t = z_t\sqrt{h_t}$ such that z_t is an idiosyncratic zero-mean and constant variance noise term, and h_t is the (time-varying conditional) volatility process to be estimated.

3.2.1 Short memory models

GARCH (p,q) model

In the standard GARCH (*p*, *q*) model, the conditional variance is given by:

$$h_t = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)h_t \qquad \ldots (5)$$

where

forecasts of volatility, are a weighted function the long run average, ω, the size of the previous period's shock (to volatility) ε_t^2, and past estimates of the conditional variance, h_t, and $\alpha(L)$ and $\beta(L)$ are polynomials of order *p* and *q*, respectively, expressed in terms of the lag operator.

In addition, the GARCH model has the property that the size of the innovations to the volatility process determines current volatility. In the basic GARCH (1,1) model, the effect of a shock on volatility decays exponentially over time and the speed of decay is measured by the extent of volatility persistence (which is reflected in the magnitude and significance of the summation of the α and β parameters). Furthermore, the sign of the shock, however, does not have any significance. The standard GARCH model is, therefore, symmetric in the sense that negative and positive shocks have the same effect on volatility.

EGARCH(p,q) model

However, returns are often found to have an asymmetric impact on volatility.[8] Stock-return volatility is frequently reported as being 'directional'- i.e., volatility responds asymmetrically to past negative and positive return shocks with negative shocks resulting in larger future volatilities. In other words, volatility is higher in a bear market than in a bull market. This feature gave rise to the development of GARCH models with asymmetric or 'leverage' effects.

In order to capture this asymmetry, Nelson (1990) proposed the exponential GARCH (EGARCH) process for the conditional variance. This model is formulated such that the leverage effect is exponential as the left hand side of the equation is the logarithm of the conditional variance. Formally, the conditional variance structure is given as:

$$\ln h_t = \omega + \left[1-\beta(L)\right]^{-1}\left[1+\alpha(L)\right]g(\eta_{t-i}) \qquad \ldots (6)$$
$$g(\eta_{t-i}) \equiv \gamma_1\eta_{t-i} + \gamma_2\left[|\eta_t| - E|\eta_t|\right]$$

where the weighted innovation $g(\eta_{t-i})$ is introduced to model the asymmetric relation between returns and volatility changes. $g(\eta_{t-i})$ captures both the size and sign effect of η_t where $\gamma_1\eta_{t-i}$ captures the sign effect and $\gamma_2\left[|\eta_t| - E|\eta_t|\right]$ depicts the size effect. Through this functional form, the EGARCH model is able to account for the asymmetric response of volatility to stock returns.

IGARCH (p,q) Model

[7]The interest surrounding volatility models has been motivated by their suitability in capturing the so-called 'stylised facts' of many financial time-series. Indeed, the GARCH models have been shown to be capable of capturing leptokurtosis, skewness and volatility clustering, which are the common attributes most often observed in high frequency financial time series data.

[8]Black (1976) proposed the 'leverage effect' to explain the asymmetry of volatility. Basically, when the price of a stock falls, its equity value also drops, and its leverage (or debt-to-equity ratio) increases. When leverage rises, the company is (typically) considered more risky and a higher degree of risk is (in finance typically) associated with higher volatility. Campbell and Hentschel (1992) provide an alternative explanation to the 'leverage effect'. They propose the 'volatility feedback hypothesis' which suggests that expected returns increase when stock price volatility increases (due to rises in future risk premia). If future dividends remain the same, the stock price should fall when volatility rises.

An Integrated GARCH model (or integrated in variance process) describes a nonstationary GARCH. This model was proposed by Engle and Bollerslev (1986) and implies that any shock to volatility is permanent and the unconditional variance is infinite. This is in contrast to the standard GARCH which assumes that shocks to volatility follow a rapid exponential decay.

The IGARCH process can be derived by expressing the GARCH (*p*,*q*) model in equation (5) as an autoregressive moving average or ARMA (*m*,*p*) process where *m* = max{*p*,*q*}, with *q* denoting the number of lags of the squared error term in the conventional GARCH (*p*,*q*) process, such that, $\left[1-\alpha(L)-\beta(L)\right]\varepsilon_t^2=\omega\left[1-\beta(L)\right]v_t$ where $v_t \equiv \varepsilon_t^2 - h_t$ and $\alpha(L)$ and $\beta(L)$ are lag polynomials. In order to satisfy the covariance-stationarity of the error process, all the roots of [1-α(*L*)-β(*L*)] and [1-β(*L*)] lie outside the unit circle. Should the autoregressive lag polynomial [1-α(*L*)-β(*L*)] contain a unit root, then the process is said to be integrated in variance. More precisely, the IGARCH model is given by

$$\varphi(L)(1-L)\varepsilon_t^2=\omega+\left[1-\beta(L)\right]v_t \qquad \text{… (7)}$$

where

$\varphi(L)=\left[1-\alpha(L)-\beta(L)\right](1-L)^{-1}$. The IGARCH model is also often given by its conditional variance representation (see Baillie, Bollerslev and Mikkelsen, 1996):

$$h_t=\omega\left[1-\beta(L)\right]^{-1}+\left(1-(1-\beta)^{-1}\alpha(L)(1-L)\right)\varepsilon_t^2 \qquad \text{… (8)}$$

In total, therefore, IGRACH processes have theoretically infinite variances, are not mean-reverting and shocks to volatility do not decay.

3.2.2 Long memory models

Long memory (or fractionally integrated) processes are distinct from both stationary and unit root processes in that they are persistent, but also mean-reverting. These processes are such that the conditional variance is characterised by a slow hyperbolic rate of decay from volatility shocks. Against this background, Baillie *et al.* (1996) proposed the fractionally integrated GARCH (FIGARCH) model by combining the fractionally integrated process for the mean with the basic GARCH process for the conditional variance.

FIGARCH (p, d, q) model

The general specification of the FIGARCH (*p*,*d*,*q*) model can be derived by introducing the fractional integration parameter, *d*, into the IGARCH model presented in equation (7). This means that the FIGARCH model can be written as:

$$\varphi(L)(1-L)^d\varepsilon_t^2=\omega+\left[1-\beta(L)\right]v_t \qquad \text{… (9)}$$

where,

$d\in(0,1)$ and all the roots of $\varphi(L)$ and $\left[1-\beta(L)\right]$ lie outside the unit circle.

From equation (9), the FIGARCH model reduces to the GARCH and IGARCH models when *d* is one and zero, respectively. The long memory process in the FIGARCH model arises because *d* is no longer constrained to an integer, but rather can take on fractional values. The conditional variance specification of the FIGARCH is also given below:

$$h_t=\omega+\left[1-\beta(L)\right]^{-1}+\left\{1-\left[1-\beta(L)\right]^{-1}\varphi(L)(1-L)^d\right\}\varepsilon_t^2 \qquad \text{… (10)}$$

For 0 < *d* < 1 the FIGARCH model implies a slow hyperbolic rate of decay for a given shock on the forecast of the future conditional variance, reflecting the highly persistent, but ultimately transitory effect of shocks to volatility.

FIEGARCH (p, d, q) model

The last model this study considers is the long memory counterpart to the EGARCH described in equation (6). The fractionally integrated EGARCH (FIEGARCH) (*p*, *d*, *q*) model proposed by Bollerslev and Mikkelsen (1996) incorporates the asymmetric behaviour of the EGARCH model (Nelson, 1991) such that the conditional variance process of the FIEGARCH can be defined by:

$$\ln h_t=\omega+\varphi(L)^{-1}(1-L)^{-d}\left[1+\varsigma(L)\right]g(\eta_{t-i}) \qquad \text{…(11)}$$

where

the fractional integration term (i.e., *d*) in equation (11) captures the long memory properties of the volatility process (Bollerslev and Mikkelsen, provide a more detailed discussion of the model's derivation).

In addition, $g(\eta_{t-i})=\gamma_1\eta_{t-i}+\gamma_2\left(|\eta_t|-E|\eta_t|\right)$ as in the EGARCH model captures both the size and sign effect associated with the asymmetric dynamics of stock-return volatility. The FIEGARCH model, therefore, entails the FIGARCH specification to modelling long memory with the EGARCH process for capturing asymmetric dynamics (i.e., the leverage effect). Furthermore, the log form of the FIEGARCH model allows estimation of the model without imposing any non-negativity conditions. In line with the other long memory models considered, the FIEGARCH model captures the highly persistent, but ultimately mean-reverting behaviour of shocks to the conditional variance process.

4. DATA AND VAR EVALUATION RESULTS

The data are obtained from Bloomberg and span the period from January 1, 1998 through to January 31, 2010, and comprise 3037 observations. The daily stock return is calculated as $r_t = \ln p_t - \ln p_{t-1}$ where $\ln p_t$ denotes the log of the stock index on day t. In this analysis, the daily closing values of ten equity markets are used.

Data from Brazil (Bovespa Stock Index), China (Shanghai Stock Exchange Composite Index), Egypt (Hermes Financial Index), India (Bombay Stock Exchange Sensitive Index), Kenya (Nairobi Stock Exchange 20), Nigeria (Nigeria Stock Exchange All Share Index), Russia (Russian Composite Index), South Africa (FTSE/JSE Africa All Share Index) and Turkey (Instanbul Stock Exchange National 100 Index) are used. In addition to these equity markets, data for the United States (S&P 500) is included as a benchmark comparator.

The diversity of emerging equity markets is illustrated in Table 1, which presents data on key market indicators, specifically, the number of listed companies, market capitalisation, market size relative to the economy and liquidity (proxied by the turnover ratio). Two distinctive features are immediately apparent.

First the African stock markets (with the exception of South Africa) are small in terms of the number of listed companies; market capitalisation and the stock market size relative to the overall economy. The emerging markets surveyed are also significantly less liquid relative to the benchmark comparator (i.e., the US). Moreover, South Africa, the largest and most developed of the African stock markets is shown to be less liquid relative to the other emerging markets examined by this study.

What is more, Nagayasu (2003) argues that equity volatility in less developed markets may show long memory behaviour given the lack of breadth and depth of these markets coupled with their less mature institutional and regulatory frameworks. Indeed, this conclusion may be relevant to the smaller African stock markets in where capital markets are still very nascent and some of the larger emerging equity markets where turnover ratios are low relative to the US and indeed some of the more liquid emerging markets (such as Taiwan and Hong Kong) which have turnover ratios in excess of 100 percent.

Following previous research (e.g., McMillan and Speight, 2007), we use the first five years of data (1256 observations) for initial model parameter estimation and the remaining observations are used to derive volatility forecasts in order to construct and evaluate VaR measures, given the models described in Section 3.[9] Given the guidelines specified by the Basle market risk framework detailed in Section 2 and the estimation methodology outlined in Section 3, initial volatility forecasts are, accordingly, derived over intervals over 60 (trading) days. Then the initial estimation sample is rolled forward and the model are updated every 60 observations before the next round of volatility forecasts are generated. This iterative procedure results in 30 sub-samples of 60 days over which VaR performance is evaluated. Model parameters are calculated using maximum likelihood estimation methods and the estimation package is G@RCH 4.0 Ox developed by Laurent and Peters (2005).

The evaluation criteria are twofold. First, we consider the ‘in-sample’ VaR failure rates which represent the VaR measures constructed using the fitted value of the volatility measure from the relevant models. Second, we examine of the ‘out-of-sample’ VaR failure rates calculated from VaR measures estimated using forecast values from the models we have analysed. The out-of-sample fit provides a forward-looking measure and, therefore, is more useful guide to the selection of the appropriate volatility model, which enhances model selection compared to model selection based on in-sample fit. Indeed, out-of-sample model selection may be of practical benefit to risk managers as they have a basis on which to make their VaR calculations operational.

Table 1 presents the in-sample VaR failure rates for the eight volatility models applied to the emerging equity markets, reported at the 95 percent, 97,5 percent and 99 percent probability levels. These results are summarised in terms of the percentage number of days for which there was an exceedance of the VaR estimate in the backtest over the 36 sub-samples, which in turn, indicates that the calculated VaR is insufficient to cover trading losses.

[9] While a variety of sample period lengths have been used or recommended in empirical work, the Basle Committee rules prescribe a minimum period of at least one year for initial parameter estimation.

Table 1: Stock market indicators of emerging markets in 2009

	Number of listed domestic companies	Market capitalisation (USD bn.)	Market capitalisation of listed companies (percent of GDP)	Turnover ratio (%)
Brazil	425	1 340,0	167,4	93,2
China	1 700	5 010,0	84,7	57,5
Egypt	306	91,1	112,6	46,9
India	4 946	1 230,0	89,3	89,5
Kenya	53	11,0	73,9	23,8
Nigeria	216	33,4	78,6	18,6
Russia	333	861,1	144,4	55,9
South Africa	411	805,6	312,6	48,7
Turkey	315	234,9	135,3	64,6
US	5 982	11 707,5	211,2	178,4

Source: World Bank; Bloomberg and author's calculations, USD and GDP denotes United States dollar and gross domestic product, respectively.

The findings of this exercise are diverse. For instance, for India, Nigeria, and South Africa, the FIGARCH model dominates performance across the three extreme percentiles this study considered. For China and Kenya, the semi-variance model is preferred at all probability levels. For Brazil and Egypt, the IGARCH and FIEGARCH processes deliver the most accurate VaR measures across all three probability levels considered. For Russia, Turkey and the US, the results suggest the selection of the preferred model is sensitive to the specification of the probability level. When the Basle Committee criterion is applied to these stock market indices (i.e., the 99 percent probability level) the results show that the RiskMetrics (Russia), FIGARCH (Turkey) and EGARCH (US) models are preferred.

These results suggest the relevance of conditional variance models – especially those incorporating a long memory component – provide the most robust VaR method compared to all the other methods investigated. Among the simple statistical methods, the semi-variance method is shown to generate the best performance. The ubiquitous RiskMetrics model is preferred only for Russia – at both 99 and 97,5 percent probability levels. Despite the popularity of the RiskMetrics approach, this study shows this model specification is often outperformed by other models. In addition, the standard GARCH model is found to be of even more limited use.

Table 2 presents the out-of-sample VaR failure rates for the eight volatility forecasting models for each of the stock indices considered. As discussed previously, these tests are performed at the 99 percent probability level as prescribed by the Basel framework. Also, consistent with previous research, evaluation is also carried out at the, 97,5 and 95 percent probability levels.

The findings of this investigation echo those of the previous exercise with some significant exceptions. For example, for India and Turkey, the FIGARCH model generates the lowest VaR failure rates, across the three probability levels considered on an out-of-sample basis (similar to the in-sample result). Meanwhile, in the case of Brazil, the IGARCH model was preferred on an in-sample basis; however, on an out-of-sample basis, the FIGARCH model provides more accurate results across all three probability levels.

Similarly, using out-of-sample VaR failure rates as the criterion, the FIEGARCH model is preferred for South Africa, given that it delivers the minimum number of exceptions, compared to the previous (in-sample) result where the FIGARCH model dominated performance across all probability levels. While the FIEGARCH model is preferred it is noteworthy that the FIGARCH model is the second best performing model for South Africa at the 99 and 97,5 percent probability levels, and the third best performing model at the 95 percent level, suggesting that this model (FIGARCH) is a close substitute of the FIEGARCH insofar as delivering low VaR failure rates.

Table 2: Value at risk failure rates – In-sample

Model	99%	97,5%	95%	99%	97,5%	95%
		Brazil			China	
SD	0,0212	0,0381	0,0453	0,0182	0,0260	0,0372
RM	0,0163	0,0219	0,0336	0,0151	0,0245	0,0411
SV	0,0195	0,0303	0,0407	0,0139*	0,0224*	0,0355*
G	0,0144	0,0280	0,0395	0,0186	0,0224*	0,0359
EG	0,0163	0,0386	0,0472	0,0195	0,0285	0,0403
IG	0,0112*	0,0188*	0,0255*	0,0171	0,0267	0,0440
FIG	0,0112*	0,0194	0,0259	0,0155	0,0224*	0,0361
FIEG	0,0142	0,0261	0,0418	0,0155	0,0262	0,0355*
		Egypt			India	
SD	0,0256	0,0494	0,0609	0,0158	0,0232	0,0319
RM	0,0152	0,0272	0,0329*	0,0160	0,0232	0,0330
SV	0,0397	0,0583	0,0618	0,0157	0,0246	0,0329
G	0,0806	0,1059	0,1127	0,0153	0,0246	0,0298*
EG	0,0294	0,0392	0,0526	0,0165	0,0251	0,0326
IG	0,0219	0,0559	0,0621	0,0179	0,0251	0,0342
FIG	0,0240	0,0419	0,0522	0,0142*	0,0230*	0,0298*
FIEG	0,0131*	0,0262*	0,0329*	0,0158	0,0244	0,0316
		Kenya			Nigeria	
SD	0,0163	0,0317	0,0402	0,0282	0,0336	0,0491
RM	0,0166	0,0312	0,0411	0,0163	0,0233	0,0322
SV	0,0118*	0,0253*	0,0324*	0,0181	0,0279	0,0349
G	0,0126	0,0281	0,0377	0,0163	0,0261	0,0376
EG	0,0137	0,0281	0,0398	0,0181	0,0318	0,0422
IG	0,0164	0,0278	0,0405	0,0138*	0,0273	0,0318*
FIG	0,0118*	0,0265	0,0358	0,0138*	0,0263*	0,0318*
FIEG	0,0153	0,0264	0,0324*	0,0165	0,0272	0,0386
		Russia			South Africa	
SD	0,0275	0,0337	0,0483	0,0184	0,0258	0,0319
RM	0,0149*	0,0228*	0,0379	0,0144	0,0239	0,0324
SV	0,0153	0,0268	0,0377	0,0115	0,0286	0,0328
G	0,0190	0,0285	0,0332*	0,0192	0,0253	0,0329
EG	0,0161	0,0236	0,0332*	0,0110	0,0281	0,0373
IG	0,0157	0,0232	0,0394	0,0115	0,0219	0,0288*
FIG	0,0153	0,0228*	0,0389	0,0104*	0,0206*	0,0288*
FIEG	0,0185	0,0259	0,0417	0,0104*	0,0209	0,0294
		Turkey			US	
SD	0,0146	0,0272	0,0363	0,0184	0,0230	0,0291
RM	0,0137	0,0254	0,0381	0,0193	0,0239	0,0302
SV	0,0169	0,0222	0,0387	0,0147	0,0230	0,0292
G	0,0143	0,0240	0,0412	0,0168	0,0256	0,0372
EG	0,0151	0,0286	0,0326	0,0119*	0,0262	0,0305
IG	0,0142	0,0219*	0,0321*	0,0168	0,0229	0,0346
FIG	0,0131*	0,0238	0,0321*	0,0125	0,0222	0,0281*
FIEG	0,0136	0,0283	0,0336	0,0125	0,0207*	0,0281*

Note: VaR failure rates give the proportion of times VaR is exceeded in the sample, '*' indicates the preferred model, SD, standard deviation; RM, RiskMetrics; SV, semi-variance; G, generalised autoregressive conditional heteroskedasticity (GARCH); EG, exponential GARCH; IG, integrated GARCH; FIG, fractionally IG; FIEG, fractionally integrated EGARCH.

In the case of Kenya, the semi-variance model continues to be the preferred model even on an out-of-sample basis, across all three probability levels. In contrast, this consistency does not hold for China, where the semi-variance model is preferred at only the 99 and 97,5 percent confidence levels, while at the 95 percent level the EGARCH model is preferred. For Russia, the RiskMetrics model is preferred at the 99 and 97,5 percent probability levels on an out-of-sample basis; while, the FIEGARCH model is selected at the 95 percent probability level. In contrast, on an in-sample basis, the RiskMetrics approach is preferred across the board for Russia. Meanwhile, for the US, the FIEGARCH model is preferred at the 99 and 95 percent probability level on an out-of-sample basis while the EGARCH is preferred at the 97,5 percent probability level. This differs slightly from the in-sample case where the EGARCH model minimises the number of exceptions at the 99 percent probability level; while, the FIEGARCH model is preferred at the 97,5 and 95 percent probability levels.

Table 3: Value at risk failure rates – Out-of-sample

Model	99%	97,5%	95%	99%	97,5%	95%
		Brazil			China	
SD	0,0226	0,0385	0,0378	0,0188	0,0273	0,0391
RM	0,0153	0,0250	0,0363	0,0172	0,0259	0,0383
SV	0,0176	0,0269	0,0362	0,0154*	0,0246*	0,0329
G	0,0166	0,0285	0,0395	0,0182	0,0275	0,0379
EG	0,0171	0,0252	0,0404	0,0195	0,0261	0,0329*
IG	0,0164	0,0248	0,0355*	0,0189	0,0274	0,0318
FIG	0,0146*	0,0237*	0,0355*	0,0163	0,0252	0,0384
FIEG	0,0181	0,0263	0,0418	0,0174	0,0288	0,0341
		Egypt			India	
SD	0,0272	0,0346	0,0439	0,0268	0,0319	0,0407
RM	0,0212	0,0302	0,0391	0,0197	0,0266	0,0344
SV	0,0270	0,0327	0,0402	0,0184	0,0253	0,0362
G	0,0397	0,0419	0,0527	0,0172	0,0251	0,0317
EG	0,0212	0,0334	0,0411	0,0193	0,0257	0,0386
IG	0,0212	0,0341	0,0359	0,0189	0,0244	0,0309
FIG	0,0208	0,0312	0,0375	0,0152*	0,0231*	0,0312*
FIEG	0,0184*	0,0269*	0,0347*	0,0161	0,0267	0,0341
		Kenya			Nigeria	
SD	0,0208	0,0277	0,0384	0,0291	0,0381	0,0429
RM	0,0391	0,0418	0,0546	0,0222	0,0316	0,0382
SV	0,0142*	0,0222*	0,0338*	0,0171	0,0251	0,0391
G	0,0161	0,0257	0,0331	0,0144	0,0284	0,0323
EG	0,0138	0,0279	0,0361	0,0150	0,0290	0,0371
IG	0,0175	0,0251	0,0338	0,0144*	0,0272*	0,0332*
FIG	0,0151	0,0240	0,0395	0,0251	0,0374	0,0461
FIEG	0,0184	0,0232	0,0400	0,0295	0,0338	0,0427
		Russia			South Africa	
SD	0,0229	0,0315	0,0372	0,0286	0,0352	0,0428
RM	0,0175*	0,0254*	0,0339	0,0190	0,0264	0,0330
SV	0,0183	0,0280	0,0388	0,0147	0,0271	0,0356
G	0,0204	0,0287	0,0394	0,0274	0,0317	0,0419
EG	0,0181	0,0294	0,0353	0,0152	0,0256	0,0371
IG	0,0186	0,0285	0,0359	0,0154	0,0258	0,0343
FIG	0,0179	0,0291	0,0381	0,0140	0,0244	0,0343
FIEG	0,0186	0,0273	0,0331*	0,0136*	0,0239*	0,0315*
		Turkey			US	
SD	0,0184	0,0281	0,0344	0,0248	0,0335	0,0442
RM	0,0203	0,0267	0,0317	0,0435	0,0513	0,0558
SV	0,0262	0,0295	0,0339	0,0219	0,0351	0,0420
G	0,0193	0,0279	0,0375	0,0238	0,0310	0,0378
EG	0,0189	0,0265	0,0393	0,0191	0,0222*	0,0315
IG	0,0175	0,0286	0,0388	0,0193	0,0233	0,0332
FIG	0,0142*	0,0255*	0,0312*	0,0169	0,0264	0,0380
FIEG	0,0154	0,0276	0,0350	0,0166*	0,0229	0,0310*

Note: VaR failure rates give the proportion of times VaR is exceeded in the sample, '*' indicates the preferred model, SD, standard deviation; RM, RiskMetrics; SV, semi-variance; G, generalised autoregressive conditional heteroskedasticity (GARCH); EG, exponential GARCH; IG, integrated GARCH; FIG, fractionally IG; FIEG, fractionally integrated EGARCH.

The in-sample and out-sample VaR failure rates point to a number of considerations. First, The ubiquitous RiskMetrics approach, while widely used, is shown to be of limited robustness in the stock markets considered in this study insofar as minimising the number of exceptions. Indeed, evidence obtained from this analysis suggests that the RiskMetric model is appropriate for only one equity index: Russia. Second, the standard deviation and basic GARCH specifications are generally outperformed across the board by the other models considered in this study. Third, in-sample model accuracy (i.e., in-sample VaR failure rate) does not imply out-sample model accuracy (i.e., out-of-sample VaR failure rate). For example, in the case of Brazil and South Africa, the IGARCH and FIGARCH models are preferred on an in-sample basis, while the FIGARCH and FIEGARCH models yield more accurate results on an out-of-sample basis.

5. DIAGNOSTIC TESTS

This study uses the standard Kupiec Lagrange Multiplier (LM) test (1995), which determines if the VaR model is correctly specified, such that, the number of exceptions occur at the specified rate. The likelihood ratio (LR) test of the null hypothesis is given by

$$LR = -2\log\left[\left(1-\hat{p}\right)^{n-x}\left(\hat{p}\right)^{x}\right] + 2\log\left[\left(1-\frac{x}{n}\right)^{n-x}\left(\frac{x}{n}\right)^{x}\right] \quad \ldots (10)$$

where n is the sample size and x is the number of failures in the sample. This test has a chi-square distribution with one degree of freedom.

In addition, this study considers the Dynamic Quartile (DQ) test developed by Engle and Manganelli (2004), which is used to check if the exceptions are independent and identically distributed. In order to test for this, they define the sequence

$$\mathrm{Hit}_k = \mathrm{I}\left(r_k \langle -\mathrm{VaR}_k\right) - \alpha \qquad \ldots (11)$$

such that this sequence assumes the value $(1-\alpha)$ every time returns, r_k, are less than the VaR quantile and $-\alpha$ otherwise and $E(\mathrm{Hit}_k)=0$. Hit_k should be uncorrelated with its own lagged values and with VaR_k and must have an expected value of zero.[10]

The results of the diagnostic tests are reported in Table 4. The results mostly indicate that the preferred models are well specified. The results of the Kupiec and DQ tests suggest rejection of the respective null hypotheses, i.e., an excessive number of exceptions or autocorrelation in the sequence of exceptions, respectively. However, for India and Kenya the results suggest that the null hypothesis cannot be rejected at the 95 percent probability level; and for Russia, the results are ambiguous at the 99 and 97,5 percent probability levels in that the test result is sensitive to the choice of the stipulated probability level. Nonetheless, in total, these diagnostic checks point to adequate model specification.

6. CONCLUSION.

An important and topical application of volatility modelling concerns the estimation of VaR. Against this background, this paper evaluates the performance of eight volatility (forecasting) models in calculating VaR for a range of emerging markets in the context of the Basle regulatory framework. Moreover, this study differs in several respects from previous endeavours because it firstly examines a broader class of GARCH-type volatility forecasting models than many previous studies. Secondly, it focuses on emerging markets –including South Africa, Egypt, Nigeria and Kenya, which account for more than 90 percent of Africa total market capitalisation. And thirdly, this study performs diagnostic checks to assess the robustness of the findings.

On an in-sample basis the results are mixed across simple statistical models and conditional variance models. The semi-variance model is preferred in China and Kenya, while the RiskMetrics approach is preferred for Russia; while, the rest of the results indicate that a range of conditional variance models especially, IGARCH, and long memory models are preferred. It is notable that the standard GARCH (1,1) model is outperformed by the rival models suggesting that this common specification is of limited value in the context of the markets considered in this study.

However, of greater interest and practical value are the out-of-sample VaR failure rates which provide a framework that can be employed by both regulators and risk managers. This study presents evidence highlighting the viability of long memory models.

For instance, the FIGARCH model delivers the most accurate VaR measures for Brazil, India, Nigeria and Turkey on an out-of-sample basis. For Egypt, South Africa and the US, the FIEGARCH process delivers the most accurate VaR estimates. These results suggest that models that incorporate a long memory or asymmetric and long memory component help deliver superior VaR performance.

Models with these attributes features deliver the most accurate VaR measures for seven of the ten stock markets evaluated. Meanwhile, in terms of the simple statistical methods utilised, the results of this analysis show that the semi-variance method delivers the most accurate VaR measures for China and Kenya across the three probability levels considered.

These results further emphasise the importance of models which can account for asymmetry in VaR estimation. The RiskMetrics method is shown to outperform all other models in delivering accurate VaR measures for Russia. These results show that a 'one-size fits all' approach to computing VaR is not always appropriate. Indeed, the ubiquitous RiskMetrics model is shown to outperform all other models only in the case of Russia. This means that risk managers, regulators and investors need to closely examine the volatility profile of their respective markets and base their VaR estimation on the specific dynamics which obtain in the relevant market.

[10] In order to perform this test, the sequence Hit_k is regressed on its lagged values and the current value of VaR. Specifically, the DQ test statistic is calculated as $DQ = \hat{\beta}' X' X \hat{\beta} / \alpha(1-\alpha)$ where X is a vector of independent variables and $\hat{\beta}$ is the vector of OLS coefficient estimates. The test statistic is chi-squared distributed with the degrees of freedom corresponding to the number of parameters.

Table 4: Kupiec and dynamic quantile results

Country	Model	99%	97,5%	95%
Brazil	FIGARCH	0,023	0,034	0,019
		(0,030)	(0,026)	(0,085)
China	Semi-Variance	0,062	0,038	0,053
		(0,055)	(0,11)	(0,061)
Egypt	FIEGARCH	0,088	0,025	0,073
		(0,039)	(0,021)	(0,44)
India	FIGARCH	0,065	0,037	0,894
		(0,12)	(0,083)	(0,33)
Kenya	Semi-Variance	0,077	0,0512	0,211
		(0,414)	(0,058)	(0,820)
Nigeria	IGARCH	0,056	0,024	0,008
		(0,060)	(0,033)	(0,027)
Russia	RiskMetrics	0,421	0, 515	0,011
		(0,539)	(0,142)	(0,052)
South Africa	FIGARCH	0,052	0,027	0,004
		(0,047)	(0,032)	(0,047)
Turkey	FIGARCH	0,042	0,518	0,0430
		(0,080)	(0,161)	(0,007)
US	FIEGARCH	0,044	0,065	0,313
		(0,086)	(0,012)	(0,038)

Notes: Entries are *p*-values associated with the Kupiec (1995) LM test, which tests the equality of the empirical failure rate to the specified statistical level; while, those in brackets are the *p*-values associated with the DQ test for autocorrelation in VaR exceptions, For China and US, the EGARCH is preferred at the 95% and 97,5% probability levels, respectively, For Russia, the FIEGARCH model is preferred at the 95% probability level,

The study shows that stock-return volatility in emerging markets is typically characterised by a long memory, or asymmetric effects or both. These features in turn can be exploited to deliver accurate VaR measures. In addition, these results may suggest that long memory behaviour deriving perhaps from the comparatively less liquidity in emerging markets (or more generally the structural make-up of these economies) may drive stock return-volatility in these markets. Similarly, the widespread use of leverage by investors (and hence the relevance of volatility models capturing asymmetries in stock return volatility) may be prevalent in the markets considered in this study.

However, the case of the benchmark comparator (i.e., the US) is somewhat curious since models incorporating both leverage and long memory generate superior volatility forecasts and hence the most accurate VaR measures. The US is the most liquid and developed market in the sample. The finding that models incorporating leverage generate superior volatility measures is not surprising, since levering is widely used by investors to increase returns on equity capital. However, the relevance of long memory models may be a specious, perhaps reflecting structural change in the underlying data (e.g., the impact of major financial crisis) which this study did not disentangle.

Furthermore, the results appear to suggest that the RiskMetrics model is of limited relevance in VaR calculation in emerging markets, while the standard GARCH model is of even more limited applicability. The findings may provide guidance on more effective prudential standards for operational risk measurement and, as such, may help to ensure adequate capitalisation and mitigate the probability of financial vulnerability. The results highlight the importance of using out-of-sample forecasting techniques and the stipulated probability level for the identification of methods that minimise the occurrence of VaR exceptions, which is a key element of a financial safety-and-soundness strategy .

Finally, while VaR was calculated in the context of emerging markets, further analysis could be conducted in a number of directions. First, these results can be augmented by a stress testing exercise in order to identify extreme events that could trigger catastrophic losses in equity investments. Second, and in view of the relative illiquidity of emerging markets, VaR measures that incorporate liquidity risk (especially in the context of African stock markets that are characterised by illiquidity linked to non-trading effects) may be beneficial for both regulators and risk managers.

REFERENCES

Assaf A and Cavalcante J. 2005. Long range dependence in the returns and volatility of the brazilian stock market. *European Review of Economics and Finance*, 5: 5-20.

Baillie RT. 1996. Long memory processes and fractional integration in econometrics. *Journal of Econometrics*, 73(1): 5-59.

Baillie RT, Bollerslev T and Mikkelsen HO. 1996. Fractionally integrated generalised autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 74(1): 3-30.

Bali T. 2003. An extreme value approach to estimating volatility and value at risk. *Journal of Business*, 76: 83-108.

Bams D, Lehnert T and Wolff CCP. 2005. An evaluation framework for alternative var-models. *Journal of International Money and Finance*, 24: 944-58.

Bank for International Settlements. 1996. Amendment to the capital accord to incorporate market risks. *Basel: Basel Committee on Banking Supervision*.

Bank for International Settlements. 2005. Committee on Global Financial System (CGFS), stress testing at major financial institutions: survey results and practice, *CGFS Publication No. 24* (Basel, January).

Bank of England. 2007. Assessing the sensitivity of value-at-risk to measures of stress. *Financial Stability Report*, 21: 33-40.

Bervas A. 2006. Market liquidity and its incorporation into risk management. *Financial Stability Review*, 8: 63-79.

Black F, 1976, Studies in stock price volatility changes, *Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Section, American Statistical Association*: 177-81.

Bollerslev T. 1986. Generalised autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31: 307-27.

Bollerslev T, Chou RY and Kroner KF. 1992. ARCH modelling in finance. *Journal of Econometrics*, 52: 5-59.

Bollerslev T and Mikkelsen HO. 1996. Modelling and pricing long memory in stock market volatility. *Journal of Econometrics*, 73: 151-84.

Brooks C and Persand G. 2003. The effect of asymmetries on stock index return value-at-risk estimates. *Journal of Risk Finance*, 4: 29-42.

Campbell J and Hentschel L. 1992. No news is good news: an asymmetric model of changing volatility in stock returns. *Journal of Econometrics*, 31: 281-318.

Christie AA. 1982. The stochastic behaviour of common stock variances: value, leverage and interest rate effects. *Journal of Financial Economics*, 10: 407-32.

Danielsson J, Jorgensen BN, Sarma M and De Vries CG 2006. Comparing downside risk measures for heavy tailed distributions. *Economics Letters,* 92: 202-08.

Ding Z, Granger CWJ and Engle RF. 1993. A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, 1: 83-106.

DiSario R, Li H, McCarthy J and Saraoglu H. 2008. Long memory in the volatility of an emerging equity market: the case of Turkey. *Journal of International Financial Markets, Institutions & Money*, 18: 305-12.

Engle RF. 1982. Autoregressive conditional hetereoskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50: 987-1007.

Engle RF and Bollerslev T. 1986. Modelling the persistence of conditional variances, *Econometric Reviews*, 5: 1-50.

Engle RF and Manganelli S. 2004. CAViaR: conditional autoregressive value at risk by regression quantiles. *Journal of Business and Economic Statistics*, 22: 367-81.

European Central Bank. 2007. Market risk measurement: beyond value at risk. *Financial Stability Review* June: 108-10.

Grootveld H and Hallerbach W. 1999. Variance versus downside risks: is there really that much difference? *European Journal of Operational Research*, 114: 304-19.

International Monetary Fund. 2007. Do market risk management techniques amplify systemic risks? *Global Financial Stability Report*, World Economic and Financial Surveys (Washington).

Jorion P. 1995a. Predicting volatility in the foreign exchange market. *Journal of Finance*, 50: 507-28.

Jorion P. 1995b. *Big Bets Gone Bad: Derivatives and Bankruptcy in Orange County*. San Diego: Academic Press.

Jorion P. 2007. *Value at Risk*, (3rd Edition). New York: McGraw-Hill.

Josephy NH and Aczel AD. 1993. A statistically optimal estimator of semi-variance. *European Journal of Operational Research*, 67: 267-71.

Kupiec P. 1995. Techniques for verifying the accuracy of risk management models. *Journal of Derivatives*, 2: 73-84.

Laurent S and Peters JP. 2005. G@ARCH 4,0 estimating and forecasting ARCH models. *Timberlake Consultants*.

McMillan DG and Speight AEH. 2007. Value-at-risk in emerging equity markets: comparative evidence for symmetric, asymmetric, and long memory GARCH models, *International Review of Finance*, 7: 1-19.

McMillan DG and Thupayagale P. 2009. The efficiency of african equity markets. *Studies in Economics and Finance,* 26(4): 275-92.

Morgan JP, 1995, 1996, 1997, *RiskMetrics: Technical Document*, New York: Morgan Guaranty Trust Company.

Nagasayu J. 2003. The efficiency of the Japanese equity market. *International Finance Review*, 4: 155-171.

Nelson DB. 1991. Conditional heteroscedasticity in asset returns: a new approach. *Econometrica*, 59: 347-70.

Poon S-H. 2005. *A Practical Guide to Forecasting Market Volatility*. New York: Wiley.

So MKP and Yu PLH. 2006. Empirical analysis of GARCH models in value at risk estimation. *Journal of International Financial Markets, Institutions & Money*, 16: 180-97.

Thupayagale P. 2010. Essays in long memory: evidence from African stock markets. *PhD Thesis*. University of St Andrews (Scotland).