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Pricing of single stock futures and dividend risk

ABSTRACT

In this paper we consider the fair pricing of single stock futures (SSFs) and the effect of dividend risk on the dividend compensation component in the pricing formulas. SSF valuation is subject to the pricing of *discrete* cash dividends (not percentages or dividend yields) in the underlying stock. Discrete cash dividends present modelling challenges which are not present when dividends are in the form of yields or percentages. Problems are created for market-makers and investors when the actual cash dividend is different from that predicted by analysts and used for pricing. We propose a new model for the fair price of a single stock futures contract which addresses dividend uncertainty.

1. INTRODUCTION

Single Stock Futures (abbreviated as SSFs) are exchange-traded derivatives with one individual share as the underlying asset. This share can provide a discrete cash dividend during the life of the futures contract.

We discuss the pricing formula of an SSF, the dividend component therein, and dividend uncertainty. This uncertainty arises because of the difference between the assumed dividend amount used in the pricing and the actual declared dividend. Banks dealing in SSFs can be arbitraged (Jeffery, 2003), and investors can be similarly disadvantaged by mis-pricing (Pengelly, 2008). A variety of products have been launched to deal with this: dividend futures in particular are sold in conjunction with SSFs to try and eliminate dividend risk.

Apart from creating new dividend-based products, the modelling of specifically discrete cash dividends in pricing formulas is an important factor in the valuation of financial products contingent on dividend-paying assets. For example, different models for the pricing of options on stocks with discrete dividends, and the inconsistency of such models, have been well studied (see references in Section 5). But, to our knowledge, the construction of new pricing models for SSFs has not been addressed. We propose an alternative formula for modelling discrete cash dividends in valuing SSFs. This new model for discrete cash dividends is consistent with the old model, but uses the actual dividend amount and thus reduces dividend risk. We believe the model may prove useful to analysts in pricing similar derivative instruments.

1.1 A short history of Single Stock Futures

The first SSFs were listed in 1994 on the Sydney Futures Exchange (Lafferty, 2002), followed by the Hong Kong Futures Exchange. In South Africa SAFEX (the South African Futures Exchange) started trading SSFs on four listed companies in 1999. This was gradually expanded so that South Africa now has one of the largest single-stock futures markets in the world with extensive on-line trading. According to *Risk* (Ferry, 2007), of 168 million equity derivatives contracts traded in South Africa between January and August 2007, 142 million contracts were SSFs.

Some stock exchanges have refused to deal in SSFs – India lifted its ban in 1998 and in the USA SSFs only started being traded in 2002 after a 18-year ban (Mitchell, 2003). The main reasons for banning SSFs have been that these contracts allow speculators to short a share without the requirements of shorting on an up-tick, and without incurring borrowing costs. Although SSFs have been extremely popular in South Africa market-makers and investors in SSFs were badly hit in 2008 and 2009 due to the increased volatility in financial markets. Margin calls could not be met and banks were forced to buy large quantities of shares (Clark, 2009).

1.2 Properties of single stock futures

A futures contract is an agreement initiated at time t = 0, to buy (long position) or sell (short position) an asset at a specified future time t = T (maturity) for an agreed futures price determined at t = 0.

An SSF is a futures contract to trade 100 shares of a specific company listed on an exchange. SSFs are quarterly contracts with maturity T = 89 days. In South Africa standard SSF contracts expire on the third Thursday of March, June, September and December, but contracts can be rolled over. Contracts can be closed out at any time t < T. SSFs also trade off-market and on-line. Market-making banks determine prices and endeavour to provide liquidity in SSF contracts.

The SSF offers an opportunity to gain exposure to the movement of a specific share without owning the share. An initial margin (deposit) is required when entering an SSF contract – usually about 10% - 25% of the cash value of the futures. Since the investor has full exposure to share price movements, he/she has gearing of roughly 4 - 10 times. Note that the price of

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the futures contract is not paid at time t = 0; only the initial margin is deposited.
Contracts are marked to market or marked to model. At the end of each day the daily loss or profit is calculated for each position and the two parties' margin accounts are adjusted. In effect the contract is closed out and rewritten every day. A maintenance margin, approximately 75% of the initial margin, is set. If the amount in either the buyer's or the seller's margin account becomes less than this maintenance margin, a margin call is made and extra funds have to be deposited by the affected party before noon the following day. Failure to make the margin call implies that the broker will close out the position.

2. PRICING FORMULAS FOR FUTURES

We present the development of the theoretical pricing formulas, with emphasis on the dividend component. It is important to note the properties of the basic pricing formulas in order to verify our proposed new model. This section may also serve to clarify formulas published by brokers and banks that do not always explain the dividend component in detail.

2.1 The basic continuous model for fair pricing

Let S(.) be the value of a non-dividend paying share. A futures contract on S initiated at t = 0, with time of delivery T, is a financial instrument such that at every time *t*, $0 \le t \le T$, there exists in the market a quoted futures price F(t;T;S) for underlying S, determined at t, for delivery at T. In particular:

a. There is a payment stream (corresponding to daily mark-to-market) so that in each interval (*t*, *t*+1] the holder receives the amount F(t +1;T;S) − F(t;T;S).

b. F(T;T;S) = S(T).

Let *r* denote the (constant) risk-free rate. The fair, no-arbitrage theoretical futures price, excluding transaction costs, is (Björk, 2004):

$$F(t;T;S) = S(t)e^{r\,(T-t)}. \qquad \dots (1)$$

Note: The payment stream in point (a) above reflects daily margin adjustments in the mark-to-market process. In total the value of the stream over [0,*T*] is:

$$\sum_{t=0}^{T} [F(t+1;T;S) - F(t;T;S)] = S(T) - F(0;T;S) \qquad \dots (2)$$

This is an extremely important property: at time T you will have paid the futures price F(0;T;S) which was agreed upon at time 0, and will have "received an asset" worth S(T). "Receiving the asset" does not necessarily imply receiving the physical underlying – through the payment stream the holder and writer have in fact fulfilled all contractual obligations.

2.2 Dividends and continuous pricing models

We now consider shares which pay dividends. Dividends are typically paid out quarterly or semi-annually. The effect of the payment is a drop in share price as the share goes ex-dividend. This drop is assumed to be of the same size as the dividend payment. The holder of the SSF does not receive the dividend but is subjected to the drop in share price. There must therefore be compensation and this forms the crucial component of the pricing formula. The continuous futures price on assets paying an income will be denoted by $F^D(t;T;S)$ where S now denotes the price of the dividend-paying stock.

Dividends are often modelled as continuous yields or percentages of stock price S. These provide the simplest analysis, but cannot be applied to SSFs where there is a single discrete cash dividend paid at t = d. The usual way of dealing with the discrete cash dividend (to effect compensation) is to deduct the present value of the dividend from the share price at times t < d and to apply the pricing formula (2.1) to the resulting capital process. Consider a discrete cash dividend with forecast value D to be paid at time d. From Jarrow and Turnbull (1995) we have the continuous-time SSF pricing formula:

$$F^D(t;T;S) = \begin{cases} e^{r(T-t)}[S(t) - e^{-r(d-t)}D] \\ \text{for } 0 \le t < d < T \\ e^{r(T-t)}S(t) \text{ for } d \le t \le T. \end{cases} \qquad \dots (3)$$

2.3 Discrete fair pricing model with dividends

The pricing formulas that are used in practise are discrete. We denote the discrete version of the futures price in Equation (3) by SSF(t). Then

$$SSF(t) = \begin{cases} S(t)[1 + r\,(T-t)] - D[1 + r\,(T-d)] \\ \text{for } 0 \le t < d < T \\ S(t)[1 + r\,(T-t)] \text{ for } d \le t \le T. \end{cases} \qquad \dots (4)$$

Formula (4) gives a fair theoretical price (excluding transactions costs) of an SSF. Times are annualised. The formula satisfies SSF(T) = S(T). The equivalent form of the important property (Equation (2)) is that the value of the payment stream over [0,T] equals:

$$\begin{aligned} \sum_{t=0}^{T} [SSF(t+1) - SSF(t)] &= SSF(T) - SSF(0) \\ &= \{S(T) + D[1 + r\,(T-d)]\} \\ &\quad - S(0)[1 + r\,T]. \qquad \dots (5) \end{aligned}$$

The investor will have paid S(0)[1 + rT] and received amount {S(T) + D[1 + r(T– d)]}. Thus, although investors do not receive the dividend as such at time d, they are compensated since at time T they will have received the future value of amount *D* via the mark-to-market payment stream.

3. DIVIDEND RISK

The first problem in the pricing of SSFs is the fact that the dividend amount D, used to establish the SSF price at time t = 0, is a forecast based on analysts' consensus and historical data. It is not necessarily the actual amount of the dividend that will be declared and paid. This introduces dividend risk. If the forecast and actual dividends differ brokers have to approach market-makers to adjust the dividend component and to re-price the SSF − a time-consuming process (Ferry, 2007; Jeffery, 2003; Pengelly, 2008). The actual time d when the dividend will be paid is similarly uncertain at time 0.

A second problem is that the forecast amount D may differ between institutions and, for some institutions, lead to the risk of being arbitraged (Ferry, 2007). It also appears (Jeffery, 2003) that unrealistically high dividend yields could be applied deliberately to lower prices of structured products (the higher the value of D, the lower the price SSF(0)). In other words, deliberate mis-pricing can be used to buy market share.

Dividends present many other challenges to pricing. Some studies (for example, Bhana, 1991; Docking and Koch, 2005) find that, although at ex-dividend date d the stock price is adjusted by amount D in theory, prices often drop by only 50% - 90% of D and the drop is usually not observed on the ex-dividend date unless D is particularly large. Furthermore, (Haug, 2003), cash dividends tend to be smaller (alternatively larger) than expected after significant drops (or increases) in share price. Tax implications due to the magnitude of D also affect the drop in share price. We will not address these issues in our analysis.

The following two sections provide examples illustrating the effect of dividend risk and uncertainty on the pricing of SSFs.

3.1 Example: Two contracts using different forecast dividends

Consider the following scenario. We have two contracts on share S that differ only in the value of forecast dividend D.

Time of pricing t = 20 June 2009.

Both SSFs on share S expire on T = 17 September 2009.

S(t) = R80,20 = spot price of S on 20 June

S(t + 1) = R72,18 = spot price of S on 21 June

r = 12% p.a.

Contract 1: D = R5,00 (the forecast dividend)

Contract 2: D* = R3,00 (alternative forecast dividend)

Expected dividend date d = 30 June 2009.

Then, using Formula 4, we can construct Table 1:

Table 1: Effect of different dividends and share price movement on SSFs

	Initial price (t = 0)	Initial margin (20%)	Price at t =1 after 10% drop in S	Change in margin account	Percentage change in margin account
Share S	80,20	n/a	72,18	n/a	n/a
Contract 1[#] D = 5,00	7 742,00	1 548,40	6 914,00	−828	−53,47%
Contract 2[#] D* = 3,00	7 947,00	1 589,40	7 119,00	−828	−52,09%

[#] Contracts on 100 shares

Discussion of results in Table 1

There is a 2,65% difference in the initial prices and initial margins of the two SSF contracts. After a 10% decrease in share price *S* over 1 day, both contracts experience the same absolute value drop in margin account and will attract a margin call. However, for Contract 1 (high dividend forecast) the percentage drop in the margin account is higher than for Contract 2 (lower dividend forecast). Therefore although buyers of Contract 1 initially paid a lower margin, they are relatively harder hit than buyers of Contract 2 by a percentage-wise higher margin call.

3.2 Example: A change in dividend amount during the life of one contract

Consider a decrease of 10% in the initial time t = 0 forecast value D so that the actual declared dividend paid at time t = d is D_{dec} = 0,9D.

Assume that r = 0,10; and (T– d) = 20/365. The change is announced at time t < d. At that time the price must therefore change from

SSF(t) = S(t)[1 + r(T – t)] – D[1 + r(T – d)]

to

SSF(t) = S(t)[1 + r(T – t)] – 0,9D[1 + r(T – d)].

The (almost instantaneous) change in theoretical value of the SSF price is:

0,1D[1 + 0,1 (20/365)] = 0,101D. This is a 10,1% change in the futures price and re-pricing will have to take place.

4. DEALING WITH DIVIDEND RISK – DIVIDEND FUTURES

Dividends in general can themselves be traded in various ways, for example with products such as contracts for difference and dividend swaps. Specifically in the case of SSFs there has been considerable demand for tools to manage dividend uncertainty. One tool is obviously to create products that can be used to hedge dividend risk. The dividend futures contract is such a product.

The dividend future (DIVF) is a derivative contract that can be used to hedge against the dividend risk that accompanies trade in SSFs. The DIVF contract is booked in conjunction with an SSF. Basically the combination becomes an SSF on a stock *without* dividends where the actual dividend amount is then exchanged. The investor is compensated in this way for the drop in share price. The price of the SSF on a non-dividend paying stock is of course simply SSF(t) = S(t)[1 + r(T – t)]. Dividends do not have to be predicted, they are simply exchanged and dividend risk is eliminated.

There is no doubt as to the importance of the DIVF market as a tool to manage dividend risk. One "disadvantage" for investors in SSFs may be that they are in effect obliged to enter into an additional contract to hedge dividend uncertainty. Another potential problem is that the dividend futures market is not always very liquid. Eurostoxx 50 dividend futures saw a collapse in May 2010 (Cameron and Wood, 2010). An illiquid DIVF market means investors may have difficulty in using dividend futures as hedging instrument.

Another way of addressing dividend risk in SSFs would be to improve the modelling of the discrete dividend component in the pricing of the SSF.

5. DIVIDEND MODELLING: A NEW MODEL FOR ESTABLISHING THE FAIR PRICE OF AN SSF

The modelling of discrete cash dividends is one of the big challenges in pricing equity derivative products: the modelling must address the need for compensation due to the drop in the underlying share price when the dividend is paid. In the case of the pricing of options on shares with discrete dividends, a variety of different pricing models have been developed. See Musiela and Rutkowski, 1997; Bos and Vandermark, 2002; Bos, Gairat and Shepeleva, 2003; Frischling, 2002; Haug *et al*., 2003; Vellekoop and Nieuwenhuis, 2006.

In SSF pricing however, all available evidence shows that discrete dividends have always been dealt with (as shown in Section 2.2) by introducing the "reduced" capital process [S(t) – $e^{-r(d-t)}$ D] for times t < d. That is, the present value of the forecast dividend is deducted from the share price S at t < d so that compensation takes place before the dividend is paid.

We suggest that the SSF pricing formula can be modified by developing a new model for discrete dividends. The new model introduces an "augmented" capital process that uses the future value of the actual declared dividend amount for compensation at times t > d.

As a first step we distinguish between assumed dividend *D* and actual declared dividend D_{dec}. We then propose a pricing model which is based on the actual dividend date d and amount D_{dec}. Let S denote the usual stock price process and let S* be the process defined by

$$S^*(t) = \begin{cases} S(t) \text{ for } 0 \le t < d \\ S(t) + D_{dec} e^{r(t-d)} \\ \text{for } t \ge d. \end{cases} \quad \ldots (6)$$

The dividend component in S*at t > d contains the future value of the actual declared dividend and is added to S(t) to compensate for the drop in share price at t = d. Process S* can be seen as an equivalent "augmented" or "dividend-compensated" process for S.

The continuous-time futures price formula for S*(t) is then: F(t, T, S*) = S*(t) $e^{r(T-t)}$. Using Equation (6) we get the new futures pricing formula F* in terms of S:

$$F^*(t, S) = \begin{cases} S(t)e^{r(T-t)} \text{ for } 0 \le t < d \\ S(t)e^{r(T-t)} + D_{dec}e^{r(T-d)} \\ \text{for } t \ge d. \end{cases} \quad \ldots (7)$$

We now use Formula (7) to propose the following discrete-form fair pricing formula for an SSF on a stock S that pays a single cash dividend during the life of the contract:

$$SSF^*(t) = \begin{cases} S(t)\ [1 + r\ (T-t)] \\ \text{for } 0 \le t < d \\ S(t)\ [1 + r\ (T-t)] + D_{dec}[1 + r\ (T-d)] \\ \text{for } d \le t \le T. \end{cases} \quad \ldots (8)$$

At time t = 0 the value of the contract is now SSF*(0) = S(0) [1 + r T]. This determines the initial margin. The dividend compensation factor only enters the valuation from time t = d when the share price drops. This means that we are able to use the actual declared dividend amount and dividend date at that stage.

It is essential to verify that the cash-flow property in Equation (5) also holds for new formula SSF*. In fact:

$$SSF^*(T) - SSF^*(0) = S(T) + D_{dec}[1 + r\,(T-d)] - S(0)[1 + r\,T] \quad \ldots (9)$$

We thus see that in both the old and new models the total cash-flow over [0, T] implies that, at time T, the investor will have paid the agreed futures price S(0)[1 + r T] and will have received the value S(T) plus the future value of a dividend. The two models are consistent in this sense.

The big difference between models (4) and (8) is that our new model uses the actual dividend D_{dec} at times t ≥ d and eliminates the dividend risk and re-pricing risk that arises from the use of forecast value D at time t < d as in the old model. See Example 5.1 below.

Of course SSFs priced by Formula (4) have a lower initial value than those priced by (8). The initial margin will thus be lower. This may at first seem advantageous to buyers, but in fact the higher initial margin of the new model reduces the negative effects of price decreases. Example 5.2 illustrates this.

Finally, the possibility of manipulating dividend estimations is also eliminated in the new model.

5.1 Example: Comparison of values for old and new models at t = 0 and t = d

We compare the prices of the SSF contract for the data given in Section 3, Example 3.1. Assume the expected and actual dividend date is d = 30 June 2009; and that the assumed dividend is D = R5,00; with actual dividend D_{dec} = R3,00.
t = 0 = 20 June.

Formula (4) (old model):
SSF(0) = 77,42

Formula (8) (new model):
SSF*(0) = 80,20[1 + 0,12(89)/365] + 0
= 82,55.

For simplicity we assume the stock price remains at S(t) = 80,20 up to the ex-dividend date. At time t_1= 29 June:

Formula (4):
SSF(t_1) = 80,20[1+0,12(80)/365]–5,00[1+0,12(79)/365]
= 77,18

Formula (8):
SSF*(t_1) = 80,20 [1 + 0,12(80)/365] + 0 = 82,31

Note that the change in futures price over period 20-29 June is exactly the same amount (0,24) for each formula, as it should be.

At time t_2 = 1 July, even though Model (4) presumed S(t_2) = 80,20 – 5 = 75,20, the observed stock price will be S(t_2) = 80,20 – 3 = 77,20. So at t_2 = 1 July:

Formula (4):
SSF(t_2) = 77,20 [1 + 0,12(78)/365] – 0 = 79,17.

Formula (8):
SSF*(t_2)= 77,20[1+0,12(78)/365]+3,00[1+0,12(79)/365]
= 82,26.

Note: The change in futures prices from 29 June to 1 July across dividend date d = 30 June differs substantially for the two pricing models. It is 1,99 (or R199 per contract) for the old model and only 0,05 (or R5 per contract) for the new model. This is a big difference and highlights the dividend risk. With the old model of Formula (4) re-pricing would have been called for.

5.2 Example: Comparison of initial margins and changes in margin accounts

The data of Examples 3.1 and 5.1 are used.

Value of contract (100 shares) for old pricing model: SSF(0) = 7 742,00.

Value of contract (100 shares) for new pricing model: SSF*(0) = 8 255,00.

Initial margin of 20% for old model: 1 548,40

Initial margin of 20% for new model: 1 651,00

Consider a drop of 10% in share price *S* over 1 day (20 June – 21 June).

Change in margin account for old model: −828,00

Percentage change in margin account for old model: −53,47%

Change in margin account for new model: −828,00

Percentage change in margin account for new model: −50,15%

Note: The higher initial price and initial margin with the new model has the effect of smaller percentage changes in the margin account than with the old model.

6. CONCLUSION

Classic pricing formulas for SSFs use assumed and not actual dividends. The standard (old) pricing models namely deduct the present value of the forecast dividend from the future stock price before the dividend date. The problem is that forecast values may differ between institutions and also differ from the actual declared dividend. This means the futures contracts are subject to dividend uncertainty. Our examples illustrate the complications in pricing that may arise with a resulting need for re-pricing the SSF.

Although dividend futures can be traded together with SSFs to manage dividend risk, it is important also to investigate the fundamental modelling of discrete dividends. We suggest a new model for pricing SSFs that is consistent with the old model, but that takes the actual declared dividend into consideration. Investors are compensated after the dividend date by adding the future value of the actual dividend to the future stock price. Dividend risk and re-pricing are minimised in this way. The higher initial margin (due to the higher price) for the new pricing model also has the effect of smaller percentage changes in the margin account.

We believe that this model for discrete dividends in SSFs may provide analysts with fresh applications in other futures and forward based products.

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